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ESTIMATION OF THE REGRESSION COEFFICIENTS IN ANGSTROM CORRELATION FROM ELEVATION IN THE RANGE 0-2000 m

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ABSTRACT

Using only the elevation of the location, equations are suggested for computing the regression coefficients a and b of Angstrom correlation. The equations for estimating a and b are developed from the global radiation and sunshine duration data reported, by various authors, for 29 locations in Africa and Europe. The developed equations are tested for applicability by computing global radiation for fifteen new locations spread all around the world. The agreement between the measured data reported for these stations and the estimated values is remarkable. It is suggested that this procedure can be employed to estimate global radiation for any location around the world.

INTRODUCTION

Estimates of monthly average daily global radiation are required for predicting the energy conversion efficiency of solar energy devices. A knowledge of global solar radiation data, gained over a long period of time, is thus essential to design any solar energy system. Measured radiation data are not available for many countries and most of them cannot afford the measurement equipment and techniques involved. Therefore, it is rather important to develop methods to estimate the solar radiation using whatever weather data are available. Several empirical correlations have been developed to calculate solar radiation using various meteorological parameters. Of all the models, the most popular one is the regression equation due to Angstrom (1924) and Prescott (1940) in the following form

$$H/Ho = a + b\,(S/SO) \qquad (1)$$

Where H is the monthly mean daily global radiation falling on a horizontal surface at a particular location, Ho is the monthly mean daily radiation on a horizontal surface in the absence of any atmosphere at a particular latitude, S is the monthly mean daily number of hours of observed bright sunshine, S_o is the monthly mean daily number of hours of sunshine in a given month between sunrise and sunset and a and b are climatologically determined regression coefficients. S/S_o is often called the percentage of possible sunshine. Equation 1 has been extensively used to estimate the global radiation.

In the above equation H_o and S_o can easily be calculated for any location on any day using a set of constants like latitude, declination, sunset hour angle etc. S values are easily measureable and long term data are available for many locations in most of the countries. However, the values reported by various authors on the regression coefficients a and b show large variations not only in different parts of the world but even in different locations in the same country. The main problem in utilizing the Angstrom type correlation is thus to obtain the proper values of the constants a and b for a given location. The regression coefficients are observed to vary with the elevation of the station above sea level (Neuwirth 1980) and it should be possible to obtain a relationship between these constants and the altitude of a location. If a general equation can thus be established connecting a and b with elevation and if the applicability of such an equation is tested for locations around the world, Angstrom correlations are more easy to apply to any location.

In the present study, reported values (Chinnery 1971; Gladius 1983; Jain 1983, 1984; Gopinathan 1986) of the regression coefficients a and b for 29 locations from Africa and Europe, in the latitude range from 34°S to 46°N, are utilized to find out the dependence of regression coefficients on elevation (h). The least square method was used to calculate the coefficients a and b for some of the locations where the reported values are not available. A quadratic relationship between the regression constants (a and b) and elevation is assumed and the constants A_0, A_1, A_2, and B_0, B_1, B_2 in the equations

$$a = A_0 + A_1 h + A_2 h^2 \text{ and}$$
$$b = B_0 + B_1 h + B_2 h^2$$

are computed using a, b and h data from the 29 locations mentioned above. The applicability of the quadratic equations for calculating a and b is then tested by estimating the regression constants a and b and the global radiation for fifteen new locations spread around the world. Neuwirth (1980) has reported such a quadratic relationship between the regression coefficients and the elevation for locations in Australia.

MATERIAL AND METHODS

The monthly mean daily values of the maximum possible number of sunshine hour S_0 and the extraterrestrial radiation H_0, in Joules per metre^2 per day, are calculated using the expressions

$$S_0 = \frac{2}{15} \cos^{-1} (-\tan \phi \tan \delta) \qquad (2)$$

$$H_0 = \frac{24 \times 3600}{\pi} \times I_{sc} \left[1 + 0.033 \cos \frac{(360\, n)}{365}\right] \times (\cos \phi \operatorname{Cos} \delta \sin \omega + \omega \sin \phi \sin \delta) \qquad (3)$$

where ω, the sunset hour angle in radians, is given by

$$\omega = \cos^{-1} (-\tan \phi \tan \delta) \tag{4}$$

the declination δ is given by

$$\delta = 23.45 \quad \sin \left(360 \times \frac{284+n}{365}\right) \tag{5}$$

I_{sc} is the solar constant and n is the day of the year.

A computer programme is written to evaluate the values of a and b for some of the locations in Africa where the regression coefficient values reported by others are not available. Another computer programme was written to find out the values of the constants A_0, A_1, A_2 and B_0, B_1, B_2 in the quadratic equation connecting a, b to the elevation of the 29 locations employed in the analysis. Three sets of equations are developed depending on elevation range of stations. The three classes considered are (1) 0-100 m (2) 100-400 m (3) 400-2000 m. Such a classification was necessary as one single set of a and b constants was not satisfying the whole range of elevation. Data from 12 stations were employed to obtain the constants of the quadratic equation in the altitude range 0-100 m, data from 7 stations in the range 100-400 m and data from 10 stations were used in the range 400-2000 m altitude. Reported data from the following locations were used to evaluate a and b constants. The quantity given in the bracket after each location is the altitude of the station, above sea level, in metres.

1. In the elevation range 0-100 m

   Pisa (1), Olbia (2), Venezia (6), Brindisi (10), Trapani (14), Pescara (16), Capetown (17), Cagliari (18), Pianosa (27), Amendola (56), Messina (59), Napoli (72).

2. In the elevation range 100-400 m

   Milano (103), Ancona (105), Roma (131), Pantelleria (170), Bolzano (241), Vigna De Valle (270), Torino (282).

3. In the elevation range 400-2000 m

   Mongu (1053), Keetmanhoop (1066), Lusaka (1154), Windhoek (1217), Mansa (1259), Pretoria (1369), Bloemfontein (1422), Harara (1471), M. Terminilli (1875), Nairobi (1959).

Methods of comparison

The accuracy of global radiation values estimated from the developed method is tested statistically. In this study, the root mean square error (RMSE), the mean bias error (MBE) and the mean relative percentage error (MPE) are used to evaluate the accuracy of the correlation described above.

The root mean square error (RMSE) is defined as

$$RMSE = \left\{ [\Sigma(H_{ical} - H_{imeas})^2]/n \right\}^{1/2} \quad (6)$$

where H_{ical} is the ith calculated value, H_{imeas} is the ith measured and n is the total number of observations.

$$MBE = [\Sigma(H_{ical} - H_{imeas})]/n \quad (7)$$

The mean relative percentage error (MPE) is defined as

$$MPE = \frac{[\Sigma H_{imes} - H_{ical} \times 100)]/n}{H_{i\ meas}} \quad (8)$$

In the third case, the sign of the errors are neglected in the summation and all the percentage errors are added up while calculating the mean. The lower the RMSE, the more accurate the estimate is. The positive MBE shows an over estimation while a negative MBE shows an underestimation.

RESULTS AND DISCUSSION

From the regression coefficients a and b and elevation the following emprical relationships connecting a and b with elevation, in kilometres, are established.

1. In the elevation range 0-100 m
   $a = 0.184 - 0.076\ h + 3.395\ h^2$
   $b = 0.662 + 1.526\ h - 17.44\ h^2$
2. In the elevation range 100—400 m
   $a = 0.077 + 1.147\ h - 3.036\ h^2$
   $b = 0.937 - 2.722\ h + 7.294\ h^2$
3. In the elevation range 400—2000 m
   $a = -0.161 + 0.568\ h - 0.184\ h^2$
   $b = 0.963 - 0.639\ h + 0217\ h^2$

Table 1 shows the estimated values of A_0, A_1, A_2 and B_0, B_1, B_2 constants for the three ranges of elevation.

Table 1. Constants A and B for the evaluation of a and b

Elevation	Ao, A1, A2	Bo, B1, B2
0 —100 m	Ao = 0.184 A1 = -0.076 A2 = 3.395	Bo = 0.662 B1 = 1.526 B2 = -17.44
100—400m	Ao = 0.077 A1 = 1.147 A2 = -3.036	Bo = 0.937 B1 = -2.722 B2 = 7.294
400—2000m	Ao = -0.161 A1 = 0.568 A2 = -0.184	Bo = 0.963 B1 = -0.639 B2 = 0.217

The verification of the obtained correlation was done by comparison of the calculated global radiation with measured data for some new stations that are not used in the regression analysis. These constants and the procedure are used to estimate monthly mean daily global radiation on a horizontal surface for fifteen locations around the world. Calculations are carried out for locations in the latitude range from 35°S to 46°N. None of these locations, selected to test the applicability of the model were included in the regression analysis for obtaining A and B values given in Table 1.

Table 2 gives the geographical location and the estimated values of a and b for the 15 locations. The a + b values that represent the clear day fraction of H_0 are also included in the table. The a and b values thus estimated are employed alongwith the present possible sunshine to compute the monthly mean daily global radiation on a horizontal surface for these 15 stations. The experimental values on global radiation and sunshine duration for these stations were obtained from the work published by various authors. (Lof et al. 1966; Chinnery 1971; Gopinathan 1985; Lewis 1983; Donald et al. 1981; Neeta 1982; Khogali 1983; Jain 1984). The accuracy of the estimated data on global radiation are evaluated by calculating MBE, RMSE, and MPE values. The estimated values of MBE, RMSE and MPE are also presented in Table 2. As seen from the table, there is a remarkable agreement between the experimental and the estimated values of global radiation for all the 12 station. The RMSE values are very low, indicating very good agreement. The average percentage error incurred in predicting global radiation is very low and lies between 0.7 for Maseru and 9.2 for Juba. The percentage error for a single month rarely exceeded 10 per cent in any of the stations. The method has thus found to perform very well in both the hemispheres. The agreement between measured and estimated data for locations that are not included in the regression analysis, for obtaining the constants of equations for a and b, demonstrate the wide applicability of the method. Furthermore all the 15 stations selected for testing the applicability exhibit entirely different climatological and geographical conditions. This procedure can thus be considered to be applicable for estimation purposes for any location at any part of the world with an accuracy of about 10 per cent.

In this study we have divided the locations into three altitude regions for estimating the regression coefficients. However, one has to justify the need to have this type of classification. We did try to obtain a single set of constants for evaluating the regression coefficients from the data of all the 29 locations mentioned earlier. The constants thus obtained were then used to estimate global radiation for different locations spread around the world. However, there was no agreement between the estimated and experimental data and the relative percentage error were more than 40 per cent in many cases. It was thus necessary to look for new equations and was observed that accurate estimations are possible if the locations are divided into different altitude regions. A minimum of three altitude regions were found to be necessary to estimate global radiation values within 10 per cent accuracy.

Table 2. Estimated values of the regression coefficients and the mean bias, root mean square and the mean percentage errors in global radiation for the 15 locations

Location	Latitude Degrees	Longitude Degrees	Elevation metres	a	b	a+b	MBE MJm^{-2}	RMSE MJm^{-2}	MPE
Buenosaires	34.59S	58.48W	25	0.184	0.689	0.873	0.827	1.053	4.9
Durban	29.83S	31.03E	5	0.188	0.670	0.858	0.476	0.778	4.3
Maseru	29.32S	27.48E	1571	0.277	0.495	0.772	0.146	0.218	0.7
Upington	28.43S	21.27E	814	0.179	0.587	0.766	0.674	0.936	3.6
Bulawayo	20.15S	28.62E	1343	0.270	0 496	0.766	-1.061	1.115	5.0
Maun	19.98S	23.42E	945	0.212	0.553	0.765	0.650	1.063	4.5
Malange	9.55S	16.36E	1151	0.249	0.515	0.764	–0.068	1.106	5.0
Juba	4.87S	31.6 E	460	0.061	0.715	0.776	-1.178	1.945	9.2
Penang	5.3 N	100.27E	3	0.184	0.666	0.850	0.944	1.017	4.9
Trivandrum	8.48N	76.97E	64	0.193	0.657	0.850	0.024	0.990	4.0
Elfasher	13.69N	25.33E	733	0.156	0.612	0.768	–0.452	1.246	4.5
Sanantonio	29.53N	92.47W	249	0.175	0.711	0.886	1.546	1.995	8.5
Srinagar	34.08N	74.83E	1593	0.227	0.497	0.724	–0.848	1.049	5.2
Ustica	38.70N	13.18E	251	0.174	0.714	0.888	0.086	0.520	2.2
Trieste	45.65N	13.77E	20	0.184	0.638	0.822	-0.205	0.424	3.1

The limiting values of the altitude given in each range here are not very critical and are the assumed approximate values. As mentioned earlier a and b values in the region 0-100 m are evaluated from data for 12 locations in the range 1 m to 72 m. (Pisa to Napoli). The applicability of the derived constants in this region is also tested on locations in altitude range 3 m to 64 m. (Penang to Trivandrum). The first altitude region is thus classified as 0-100 m. Data from Minalo to Torina are used to obtain the second set of constants and are tested on locations in that range. This region is classified as 100-400 m region. Though the third set of constants were derived from locations in the altitude range 1053 m to 1959 m (Mongu to Nairobi) they are found suitable, for evaluation purposes, even for locations of lower elevation, ranging from 400 m onwards. This is well demonstrated by the accuracy of the estimated data for Upington (814 m), Juba (460 m) and Elfasher (733 m). The third set of constants are thus recommended to evaluate global radiation for stations 400-2000 m. Calculations were also carried out to test whether stations like Juba, with an elevation of 460 m, should be included in the second category (100 m to 400 m range) or not. The errors involved in the estimated values, when calculated from the second set of equations, were very high and above 50 per cent in many cases. However, the errors from the third set of constants were always within 10 per cent and the third classification is thus set for stations in the elevation range 400-2000 m.

It is thus possible to estimate monthly mean daily global radiation on a horizontal surface by only knowing the duration of sunshine and the altitude above sea level. The errors involved are fairly small and are within 10 per cent. Depending on the elevation of the station to be studied the proper equation for a and b should be chosen from the above three classes.

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