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Growth and Yield Studies in the Pure Even-aged Plantations in IGNP Area of Rajasthan for their Sustainable Management

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Abstract: Growth and yield studies based on the permanent sample plot (PSP) data are required for the sustainable management of plantations. In this study, volume functions, site index equations, potential stand density, stem number and basal area development models in pure even-aged stands of *Dalbergia sissoo*, *Tecomella undulata* and *Eucalyptus camaldulensis* planted in IGNP area of Rajasthan have been developed. Various volume equations were tried to arrive at the best equation for predicting volume yield in the stands. The volume equations assume importance in projecting the total volume at different stages as the plantations mature. Some base-age variant and invariant site-index models were used and compared for their relative accuracy to assess the productive capacity of the sites in relation to different tree species. Relationships between quadratic mean diameter and stems ha^{-1} were developed to establish the limiting density line which is useful in generating information about the maximum number of trees ha^{-1} that can be retained in the stands at a given mean stand diameter. Two different models were compared to describe the mortality and the model with site index as one of the variables performed better compared to the model without site index. Seven different stand level models also were compared for predicting basal area in the stands. The models tested belong to the path invariant algebraic difference form of a non-linear model and can be used to predict future basal area in the stands. These can be used to analyze the relationship between stand density and tree growth. In combination with the stand density model, the proposed basal area models may also be used to define the type and weight of thinnings in the stands and are crucial for evaluating different silvicultural treatment options. The performance of the models was compared using different quantitative tests and best models were selected based on the fit statistics and model evaluation criteria for their wider applicability and sustainable management of *D. sissoo, T. undulata* and *E. camaldulensis* plantations in the study area.

Key words: Growth and yield models, model evaluation, IGNP area, *D. sissoo, T. undulata, E. camaldulensis.*

Forest menstruation is one of the most fundamental disciplines within forest and related sciences. It deals with the measurement of trees and stands and the analysis of the resultant information. During the early days of sustained forest management, simple measurement and estimation methods and analysis of inventory and research data were available. The middle of last century, however, witnessed a worldwide increase in the need for more quantitative information about trees and stands. This generated the need for more sophisticated methods to obtain and analyze forest data. This development was followed by a phenomenal explosion of information. The key to successful forest/timber management is a proper understanding of growth processes, and one of the objectives of modelling forest development is to provide the tools that enable foresters to compare alternative silvicultural treatments. Development of sound management practices is one of the major priorities of the forestry sector. Accurate predictions of stand growth and yield are needed for determining sustainable harvests.

To improve the agricultural productivity and the living conditions of the people in the arid parts of Rajasthan State in India, the Indira Gandhi Canal was constructed. Large-scale afforestation activities were taken up by the State Forest Department to combat desertification and plantations of various tree species like *Dalbergia sissoo*, *Eucalyptus camaldulensis* and *Tecomella undulata* were established using the water from the canal.

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The plantations were of different age groups with varying stand densities. Information about the growth and yield of these species is scarce and hence growth models have become very important. Robust models are not available for estimating productivity and projecting basal area and mortality for these species which is crucial for evaluating different silvicultural treatment options.

Volume equations play a crucial role in forest management. Accurate estimates of tree volume are fundamental for forest ecosystem modelling and regional carbon accounting. Volume equations are critical starting points if forest management is to be successful and efficient. Allometric equations for predicting wood volume play a critical and obvious role in the management of any silvicultural system, and their absence would represent an impediment to developing and implementing management plans geared towards the harvest and utilization of wood products. The aim of any volume equation is to provide accurate estimates with acceptable levels of local bias over the entire range of tree size in the data. The accurate prediction of intermediate and final harvests in the construction of yield tables depends on the accuracy of individual-tree volume equations (Perez and Kanninen, 2003; Bi and Hamilton, 1998).

Stand growth and yield depend on the productivity of the site, thus a variety of methods for estimating site productivity has been developed (Clutter *et al.,* 1983; Prodan *et al.*, 1997). The most common approach involves the use of site index equations. Numerous studies have focused on the methodical aspects of constructing site index equations (e.g. Grut, 1977; Ortega and Montero, 1988; Zeide, 1993). Site index, defined as the dominant height at a given reference age, is the most commonly used variable for evaluating forest productivity or for relating productivity to ecological variables (Clutter *et al.*, 1983; Carmean and Lenthall, 1989; Payandeh and Wang, 1994). Also, site index is widely used in stand growth models to estimate forest-level yields (e.g. Pretzsch, 1995; Sterba, 1995; Gadow and Hui, 1999). Thus, the development of site index equations is a fundamental task in forest management.

Populations of trees growing at high densities are subject to density-dependent mortality or self-thinning (Yoda *et al*., 1963; Westoby, 1984). For a given average tree size, there is a limit to the number of trees ha^{-1} that may co-exist in an even-aged stand. The relationship between the average tree size and the number of live trees per unit area may be described by means of a limiting line. Estimating the potential density of forest stands, in terms of the surviving trees per ha, is a central element of growth modelling. It is also one of the most difficult problems to solve, mainly because suitable data from untreated, densely stocked stands are rarely found.

Furthermore, the natural decline of the number of surviving trees in an unthinned forest is usually characterized by intermittent brief spells of high mortality, followed by long periods of low mortality (Gadow and Hui, 1999), which makes estimation of tree survival a real challenge.

The stand basal area is an important density measure, which simultaneously takes into account the average tree size and the number of trees per unit area. Basal area is being used to analyze the relationship between stand density and tree growth (Assmann, 1961). Moreover, in combination with the number of trees, basal area can be used to define the type and weight of a thinning (Gadow and Hui, 1999; Staupendahl, 1999). Models for stand basal area development have been generated by various workers using a differential equation or the path invariant algebraic difference form of a non-linear equation (Pienaar and Shiver, 1986; Souter, 1986; Hui and Gadow, 1993; Garcia, 1994; Rodríguez, 1995; Kvist Johannsen, 1999).

In this article, some equations and models developed for predicting volume, site index, potential density, mortality and basal area growth in the pure even-aged stands of *D. sissoo, E. camaldulensis* and *T. undulata* available in the Indira Gandhi Nahar Priyojana (IGNP) area of Rajasthan have been presented.

Materials and Methods

Study area

The study area (IGNP area Stage-I and Stage-II) is characterized by a large variation in the diurnal and seasonal temperatures. The summer temperature often exceeds 46-48°C, especially during May-June. During December-January, the night temperature occasionally reaches

0°C owing to cold spells associated with the western disturbance and cause frost conditions. The mean monthly temperature in the area varies between 39.5 and 42.5°C while the mean monthly minimum temperature varies between 14 and 16°C. The mean annual rainfall varies between 150 to 300 mm. The major quantity of rainfall is received during the south-west monsoon (July-September). The mean monthly relative humidity fluctuates greatly during the year between 15 to 80%. The mean evaporation varies from 2.7 to 4.7 mm per day in winter and from 13.2 to 15.3 mm per day in summer. Wind speeds as high as 130 km per hour may be experienced during the summer months. Dust storms are also common in the area. The terrain is very undulating consisting of moving sand dunes, dry undulating plains of hard sand and gravelly soil and rolling plains of loose sand. The soil is rich in potash but poor in nitrogen and low in organic matter. The soils are coarsely textured and the water retention capacity is low.

Data and field procedure

For conducting growth and yield studies, 30 sample plots of *D. sissoo*, 35 of *E. camaldulensis* and 22 of *T. undulata* were laid out at various locations in IGNP area covering the available age groups (3 to 33 yrs. for *D. sissoo*, 3 to 31 yrs. for *E. camaldulensis* and 14 to 20 yrs. for *T. undulata*) and stand densities (342 to 2632 trees ha^{-1} for *D. sissoo*, 439 to 3257 trees ha^{-1} for *E. camaldulensis* and 450 to 2188 trees ha^{-1} for *T. undulata*). The plantation spacing adopted initially for *E. camadulensis* and *D. sissoo* was 2 m x 2 m and 3 m x 3 m which were later changed to 3 m x 4 m. For *T. undulata*, it was mainly 2 m x 3 m though few plantations were done at 2 m x 2 m, 3 m x 3 m and 3 m x 5 m too.

The plots, representative of the growing conditions in the area, were scattered in IGNP Stage-I (0 RD to 620 RD) and Stage-II (620 RD to 1458 RD) for *E. camaldulensis* and *D. sissoo* while the plots of *T. undulata* were located at 683, 710, 1055, 1125, 1206, 1246, 1265, 1284, 1325, 1333, 1365, 1406, 1409, 1414, 1417, 1437, 1438, 1450, 1458 RD of main canal, 6 RD JJWD, 18 RD SBS and 35 RD of SBS in Stage-II. The plots were rectangular and their size ranged from 0.04 to 0.1 ha, depending on stand density, in order to achieve a minimum of 30 trees per plot. For identification and demarcation, trenches were dug at the four corners of the plot and the trees inside were numbered and enumerated. The check trees surrounding the plots were marked with rings. Stratified multistage sampling procedure was used to lay out the plots. Post stratification was done as the complete information on sampling frame was not known. Annual measurements were taken continuously for 5 years in *D. sissoo* and *E. camaldulensis* plots laid out in INGP Stage-I and II, and for 3 years in *T. undulata* plots established in IGNP Stage-II.

Two-phase sampling was adopted for enumeration of trees in each plot. First the diameters of all the trees within the plots were measured and then a sub-sample of diameter-height pairs was taken which was subsequently used to determine the height and dbh regression. The height of the other trees in the sample plots was estimated using this regression equation. The same sampling procedure was adopted for volume estimation of the trees in the plot. Trees, covering different diameter classes in the plots, were felled from the surround of the plots and measured for D, H and timber and wood volumes (over-bark and under-bark). Allometric relationship between volumes and D^2H were derived for volume predictions. The volume equation developed was applied on the trees within the plots to estimate wood volume per hectare. The trees of desired diameter class were felled from the surrounding of the plot to keep the sample plots undisturbed. The plot data included a record of the age (A), dominant stand height (H), quadratic mean diameter (D_g), stems ha^{-1} (N), basal area ha^{-1} (BA) etc. Though the harvestable rotation of *E. camaldulensis* is less than 15 years in the study area, however, older plantations available in the area were also considered to avoid the extrapolation of the models/equations if applied on these plantations and also to help forest department in sustainable harvesting and management of the plantations at a later stage. Also, growth and yield studies typically require a full range of ages, densities and growing sites. The summary statistics of the pooled data of all the plots are given in Table 1. A total of 71 sample trees of *D. sissoo*, 91 of *E. camaldulensis* and 75 of *T. undulata* were felled from different plantations for constructing volume equations. No thinning was done in the stands and the decline in the stem numbers ha^{-1} was caused by

Table 1. Summary statistics for the pooled data of the 30 plots of D. sissoo, 35 plots of E. camaldulensis and 22 plots of T. undulata

Attributes	Minimum	Maximum	Mean	Standard deviation
D. sissoo				
Age (years)	3.20	33.40	12.30	6.57
Dominant height (m)	8.71	22.78	14.40	3.22
Stand density (stems ha^{-1})	342	2632	1465	553.36
Quadratic mean diameter (cm)	5.76	29.83	13.29	5.45
Basal area (m^2 ha^{-1})	4.82	32.80	17.61	5.64
Site index* (m)	8.65	18.68	14.46	2.77
E. camaldulensis				
Age (years)	3.20	31.40	12.18	0.49
Dominant height (m)	8.16	31.13	17.83	4.98
Stand density (stems ha^{-1})	439	3257	1636	608.12
Quadratic mean diameter (cm)	4.74	35.88	12.91	5.96
Basal area (m^2 ha^{-1})	3.45	58.70	19.27	0.92
Site index* (m)	8.48	20.55	15.31	3.51
T. undulata				
Age (years)	14	20	17.49	1.43
Dominant height (m)	4.43	8.58	6.04	1.17
Stand density (stems ha^{-1})	450	2188	1150.28	435.64
Quadratic mean Diameter (cm)	6.12	12.32	8.40	1.63
Basal area (m^2 ha^{-1})	1.84	14.31	6.59	3.53
Site index* (m)	4.40	8.60	6.00	1.24

*at base age 15, 10 and 17 years for *D. sissoo, E. camaldulensis* and *T. undulata,* respectively

the natural mortality or self-thinning. The other probable reasons of mortality were recurrent droughts, water-logging and pathogenic problems at some locations.

Volume equations

Linear and non-linear equations were used to model the relationship of total volume with dbh, and with dbh and total height. A total of 8 volume equations (Table 2) were selected from the literature based on their wide application (Spurr, 1952; Loetsch *et al.*, 1973; Clutter *et al.*, 1983; Ramnaraine, 1994; Chakrabarti and Gaharwar, 1995; Moret *et al.*, 1998; Perez and Kanninen, 2003) and fitted to get suitable function.

Table 2. Equations for the total volume tested in the study

Equation type	Equation no.
$V = a+bD^2H$	I
$V = a+bD^2$	II
$V = a+bD+cD^2$	III
$V = a+bD+cD^2+dDH$	IV
$\sqrt{V} = a+bD$	V
$Ln (V) = a+bDH+cD^2H$	VI
$V = aD^b$	VII
$V = aD^bH^c$	VIII

Site index equations

Several models developed by various workers, based on some extension of Chapman-Richards function (Richards, 1959; Chapman, 1961) are reported in literature. The following four models were used and compared to develop site index equations:

Ek (1971):

$$H = as^b(1 - e^{-ct})^d + \varepsilon \quad (1)$$

Newnham (1988):

$$H = aS^b\left(1 - e^{-ct}\right)^{dS^e} + \varepsilon \quad (2)$$

where, $d=\ln[S/(aS^b)]/\ln[1-\exp(-ct_1)]$; t_1=index age, a, b, c are the parameters to be estimated; H is the top height at age t; S is the site-index.

Goelz and Burk (1992):

$$H_2=H_1\left[\frac{1-\exp\left\{-a\left(\frac{H_1}{t_1}\right)^b t_1^c t_2\right\}}{1-\exp\left\{-a\left(\frac{H_1}{t_1}\right)^b t_1^c t_1\right\}}\right]^d+\varepsilon \qquad (3)$$

Payandeh and Wang (1994):

$$H_2=aH_1^b\left(1-e^{-ct_2}\right)^d+\varepsilon \qquad (4)$$

where, $d=\ln[H_1/(a\ \ H_1^b)]/\ln[1-\exp(-ct_1)]$; t_1, t_2=ages (years) at periods 1 and 2 and H_1, H_2=heights at t_1 and t_2, respectively.

Equations 1 and 2 are base-age variant models. Model 1 is an extension of Chapman-Richards function (Richards, 1959; Chapman, 1961), proposed by Ek (1971) to account for the polymorphic growth pattern of the species. Model 2 is a constrained version of model 1 used by Newnham (1988), which has the property of H=S at the index age i.e. for $t=t_1$. Models 3 and 4 are well-behaved base-age invariant functions, which can be used to estimate height at any age given the height at any other age. Among a few well-behaved models, Goelz and Burk (1992) selected model 3 based on its superiority with regard to bias and RMSE for the data. Here, the base-age was selected as 10 years for *E. camaldulensis*, 15 years for *D. sissoo* and 17 years for *T. undulata*. The site index obtained for the plots at these base-ages varied between 10 and 20 meters, 9 to 19 meters and 4.4 to 8.6 meters, respectively. Accordingly, five site classes/qualities were defined at 2 m intervals for *E. camaldumensis* and *D. sissoo* and 1.4 m interval for *T. undulata*.

Potential density

The relationship between the quadratic mean diameter (D_g), dominant height (H) and the number of stems per unit area (N) may be presented as follows (Goulding, 1972):

$$D_g=\frac{1}{\alpha_0 H^{\alpha_1} N+\beta_0 H^{\beta_1}} \qquad (5)$$

α_0, α_1, β_0, β_1 are parameters that are to be estimated.

The per-hectare basal area is:

$$BA=\frac{\Pi}{4}D_g^2 * N=\frac{\pi * N}{4\left[\alpha_0 H^{\alpha_1} N+\beta_0 H^{\beta_1}\right]^2} \qquad (6)$$

The stems per ha at maximum basal area may be obtained by setting the first derivative of this equation with respect to N equal to zero (Sterba, 1975):

$$N_{G\max}=\frac{\beta_0}{\alpha_0}H^{(\beta_1-\alpha_1)} \qquad (7)$$

Substituting N in equation 7, one can obtain the quadratic mean diameter at maximum basal area:

$$D_{g_{G\max}}=\frac{1}{2\beta_0 H^{\beta_1}} \qquad (8)$$

Solving the equation 8 for H and substituting the expression in equation 3 we finally obtain the stems per ha at maximum basal area (Sterba, 1987):

$$N_{G\max}=\frac{\beta_0}{\alpha_0}(2\beta_0)^{\frac{\alpha_1}{\beta_1}-1}D_{g_{G\max}}^{\frac{\alpha_1}{\beta_1}-1} \qquad (9)$$

The resulting equation 9 represents the limiting relationship.

Stem number development

In practice the limiting relationship is difficult to determine, because some trees may die although the limiting density has not been reached (Gadow and Bredenkamp, 1992). Two different equations have been used here to model intermediate mortality in the stands.

Clutter and Jones (1980) used an ordinary differential equation for modelling the rate of change in the number of stems per ha as a function of stand age and arrived at following equation:

$$N_2=\left[N_1^{\ a}+b(A_1^{\ c}-A_2^{\ c})\right]^{\frac{1}{a}} \qquad (10)$$

where, N_1 and N_2 are stems per ha at stand ages A_1 and A_2, and a, b, c are model parameters.

A slightly extended version of equation 6 was used by Pienaar *et al.* (1990) by including site index as an independent variable:

$$N_2 = \left[N_1^{a} + \left(b + \frac{c}{SI} \right) \left(\left[\frac{A_2}{10} \right]^{d} - \left[\frac{A_1}{10} \right]^{d} \right) \right]^{\frac{1}{a}} \tag{11}$$

where, N_1 and N_2 are stems per ha at stand ages A_1 and A_2, SI is the site index and a, b, c, d are model parameters.

Basal area models

In modelling basal area, the path invariant algebraic difference form of growth functions has been applied. Seven such equations were selected from literature based on their wider applicability.

Pienaar and Shiver (1986) developed a growth function to forecast basal area at a given age as a function of previous basal area, age, height and stem number:

$$\begin{aligned} \ln(BA_2) = \ln(BA_1) + \alpha * \left(\frac{1}{A_2} - \frac{1}{A_1} \right) \\ +\beta * (\ln N_2 - \ln N_1) + \gamma * (\ln H_2 - \ln H_1) \\ +\delta * \left(\frac{\ln H_2}{A_2} - \frac{\ln H_1}{A_1} \right) \end{aligned} \tag{12}$$

where

BA_1 and BA_2	= basal area at age A_1 and A_2
H_1 and H_2	= top height at age A_1 and A_2
N_1 and N_2	= Number of stems at age A_1 and A_2
α, β, γ and δ	= model parameters

Garcia (1994) presented a differential form of an equation, which may be modified to correspond to the difference form as follows:

$$BA_2 = BA_1 + \left[\alpha * BA_1 * (\gamma - \ln BA_1) * \frac{1}{\alpha * A_1 + \delta} \right] * \Delta t \tag{13}$$

where, Δt is the interval between measurements.

Kvist Johannsen (1999) developed another differential model for oak in Denmark that can be simplified to yield the following difference form of equation:

$$BA_2 = BA_1 + \left[\alpha * BA_1^{\beta} * \exp(-\gamma * BA_1 - \delta * H_1) \right] * \Delta t \tag{14}$$

Hui and Gadow (1993) developed the following equation for projecting a known basal area for stands of *Cunninghamia lanceolata* of varying density:

$$BA_2 = BA_1 * N_2^{1-\alpha * H_2^{\beta}} * N_1^{\alpha * H_1^{\beta} - 1} * \left(\frac{H_2}{H_1} \right)^{\gamma} \tag{15}$$

The differential form of the model proposed by Rodríguez (1995) for *Pinus pinaster* in Spain can be expressed in the difference form as follows:

$$BA_2 = BA_1 + \left(\alpha * BA_1^{\beta} * A_1^{\gamma} \right) * \Delta t \tag{16}$$

Schumacher (1939) proposed following age dependent basal area model that was later used by Schumacher and Coile (1960), Clutter (1963) and Sullivan and Clutter (1972):

$$\ln(BA_2) = \alpha + (\ln BA_1 - \alpha) * \left(\frac{A_1}{A_2} \right) \tag{17}$$

Souter (1986) presented the following model based on Schumacher's equation:

$$\begin{aligned} \ln(BA_2) = \left(\frac{A_1}{A_2} \right) * \ln BA_1 + \alpha * \left(1 - \frac{A_1}{A_2} \right) \\ +\beta * \left(\ln N_2 - \left(\frac{A_1}{A_2} \right) * \ln N_1 \right) \end{aligned} \tag{18}$$

For fitting the equations, we used all possible intervals instead of only the interval data of successive measurements.

Because of repeated measurements in the plots, observations are correlated. The problem of autocorrelation may invalidate standard regression hypothesis testing procedures. Autocorrelation was modelled as a first-order autoregressive process where the error term was expanded to represent the autocorrelation structure inherent in fitting basal area models to growth interval data structure (Parresol and Vissage, 1998). The data analysis was done using SAS statistical software (SAS Institute Inc., 2000).

Model evaluation

The quantitative evaluation of models is a very important part of growth modelling. The mean residual (MRES), a measure of average model bias, describes the directional magnitude, i.e. the size of expected under- or overestimates. Indices of model precision are the root mean square error (RMSE), the model efficiency (MEF) and the variance ratio (VR). The root mean squared error is based on the residual sum of squares, which gives more weight to the larger discrepancies. The model efficiency index is analogous to R^2 and provides a relative measure of performance. The variance ratio measures the estimated

Table 3. Criteria for evaluating model performance (y=observed values; $\hat{y}$=predicted values; (y-$\hat{y}$)=residuals; p=number of model parameters)

Criterion	Formula	Ideal value
Mean residual	$MRES = \frac{\sum(y_i - \hat{y}_i)}{n}$	0
Root mean square error	$RMSE = \sqrt{\frac{\sum(y_i - \hat{y}_i)^2}{n-1-p}}$	0
Model efficiency	$MEF = \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2}$	0
Variance ratio	$VR = \frac{\sum(\hat{y}_i - \bar{\hat{y}}_i)^2}{\sum(y_i - \bar{y})^2}$	1

variance as a proportion of the observed one (Gadow and Hui, 1999). The criteria, their formula and ideal values are shown in Table 3.

Results and Discussion

The data from the 35 PSPs of *E. camaldulensis* showed that the tallest dominant tree recorded was 31.13 m at the age of 31.4 years. At the age of 11.6 years, the better sites recorded a dominant height of 21.93 m while on poor site it was only 11.66 m. Also, at the age of 18 years the dominant height was 18.61 m on poor site and 30.18 m on better site. The maximum average diameter monitored in the PSP was 35.88 cm at the age of 24.8 years. All the six PSPs exceeding 20 cm as average DBH at final age came from stands with low value of stocking. In 9 plots, which were 15 years old or older, two have volume yield between 500-600 m^3 ha^{-1}, two have between 250 and 350 m^3 ha^{-1}, four have between 150 and 250 m^3 ha^{-1} and one has about 55 m^3 ha^{-1}. The range of initial stocking varied between 481 and 3257 stems ha^{-1}. The data collected from the sample plots showed that an average height growth of 1.72 m per year was observed for the trees on better sites while it varied from 76 cm to one meter for poorer sites in the plantations of 8 to 12 years of age. During the initial 5 years of age, height growth as high as 2.55 m was recorded. During 8-12 years of age, the diameter growth of 1.8 cm yr^{-1} was maintained on the best site while on poorer sites it ranged from 0.6 cm to 1 cm. During the first five years of age, diameter growth as high as 2.2 cm was observed. The volume yield ranged from 1.82 to 24.82 m^3 ha^{-1} yr^{-1} depending upon age, density and site quality. *E. camaldulensis* planted at 2 m x 2 m spacing produced volume yield of 1.82 m^3 to 15.46 m^3 at the age of 8 years according to site quality. At a spacing of 3 m x 3 m, the plantations of age 7 years produced MAI of 4.76 m^3 on poorer site while on better site it was 11.35 m^3. The ratio of best to worst volume yield at the age of 8 years with 2 m x 2 m spacing was found to be 8:1.

Similarly, the data from 30 PSPs of *D. sissoo* revealed that the tallest dominant tree recorded was 22.78 m at the age of 27.7 years. At the age of 11.3 years, the better sites recorded a dominant height of 18.48 m while on poor site it was only 9.94 m. Also, at the age of 8.3 years the dominant height was 12.30 m on poor site and 18.21 m on better site. The maximum average diameter monitored in the PSP was 29.84 cm at the age of 33.4 years. All the three PSPs exceeding 20 cm as average dbh at final age came from stands with low values of stocking. Of the three plots, which were 25 years old or older, two had volume yield between 190-215 m^3 ha^{-1} and one was about 300 m^3 ha^{-1}. The range of initial stocking varied between 356 and 2632 stems per hectare. The data collected from the sample plots showed that an average height growth of 1.68 m yr^{-1} was observed for the trees on the better sites while it varied from 76 cm to 1 meter on poorer sites in the plantations of 8 to 12 years of age. During the initial 5 years of age, annual height growth as high as 2.18 m was recorded. During 8-12 years of age, a diameter growth of 1.6 cm yr^{-1} was maintained on the best site while on poorer sites it ranged from 0.8 cm to 1 cm. During the first five years of age, annual diameter growth as high as 1.8 cm was observed. The MAI ranged from 2.10 to 19.90 m^3 ha^{-1} yr^{-1} depending upon age, density and site quality. *D. sissoo* planted at 2 m x 2 m spacing produced volume yield of 5.98 m^3 to 15.25 m^3 at the age of 8.3 years according to site quality. At a spacing of 2 m x 3 m, the plantations of age 8.6 years produced MAI of 2.23 m^3 on poorer site while on better site it was 16.04 m^3. The ratio of best to worst volume yield at the age of 8.5 years at 2 m x 2 m spacing was found to be about 7:1.

The analysis of the data collected from the 22 sample plots of *T. undulata* laid out in IGNP Stage-II indicated that depending upon age, site and density, average height in the stands varied from 3.45 to 6.24 m, mean quadratic diameter from 6.30 to 12.28 cm, dominant height from 4.56 to 8.54 m, basal area from 1.94 to 14.21 m^2 ha^{-1}, and volume yield from 4.20 to 44.10 m^3 ha^{-1}. The MAI in height, dbh and volume ranged from 0.19 to 0.37 m yr^{-1}, 0.36 to 0.64 cm yr^{-1} and 0.22 to 2.47 m^3 ha^{-1} yr^{-1}, respectively.

During the data analysis for model development, the fit statistics showed that the R^2 values were generally high and acceptable for all the equations while RMSE values were very low. The standard errors for the parameter coefficients of different equations indicated that all the regression coefficients were significant.

Total wood volume equations

The combined variable equation performs well for *E. camaldulensis* and *D. sissoo* while in case of *T. undulata*, non-linear functions performed better. Height is often difficult to measure accurately and may not always be available. In such cases, single-entry (volume-diameter) equations are the best alternative. The equations developed and selected based on the fit statistics given in Table 4 are as follows:

E. camaldulensis

$$V = 0.000169*D^{2.41298}$$

$$V = -0.00226 + 0.0000333\ D^2H$$

D. sissoo

$$V = 0.01328 - 0.00538\ D + 0.000760\ D^2$$

$$V = -0.0023 + 0.0000364\ D^2H$$

T. undulata

$$V = 0.000088\ D^{2.381398}$$

$$V = 0.000066\ D^{2.100121}H^{0.553696}$$

where, V is the total wood volumes (in m^3) over-bark, D is the dbh in cm and H is the total tree height in m.

The combined variable equation has been well recognized in volume predictions of many tree species with R^2 usually above 95% (Avery and Burkhart, 1994). The non-linear equations have more biological logic as volume would be zero when D=0 and H=0 (Perez and Kanninen, 2003).

Site index equations

At first, the four algebraic difference equations were fitted without the autocorrelation parameters using generalized nonlinear least squares and residual were tested for autocorrelation using Durbin-Watson test (Durbin and Watson, 1971). Then the models were refitted, including the autocorrelation parameters, with the SAS/ETS @ MODEL procedure (SAS Institute Inc., 2000).

The Goelz and Burk model, which is based on the algebraic difference form of Chapman-Richards function, produced the best fit for the development of site index equation for *D. sissoo* while Payandeh and Wang function, which is a constrained version of Chapman-Richards model, performed better in case of *E. camaldulensis* and *T. undulata*. Both the models possess all nine desirable properties discussed by Goelz and Burk (1992). The site index models, developed and selected base on the fit statistics for model evaluation presented in Table 5, for the three species are presented below:

E. camaldulensis

$$H_2 = 3.9569 H_1^{0.7584} \left(1 - e^{-0.0278 t_2}\right)^d$$

$$\text{where,}\quad d = \frac{\ln\left[H_1 / 3.9569 H^{0.7584}\right]}{\ln\left[1 - \exp\left(-0.0278 * t_1\right)\right]}$$

Table 4. Fit statistics for volume equations tested in the study for different species

Equation no.	*E. camaldulensis*		*D. sissoo*		*T. undulata*	
	R^2	RMSE	R^2	RMSE	R^2	RMSE
I	0.998	0.00001	0.998	0.00004	0.818	0.01293
II	0.938	0.00001	0.951	0.00001	0.937	0.00762
III	0.966	0.00005	0.983	0.00004	0.940	0.00741
IV	0.991	0.00001	0.993	0.00004	0.944	0.00718
V	0.956	0.00819	0.961	0.00917	0.946	0.00751
VI	0.932	0.10240	0.952	0.00114	0.870	0.02409
VII	0.985	0.00022	0.966	0.00378	0.942	0.00736
VIII	0.991	0.00017	0.993	0.00149	0.944	0.00717

Table 5. Fit statistics for evaluating and selecting site index equations for different species

Equation no.	*E. camaldulensis*		*D. sissoo*		*T. undulata*	
	R^2	RMSE	R^2	RMSE	R^2	RMSE
1	0.820	2.14	0.769	1.60	0.978	0.1724
2	0.847	1.96	0.815	1.39	0.975	0.1841
7	0.982	0.67	0.975	0.52	0.978	0.1728
8	0.983	0.66	0.972	0.55	0.979	0.1706

D. sissoo

$$H_2 = H_1 \left[\frac{1-\exp\left\{-0.00004\left(\frac{H_1}{t_1}\right)^{-7.4008} t_1^{1.6645} t_2\right\}}{1-\exp\left\{-0.00004\left(\frac{H_1}{t_1}\right)^{-7.4008} t_1^{1.6645} t_1\right\}} \right]^{0.2478}$$

T. undulata

$$H_2 = 14.9078 H_1^{-0.0539}\left(1-e^{-0.0182 t_2}\right)^{d}$$

where, $$d = \frac{\ln\left[H_1 / 14.9078 H^{-0.0539}\right]}{\ln\left[1-\exp\left(-0.0182 * t_1\right)\right]}$$

Although both the models are base-age invariant, their prediction accuracy may vary considerably depending on the predicted age t_1 (Goelz and Burk, 1992). Error structure has a large impact in model selection for development of site-index equations, especially when it includes testing significance of parameters and the parameters of a difference equation depends on which differences are used to fit the equation (previous measurement vs. all possible differences). To estimate the parameters in these models, we have used only two adjacent heights i.e., each height is predicted by the height one interval away from it. This significantly reduces sample size and computational time required.

To use these models to estimate stand height (H) for some desired age (t), given site index (S) and its associated base age (t_b), simply substitute S for H_1 and t_b for t_1. Similarly, to estimate site index at some chosen base age, given stand height and age, substitute S for H_2 and t_b for t_2 in the models.

Height growth models often predict heights at some age based on height at some other age (site index). However, standard procedures assume that predicted variables are constant. By fitting base-age invariant site index equations, site index and height prediction equations are fit simultaneously. It is known that individual equations will have lower variance (Curtis *et al*., 1974) but in this case neither the height prediction equation nor the site index equation will possess a shape that represents the true relationship between height and age across levels of site-index (Goelz and Burk, 1992). This behavior has been avoided by fitting difference equations. The height growth curves for *D. sissoo* and *T. undulata* plantations in IGNP area for different site classes are presented in Fig. 1.

Potential density (limiting line)

The data collected from the annual measurements in the plots of *E. camaludensis, D. sissoo* and *T. undulata* were used to fit equation 5 to develop the relationship between the quadratic mean diameter, dominant height and number of stems ha^{-1}. The estimated parameters were further used to obtain limiting line of maximum basal area through equation 9. The equations developed are presented below:

E. camaldulensis

$$N_{G\max} = 35835.69 D_{G\max}^{-1.1905}$$

R^2 = 0.899; RMSE = 1.92610

D. sissoo

$$N_{G\max} = 68761.87 D_{G\max}^{-1.5109}$$

R^2 = 0. 961; RMSE = 1. 11837

T. undulata

$$N_{G\max} = 57629.65 D_{G\max}^{-1.4697}$$

R^2 = 0.703; RMSE = 0.86309

The relationship between the quadratic mean diameter and the number of living trees per unit area along with the limiting

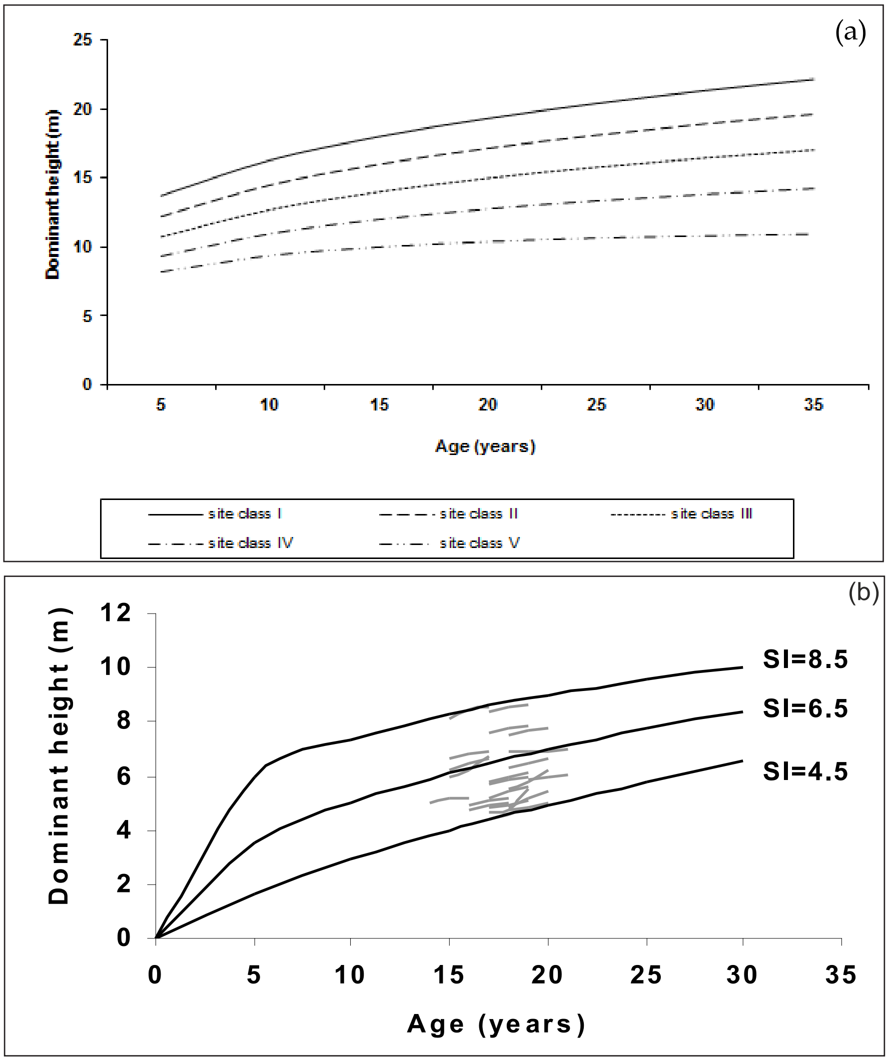

Fig. 1. Site index curves for (a) D. sissoo and (b) T. undulata plantations in IGNP area for different site classes.

line is shown in Fig. 2. The solid line in the figure represents the potential density in the plots, i.e., the maximum stems ha^{-1} the plots can have with respect to the quadratic mean diameter at maximum basal area. In case of *E. camaldulensis*, for the diameter range of 10 to 20 cm, in some plots the observed number of stems ha^{-1} exceeds the maximum limit shown by the limiting line (Fig. 2b) while in case of *D. sissoo*, most of the plots having mean tree diameter between 10 and 15 cm are overcrowded (Fig. 2a). In many of these stands natural mortality has already started as evidenced from Fig. 2. However, in practice the limiting relationship is difficult to determine, because some trees may die although the limiting density has not been reached (Gadow and Bredenkamp, 1992). The similar potential density model was applied on the even-aged stands of *Azadirechta indica* stands in Gujarat (Tewari, 2004) with a good fit (R^2 = 0.979 and RMSE = 0.9113).

Stem number development

The data were used to model the decrease in the number of trees ha^{-1} by applying equations 10 and 11. For the purpose, we used the interval data of successive measurements instead of considering all possible intervals, which means only differences between the measurements in the year 2 and year 1, and so on, were considered. Differences such as those between measurements in year 3 and year 1 were not considered. The equations for estimating the rate of change in the number of stems per ha in the stands are:

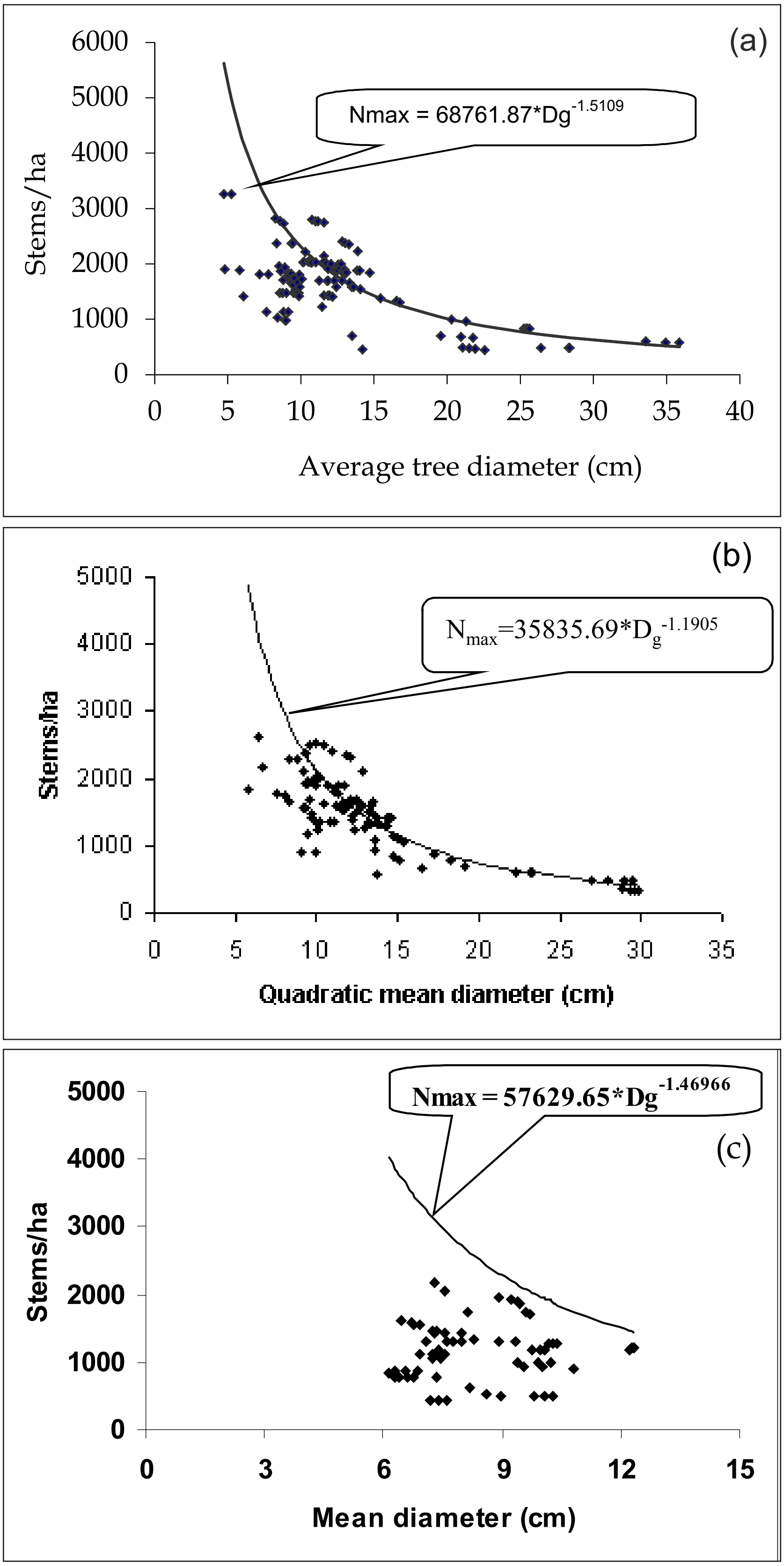

Fig. 2. Relationship between stems ha^{-1} and mean diameter for (a) D. sissoo, (b) E. camaldulensis and (c) T. undulata. The solid line is limiting line. The equation is derived from equation 9.

E. camaldulensis

$$N_2 = \left[N_1^{0.5027} + \left(-3.0526 + \frac{21.3766}{SI} \right) \left(\left[\frac{A_2}{10} \right]^{2.4940} - \left[\frac{A_1}{10} \right]^{2.4940} \right) \right]^{\frac{1}{0.5027}}$$

$R^2 = 0.991$; RMSE = 57.79980

D. sissoo

$$N_2 = \left[N_1^{0.14148} + 0.00686(A_1^{1.27431} - A_2^{1.27431}) \right]^{\frac{1}{0.14148}}$$

$R^2 = 0.952$; RMSE = 114.42545

Gadow and Hui (1993) applied equation 10 on the data set from unthinned *Cunninghamia lanceolata* growth trials in central China to model the natural decline of stem number ($R^2 = 0.93$). The average relative discrepancy between observed and expected values was 0.07%. Forss *et al.* (1996) applied equation 11 to *Acacia mangium* stands in Indonesia to generate equation for predicting the natural decline of stem numbers in the stands. The error of prediction was 5.3%. Thus, in both the cases a good fit was observed. Tewari (2004) applied both the equations on the unthinned stands of *Azadirechta indica* stands in Gujarat State of India and achieved a good fit for both equations (R^2=0.999 and RMSE=30.9 for both the equations).

At the limiting density, the physiological point of no return is reached in the balance between photosynthesis and respiration, giving the 3/2-power law (Yoda *et al.*, 1963). The results from spacing trials (Gadow and Hui, 1999) show that mortality processes are effective well before the maximum basal area is attained.

Basal area prediction model

In the analysis of basal area growth, the path invariant algebraic difference form of seven growth functions, widely used in literature, has been applied. The superiority of any

Table 6. The estimated values for the statistical criteria considered for testing the predictive abilities of the basal area equations used in the study

Equation no.	R^2	MRES	RMSE	MEF	VR
E. camaldulensis					
12	0.993	0.0332448	0.00876	0.00660	0.9849
13	0.993	0.0336109	0.00871	0.00660	0.9846
14	0.990	-0.100801	0.01049	0.00968	0.9704
15	0.991	0.207278	0.00987	0.00858	0.9849
16	0.989	0.1007047	0.01131	0.01138	0.9946
17	0.979	-0.244540	0.01549	0.02087	0.8112
18	0.980	-0.203517	0.01503	0.02009	0.8082
D. sissoo					
12	0.962	-0.01672	1.22617	0.03745	0.93042
13	0.796	-0.15658	2.82833	0.20082	0.87901
14	0.799	-0.32771	2.80858	0.19648	0.72074
15	0.951	0.16918	1.38310	0.04802	0.90680
16	0.803	-0.04765	2.78068	0.19411	0.73208
17	0.790	-0.18109	2.87030	0.21006	0.56423
18	0.905	0.02498	1.93537	0.09477	0.57488
T. undulata					
12	0.995	-0.00557	0.25638	0.00535	0.99337
13	0.987	0.01134	0.26392	0.00567	1.01499
14	0.988	0.01093	0.2721	0.00556	1.01138
15	0.995	-0.00450	0.25106	0.00514	0.99670
16	0.991	0.00938	0.2609	0.01246	0.99018
17	0.983	-0.03600	0.45148	0.01660	0.95076
18	0.986	-0.01237	0.98518	0.01369	0.98518

model cannot be established only on the basis of the fit statistics. Therefore, all the models were evaluated using quantitative evaluation based on the statistical criteria (Table 6) to test their predictive abilities. The estimated values of the statistical criteria considered for testing the predictive abilities of the basal area equations are presented in Table 4. Pienaar and Shiver model (equation 12) performed better for *E. camaldulensis* and *D. sissoo* while Hui and Gadow model (equation 15) performed better in case of *T. undulata* compared to other models based on the selected quantitative statistical criteria The models selected for different species are as follows:

E. camaldulensis

$$\ln\left(BA_2\right) = \ln(BA_1) - 3.0537 * \left(\frac{1}{A_2} - \frac{1}{A_1}\right) + 0.5291 * \left(\ln N_2 - \ln N_1\right) + 0.6512 * \left(\ln H_2 - \ln H_1\right) - 8.5422 * \left(\frac{\ln H_2}{A_2} - \frac{\ln H_1}{A_1}\right)$$

D. sissoo

$$\ln\left(BA_2\right) = \ln(BA_1) + 2.3593 * \left(\frac{1}{A_2} - \frac{1}{A_1}\right) + 0.7704 * \left(\ln N_2 - \ln N_1\right) + 0.8095 * \left(\ln H_2 - \ln H_1\right) - 2.1481 * \left(\frac{\ln H_2}{A_2} - \frac{\ln H_1}{A_1}\right)$$

T. undulata

$$BA_2 = BA_1 * N_2^{1-0.1940*H_2^{0.5632}} * N_1^{0.1940*H_1^{0.5632-1}} * \left(\frac{H_2}{H_1}\right)^{3.9439}$$

All the basal area prediction models used in this study were, earlier, successfully used for basal area predictions in even-aged *Picea abies* (Gurjanov *et al*., 2000; Sánchez Orois *et al*., 2001) and *A. indica* (Tewari and Gadow, 2005) stands. There are also some limitations to the basal area models developed in this study. The data used were not from thinned stands and observed decrease in the trees in the plots was due to natural mortality, removal of trees by the villagers or insect-pest and disease problems. The models may be less accurate when used for predictions when natural mortality is very significant (high density plantations and for long projection intervals). Moreover, most of the data available for *E. camaldulensis* and *D. sissoo* are confined between 6 to 13 years of age and for *T. undulata* were for a limited age group between 14 to 21 years of age. Hence, density is confounded with age. If there is no or low mortality, which may be a consequence of heavy or closely spaced thinning events, we may approximate that there will be no change in number of stems ha^{-1} at age A_1 and age A_2 and $N_1 = N_2$. In this case basal area equations used may further be simplified.

Conclusions

Single-entry and double-entry volume equations have been constructed for all the three species selected in this study and can be used to predict volume in these species for in the study area. These equations have a very crucial role in forest management and assume importance in projecting the total volume at different stages as the plantations mature. These can also be applied with greater accuracy on any population of these species available in the study area for yield estimation.

The site index equations developed are base-age invariant and polymorphic in nature. These are helpful in assessing productive capacity of site and also to select sites suitable for a particular species. These are also useful in estimating site index at base-age given height at some other age as well as estimating height at some desired age given the site index.

Equation 5 was used to fit the data collected from the sample plots laid out in pure even-aged stands of *E. camaldulensis, D. sissoo* and *T. undulata*, for developing the relationship between mean tree diameters and surviving stems per hectare. The estimated coefficients, in turn, were used to construct the limiting line indicating the maximum number of stems expected in the stand with respect to the quadratic mean diameter at maximum basal area. This relationship is helpful in generating information about the number of trees ha^{-1} that should remain in the stands given the mean diameter of the trees in the stands. If this prescription is followed, natural mortality due to over-crowding and resulting economic loss can be avoided.

The equations to model the natural decline of stem number in the stands have also been developed. The model can be used to generate survival curves for *E. camaldulensis* and *D. sissoo* plantations for different site indices. The model predicts the mortality of the unthinned plantations at a given age and hence could be helpful in deciding appropriate thinning regimes to avoid unnecessary mortality and loss of production.

The basal area is an important density measure to assess the stocking in the stands. The basal area prediction models developed are useful in analyzing the relationship between stand density and tree growth. In combination with the stand density model, the basal area projection models developed may effectively be used to define the type and intensity of thinnings in the stands. Thus, the models presented are very crucial in evaluating silvicultural treatment options.

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