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Dry Spells, Wet Spells and Rainfall Persistency at São Gonçalo (NE Brazil)

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Abstract: Results of a study of dry spells, wet spells and rainfall persistency at São Gonçalo, a semi-arid station in Northeast Brazil are presented in this paper. Daily rainfall data for a period of 40 years is used in the study. Frequency distribution of dry spells of different durations at the station is compared with those derived from two statistical models. It is found that the Eggenberger-Polya model provides better results than the logarithmic model. Rainfall persistency is investigated using Erickson's (1965) and Jorgensen's (1949) models. The incidence of wet spells lasting one, two and three days is less than expected on chance and spells lasting longer than three days are more frequent than expected on chance basis. Rainfall persistency reaches a maximum value on the third day of a rainy spell.

Key words: Rainfall, persistency, dry spells, wet spells, probability of rain.

The semi-arid zone of north-east Brazil is 860,000 km^2 in area and contains nearly 10% of the country's population. The main climatic characteristics of this region are: annual rainfall of 400-800 mm with a coefficient of variability of 60%, high air temperature and high potential evapotranspiration. In the semi-arid region the main constraint to crop production is the low rainfall and its extreme variability.

Information on the occurrence of dry and wet spells is a matter of much importance in regions with limited water resources. This is particularly so in the estimation of crop growing periods and in irrigation planning (Robertson, 1985, 1970).

Various studies have been carried out in the past to estimate the frequency distribution of sequences of dry and wet days. The model most often used is the simple Markov chain, which assumes that the probability of any particular day being dry or wet depends only on the nature of the previous day (Gabriel and Newmann, 1962; Caskey, 1963; Weiss, 1964). Higher order Markov chain models have been used by Feyerherm and Bark (1967). The logarithmic series has been suggested by Williams (1952) as a fit to dry and wet spell distribution. The Eggenberger-Polya (1923) model has been employed by Berger and Goossens (1983) to estimate dry and wet spell frequencies. Racsko *et al*. (1991) found that the mixture of two geometric models fitted well to the distribution of dry spells in Hungary. The negative binomial distribution has been fitted successfully to the distribution of wet and dry spells in Greece (Anagnostopoulou *et al*., 2003; Tolica and Maheras, 2005). Deni and Jemain (2008) suggested that the mixed geometric model is adequate for fitting the sequence of dry days over peninsular Malaysia.

Results of a study of dry spells, wet spells and rainfall persistency at Sao Goncalo (38°13′ W, 6°45′ S) a semi-arid station in north-east Brazil are reported in this paper.

Frequency distribution of dry and wet spells at Itabaina (35°20′ W, 7°20′ S), another semi-arid station in this region has been discussed in a previous paper (Karuna Kumar and Ramana Rao, 2006).

Sao Goncalo has mean annual precipitation of 910 mm and potential evapotranspiration of 1625 mm. According to Thornthwaites classification (Thornthwaite, 1948; Thornthwaite and Mather, 1955) its climatic formula is D A′ d a′. Cotton and corn are the main crops grown in the area.

Materials and Methods

Daily rainfall data for the months of February-April during the period 1941-1980 is used in this study. In this paper a dry day is defined as a day with no rainfall. A dry spell is a sequence of dry days bracketed by wet days on both sides. Daily rainfall data at Sao Goncalo for the months of February to April

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during the study period is used to obtain the frequency distribution of dry spells of different durations. These frequencies are compared with those computed using (a) logarithmic model, and (b) Eggenberger-Polya model.

The logarithmic series is mathematically written as (Williams, 1952):

$$\alpha x, \alpha \frac{x^2}{2}, \alpha \frac{x^3}{3}, \alpha \frac{x^4}{4} \ldots$$

where, each term, in the case of dry spell series represents the number of spells of dry days lasting over 1, 2, 3, etc. days.

The two constants x (which is always less than unity) and α can be computed from the two simultaneous equations:

$$S = \alpha \ln\left(1 + N/\alpha\right)$$

$$N = \frac{\alpha x}{(1-x)}$$

where, S is the total number of dry spells and N is the total number of dry days involved in the study period. For the estimation of α and x, the procedure given by Williams (1947) is used.

In the Eggenberger-Polya (1923) model the frequencies (Ø) of dry spells of 1, 2, 3, etc. days are given by

$$\phi_1 = \frac{S}{(1+d)^{m/d}}, \quad \phi_2 = \phi_1 \frac{m}{1+d}, \quad \phi_3 = \phi_2 \frac{m+d}{2(1+d)}$$

where, $d = \frac{\sigma^2}{m} - 1$, (m+1) is the mean length of a dry spell and S is the total number of dry spells.

Daily rainfall data for the months of February to April during the period 1941-1980 are also used for the study of wet spells and rainfall persistency. These months represent the wettest part of the year at this station.

A wet spell is defined as a sequence of rainy days bracketed by dry days on both sides. From the daily rainfall data frequencies of wet spells of different durations are obtained. Frequencies of occurrence of rain on the first, second and third, etc. days following rainy periods of various durations are also derived. This information is used to study the persistency of rainfall at the station.

Results and Discussion

The total number of dry days (Nd) is 1966 or approximately 55% of the days considered in the study. The number of dry spells of all durations is 651. The number of dry spells of more than seven days duration is 8% of this total. Dry spells which occur rarely are not of much importance and observed frequencies of spells of up to seven days duration are compared with frequencies derived from the two models described above (Table 1). Parameters of the two models are also included in the table. It can be seen that the Eggenberger-Polya model yields better results than the logarithmic model. According to the EP model the total number of dry spells is 643 and 41% of the first dry days will be followed by at least one more dry day. Similarly 60% and 66% of the second and third dry days will be followed by at least one more dry day. The chance of a four day dry spell lasting for one more day is 70%

Table 1. Observed and theoretical frequencies of dry spells at Sao Goncalo

Duration of dry spells in days	Observed frequencies	Theoretical frequencies	
		E P	Log
1	279	287	289
2	118	118	123
3	72	71	70
4	49	47	45
5	35	33	31
6	27	24	22
7	16	17	16

α= 340; d= 3.8282; x= 0.852; m= 1.9892; S = 651.

The total number of days considered in the study is 3570 and the number of rainy days (Nw) is 1604. The probability on chance of rain on any day (P) is 0.45 and that of a dry day (Q) is 0.55. The probability of one, two, three, etc., wet days in sequence is 0.45, 0.20, 0.09 and so on.

The number of wet spells (Sw) is 652 and the number of dry spells (Sd) is 651. The conditional probabilities P (d/d) and P (w/w) are

P (d/d) = 1 - Sd/Nd = 0.67

P (w/w) = 1 - Sw/Nw = 0.59

If P is the general probability of an event and P_1 is the probability that the event will occur after an occurrence on the preceding occasion, then the persistence as defined by Bessen (1924) is:

$$R_b = \left(\frac{1-P}{1-P_1}\right) - 1$$

This expression is zero when there is no persistence i.e., if P_1=P and will become infinite if occurrence of an event were always followed by another occurrence i.e., if P_1 = 1.

The 95% confidence limits of the persistence ratio are (Brooks and Carruthers 1953):

$$\frac{Q}{Q \pm 1.96\sqrt{\frac{PQ}{N}}}$$

where, N is the total number of possible occurrences and Q=1-P

In the present case for wet days at Sao Goncalo R_b is 0.34 and the value of the persistence ratio (1+R_b) is well above the upper limit (1.03) and the hypothesis of no persistence can be discarded.

To appreciate the effect of persistence we can compute the number of wet spells of different lengths if there is no persistence and compare them with the observed frequencies of wet spells. Frequencies of occurrence of wet spells of different durations expected on chance (no persistence) are computed using the expression: NP^rQ^2 (Berger and Goossens, 1983), where N is the total number of days considered in the study.

The frequencies obtained are given in Table 2 together with the observed frequencies of wet spells. Also included in the table are cumulative totals of observed wet spells from the longest to the shortest. The occurrence of wet spells lasting one, two and three days is less than that expected on chance and wet spells lasting longer than three days are more frequent than expected on chance basis.

Rainfall persistency can also be studied by means of Erickson's (1965) model.

If the probability of rain on day n is Pn if day 0 is rainy and pn if day 0 is dry,

$$p_n = p_{n-1}P(w/w) + (1-p_{n-1})P(w/d)$$

where $P_1 = P(w/w)$ and

$$p_n = p_{n-1}P(w/w) + (1-p_{n-1})P(w/d)$$

where $p_1 = P(w/d)$

$$\lim_{n\to\infty} P_n = \lim_{n\to\infty} p_n = P$$

Similarly if the probability of dry weather on day n is Qn if no rain occurs on day 0 and qn if day 0 is wet

$$Q_n = Q_{n-1}P(d/d) + (1-Q_{n-1})P(d/w)$$

where, $Q_1 = P(d/d)$, and

$$q_n = q_{n-1}P(d/d) + (1-q_{n-1})P(d/w)$$

where $q_1 = P(d/w)$

$$\lim_{n\to\infty} Q_n = \lim_{n\to\infty} q_n = Q$$

In practice the value of n for which

$P_n \approx p_n = P$ and $Q_n \approx q_n = Q$

gives a measure of the length of the period over which the system is still remembering the state of day 0.

Values of Q_n, q_n, P_n and p_n for Sao Goncalo (Table 3) show that after three to four days the persistency effect has ebbed out. At this station studies using a second order Markov chain model can be expected to provide better results than those based on a simple Markov chain model.

Rainfall persistency is defined by Jorgensen (1949) as the observed percentage frequency in excess of chance that a period of M rainy days

Table 2. Wet spells at São Gonçalo - PB

	Duration of wet spells (days)																
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Observed Frequencies	296	147	73	58	34	13	7	5	5	2	3	3	0	3	0	1	2
Frequencies expected on chance	486	219	98	44	20	9	4	2	1	0	0	0	0	0	0	0	0
Cumulative totals of observed frequencies	652	356	209	136	78	44	31	24	19	14	12	9	6	6	3	3	2

Table 3. Probabilities of Erikson's model together with P and Q

	n					P or Q
	1	2	3	4	5	
P_n	0.59	0.48	0.45	0.45		0.45
p_n	0.33	0.42	0.44	0.44		
Q_n	0.67	0.58	0.56	0.55		0.55
q_n	0.41	0.52	0.54	0.55		

is followed by N or more consecutive days with rain beginning L days later.

From Table 2 it is seen that out of 652 cases of wet spells of at least one day duration 356 or about 55% are of at least two-day duration, 209 cases or 32% are of minimum three days duration etc. Of the 356 cases of wet spells of at least two days length 209 or 59% are of at least three days duration, 38% are of minimum four days duration etc. Such information is presented in Table 4 together with percentage frequencies of a wet spell continuing for 1, 2, 3, etc. days on a chance basis.

Table 4. Percentage frequencies of occurrence of rain on N or more days following a wet spell of M days. Probabilities of N consecutive rainy days expected on chance are given at the top

M-Length in days of preceding wet spell	45%	20%	9%	4%	2%	<1%
5	56	40	31	24	18	15
4	57	32	23	18	14	10
3	65	37	21	15	11	9
2	59	38	22	12	9	7
1	55	32	21	17	7	5
	1	2	3	4	5	6

N – Number of rainy days following a wet spell of M days

The highest value of probabilities of rain continuing for at least one more day occurs after a wet spell of three days with a value of 65% as compared with 45% expected on chance. Maximum value of probability of a wet spell continuing for at least two more days occurs after a wet spell of five days. According to the definition given above, rainfall persistency is given by the difference between the observed probabilities of rain continuing for one, two, three, etc days and the corresponding probabilities based on chance. Persistency values for L=1 and N=1 are shown in Fig. 1.

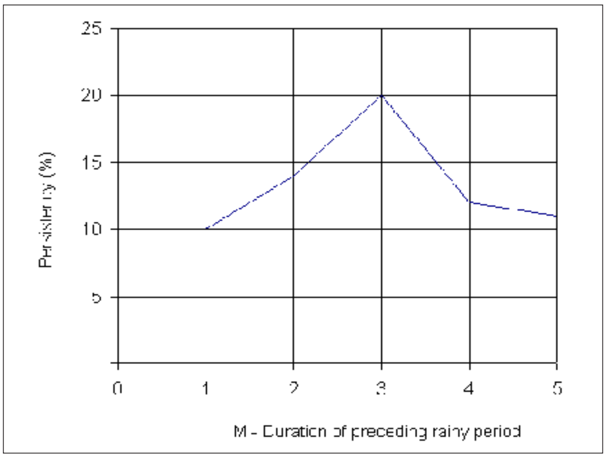

Fig. 1. Rainfall persistency at Sao Goncalo with L=1 and N=1.

It is seen that persistency reaches a maximum value of 20% on the third day of a wet spell. It has a low value of 10% on the first day of a wet spell.

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