On the Problem of Nonadditivity in Two-way Analysis of Variance of Data for Comparing the Efficiency of Fishing Gears

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To bring out the relative efficiency of various types of fishing gears, in the analysis of catch data, a combination of Tukey's test, consequent transformation and graphical analysis for outlier elimination has been introduced, which can be advantageously used for applying ANOVA techniques, Application of these procedures to actual sets of data showed that nonadditivity in the data was caused by either the presence of outliers, or the absence of a suitable transformation or both. As a corollary, the concurrent model: $X_{ij} = \mu + \alpha_i + \beta_j + \lambda \alpha_i \beta_j + \epsilon_{ij}$ adequately fits the data.

The difficulties in using analysis of variance (ANOVA) F-test for comparing the efficiency of fishing gears have been discussed by Nair (1982) and Nair & Alagaraja (1982). Broadly, these problems arose from the lack of satisfaction of the assumptions underlying analysis of variance. The importance of each assumption has been clearly discussed (Eisenhart, 1947). Kempthorne (1967) has indicated that the main requirements on the usefulness of a model are the additivity of treatment effects and homogeneity of errors and that of these two, additivity is more important. Treatment of nonadditivity in two-way classification has received much attention (Tukey, 1949; Mandel, 1961; Daniel, 1976; Johnson & Graybill, 1972a, b; Krishnaiah & Yochmowitz, 1980; Marasinghe & Johnson, 1981, 1982; Bradu & Gabriel, 1978 and Snee, 1982). Snedecor & Cochran (1968) describe the usefulness of Tukey's (1949) test of additivity " (i) to help decide if a transformation is necessary (ii) to suggest a suitable transformation and (iii) to learn if a transformation has been successful in producing additivity." Federer (1967) has observed that Tukey's sum of squares for nonadditivity is increased when one or more observations are usually discrepant and when the row and column effects are not additive and that nonadditivity could arise from more than one source. Johnson & Graybill's (1972b) and Rao's (1974) methods of derivation and interpretation of Tukey's test show that when the above type of nonadditivity is present, the model is:

$$X_{ij} = \mu + \alpha_i + \beta_j + \lambda \alpha_i \beta_j + \epsilon_{ij}$$

and that Tukey's test correspond to testing $\lambda = 0$. Xij stands for catch on the ith day for the jth gear, μ is the overall mean catch, α_i and β_j are the effects due to the ith day and jth gear respectively, λ a constant and ϵ_{ij} is the error team. Mandel, as quoted by Krishnaiah & Yochmowitz (1980), identified this model as the concurrent model and the concurrent model can be tested effectively by using Tukey's test for nonadditivity. Johnson & Graybill (1972b) and Hegemann & Johnson (1976b) have discussed that when Tukey's test shows significant nonadditivity, that is when the model given above describes the data, then the best way to analyse the data may be to find a transformation that will restore additivity. Bartlett (1947) gives a number of transformations suitable for various forms of relationship between the variance in terms of the mean and the distribution for which those are appropriate. He recommended logarithmic transformation for certain type of data with considerable heterogeneity. Nair (1982) has found that for data on fishing experiments with trawl nets logarithmic transformation did not stabilize the variance. Also application of Tukey's test to the data after logarithmic transformation showed highly sugnificant nonadditivity ($p < 0.001$). Cochran (1947) has observed that nonadditivity tends to produce heterogeneity of the error variance. Snee (1982) discusses procedures to examine whether non-additivity is caused due to non-homogeneous variance or interaction between row and column factors. These show the relative importance of the assumption of additivity and this communication presents the results of an investigation on non-additivity in trawl net-catch data on comparative fishing efficiency studies and procedures to tackle the problem using graphical analysis and transformation.

Materials and Methods

To decide whether a transformation is necessary and if required what would be the appropriate one, Tukey's (1949) test of additivity was applied to the four sets of data given in Nair (1982). Graphical analysis of nonadditivity (Tukey, 1949) was applied to these data to check whether the nonadditivity was due to analysis in the wrong form or due to one or more usually discrepant values. Tukey's test of additivity leads to transformation of the form $Y = X^p$ in which X is the original scale. The procedure followed in Snedecor & Cochran (1968) was applied to determine 'p', to which X, the observation must be raised to produce additivity. 'p' is estimated by $(1-B\bar{X}..)$, where B is the regression coefficient in the linear regression of the residual $(X_{ij}-\hat{X}_{ij})$ on the variate $(\bar{X}_{i}.-\bar{X}..)(\bar{X}.j-X..)$. An estimate of B is obtained from $B = \frac{N}{D}$, where $N = \sum_i w_i d_i$, $w_i = \sum_j X_{ij} \cdot d_j$, $d_i = (\bar{X}_i.-\bar{X}..)$, $d_j = (\bar{X}._j-\bar{X}..)$ and $D = (\sum d_i^2)(\sum d_j^2)$; $\bar{X}_i.$, $\bar{X}._j$ and $\bar{X}..$ refer to the row (block) means, column (treatment) means and grand mean respectively. Tests for nonadditivity is given by F, where F follows Snedecor's F distribution with 1 and [(r-1) (c-1) -1] degrees of Freedom, r and c indicating numbers of rows and columns, respectively. Tukey (1949) discusses transformations which are additive for $O \leq P < 1$, $p = 1$ and $1 < p$ and $\log(x+a)$ corresponding to none of these. Snedecor & Cochran (1968) stated that when $p = -1$, it is a reciprocal transformation analysing $1/X$, instead of X. ($p = o$ corresponds to logrithmic transformation because for p very small X^p behaves like log X)

Results and Discussion

Application of Tukey's test of additivity for the four sets of data on trawl catch (Nair, 1982) showed that there was significant non-additivity in all the sets (Table 1). For sets 1-3 (that is for the actual data), nonadditivity was found to be very highly significant with $p < 0.001$. Tukey's (1949) procedure was followed to check whether nonadditivity was caused by the presence of one or more discrepant observations or due to the need for a transformation. His method of graphical analysis for detecting the discrepant observations (outliers) was applied to the four sets of data. The method involves in plotting w_i against the block means. According to Tukey, "a usually discrepant observation will tend to be reflected by one point high or low and the others distributed around a nearly horizontal line. An analysis in the wrong terms will tend to be reflected by a slanting regression line." To determine the points high or low Tukey provided a 2s limit, namely,

Table 1. *Test for nonadditivity of the four sets of data*

	F for nonadditivity	Degrees of freedom
Set 1	38.64***	1,67
Set 2	63.87***	1,67
Set 3	87.70***	1,67
Set 4	4.80*	1,18

$$(\text{average cross product}) \pm 2 \left\{ \begin{array}{l} \text{sums of squares} \\ \text{of deviations of} \\ \text{column means } (= \sum d_{.j}^2) \end{array} \right\}^{\frac{1}{2}} \left\{ \begin{array}{l} \text{Means square for} \\ \text{balance} \end{array} \right\}^{\frac{1}{2}}$$
$$(= \sum w_i/\text{no. of rows})$$

The plots of w_i against the row means with the 2s limits for sets 1-4 are presented in Figs. 1-4. The figures show the presence

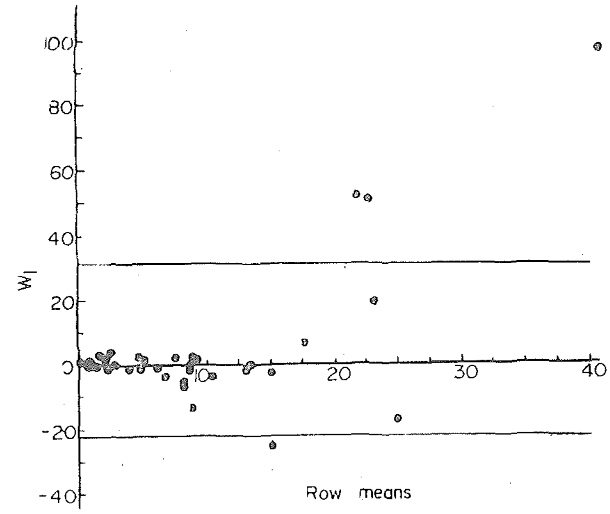

Fig. 1. Plot of w_i on row means with the 2 s limits for set 1

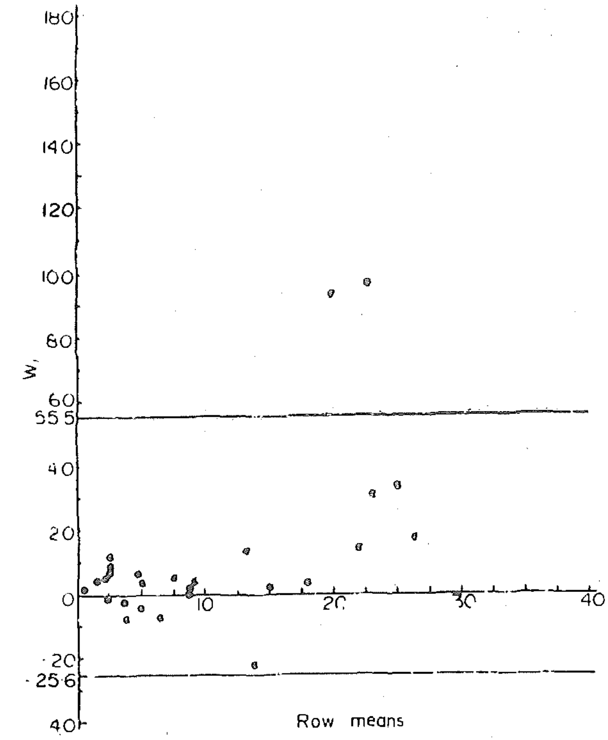

Fig. 2. Plot of w_i on row means with the 2 s limits for set 2

of outliers in all the four sets ranging from 1 to 5 in number. It is clear from the figures that the points excluding the outliers are distributed on a nearly horizontal line for set 1 and on a slanting regression line for

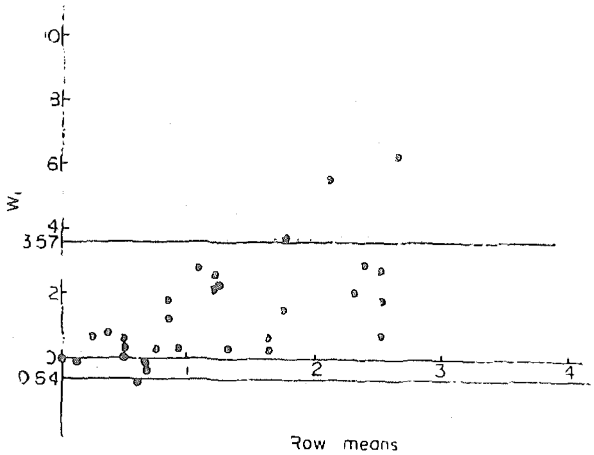

Fig. 3. Plot of w_i on row means with 2 s limits for set 3

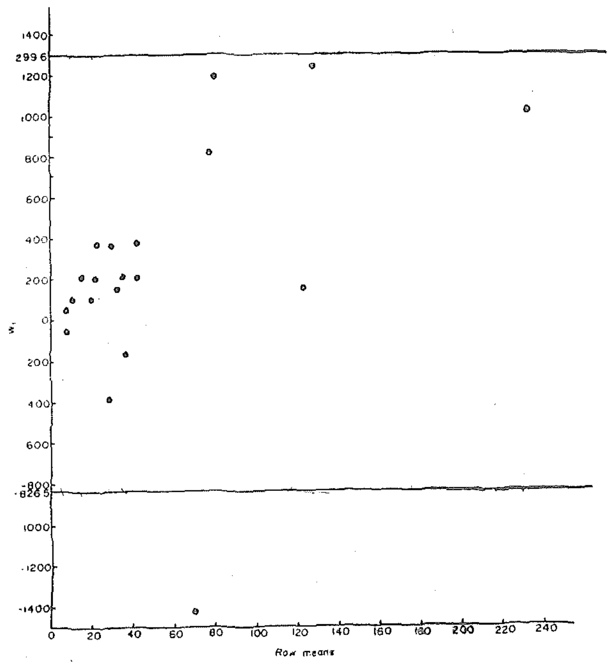

Fig. 4. Plot of w_i on row means with 2 s limits for set 4

sets, 2 to 4. This shows that no transformation is required for set 1, after removing the outliers while it is required for the other sets. This was confirmed by applying Tukey's test to the outlier-eliminated data (Table 2). Sets 2-4 showed the presence of nonadditivity indicating the need for a transformation for these sets.

The power transformation $Y = X^p$, suggested by Tukey's test of additivity were worked out for sets 2-4. These have been

Table 2. *Test for nonadditivity of the outlier-eliminated data*

	F for nonadditivity	Degrees of freedom
Set 1	0.02 (not significant)	1,59
Set 2	9.90**	1,61
Set 3	34.37***	1,57
Set 4	15.23**	1,17

*Significant at 5% level,
**Significant at 1% level,
***Significant at 0.1% level

presented in Table 3 along with the estimated values of B and P. For set 2, the transformation worked out to $Y=X^{-0.31}$, which is a reciprocal transformation. For set 3, the transformation obtained was $Y = X^{0.0618}$ and for set 4, $Y = X^{0.1594}$.

Table 3. *Tukey's transformation after eliminating the outliers*

	B	P	$Y=X^p$
Set 1	Data additive after exclusion of outliers		
set 2	0.1594	-0.31	$X^{-0.31}$
set 3	1.0335	0.0618	$X^{0.0618}$
set 4	0.0166	0.1594	$X^{0.1594}$

The data were analysed after carrying out these transformations. Tukey's test of additivity now showed, nonadditivity to be insignificant for all the sets (Table 4). The reduction by 4 in the lower d.f. for set 2 is due to omission of two rows where one observation each was zero. Though p was as small as 0.0618 for set 3, logarithmic transformation did not remove nonadditivity, F for nonadditivity being 12.97***, which is highly significant for 1 and 57 degrees of freedom. Thus application of the power transformation suggested by Tukey's test to the data after eliminating the outliers has been found to be effective in making the data additive. In case where nonadditivity is not accounted for by Tukey's transformation and outlier elimination by graphical analysis or in other words where the concurrent model does not describe the data, there are other methods for testing the structure of interaction and testing the main effects, for instance, methods mentioned by Marasinghe & Johnson (1982) (for a multiplicative interaction structure) and Krishnaiah & Yochmowitz (1980). The work in this line would be considered later.

Table 4. *Test for nonadditivity of the outlier-eliminated and transformed data*

	F for nonadditivity		Degrees of freedom
Set 1	Not applicable as data is additive after exclusion of outliers		
Set 2	2.55	Not significant	1,57
Set 3	0.05	,,	1,57
Set 4	0.13	,,	1,17

Daniel (1976) points out that nonadditivity is often associated with a few rows or columns of the two-way table. Snee (1982) states that nonadditivity in a two-way classification with one observation per cell may be either due to nonhomogeneous variance or interaction and the data may not be sufficient to distinguish between these two. However, ways and means for interpretation of the observed nonadditivity has been discussed by him. Federer (1967) states that the sum of squares associated with Tukey's one degree of freedom for nonadditivity gives the linear row by linear column interaction. Nair (1982) reported the dependence of standard error per unit on the average catch. A look at the model considered in this paper will show that when the availability of fishes changes over period of days, the α_i's may change, for different periods causing this situation. (The dependence of variance on the mean also suggests nonnormality).

Apart from graphical procedure, much work has been done on the rejection of outliers. Rules for rejection has been discussed by Anscombe (1960), Anscombe & Tukey (1963) and Snedecor & Cochran (1968). Lately, Gaplin & Hawkins (1981) have presented bounds for the fractiles of maximum normed residuals (MNR). The present procedure is convenient to apply along with additivity test because the steps involved in testing provide the material for graphical analysis.

The present study shows that elimination of the outliers by graphical analysis and application of Tukey's test of additivity can be adopted to tackle the problem of nonadditivity in the analysis of catch data. Nair & Alagaraja (1982) suggested Wilcoxon's matched-pairs signed-rank test as an appropriate procedure for comparing the efficiency of two fishing gears and illustrated with a set of data the superiority of this method over ususal ANOVA. (Ordinary ANOVA was less sensitive in this case). The same set of data was analysed using the above procedure (that is outlier-elimination and application of Tukey's test of additivity and the consequent transformation as introduced and discussed in this paper) and the same result as that given by Wilcoxon's test was obtained. This shows the usefulness of this combination procedure in statistical comparison of the efficiency of fishing gears.

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