Indian Journal of Agricultural Sciences **83** (1): 99–102, January 2013/Article

Comparison of few statistical models describing castor (*Ricinus communis*) seed yield response to phosphate fertilizer

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Received: 31 May 2011; Revised accepted: 30 October 2012

ABSTRACT

Five statistical models, linear response and plateau (LRP), quadratic response and plateau (QRP), logistic, quadratic and square root were compared to describe castor seed yield response to phosphate fertilization under irrigated conditions of Junagadh and S K Nagar regions. Adjusted R^2 values explained the variability in castor seed yield response to P fertilization, but the optimum phosphorus (P_{op}) values calculated by these models varied greatly. Overall the QRP followed by LRP model fitted the data with less bias than the other models, and calculated Pop values for Junagadh and S K Nagar (37.1 and 25.6 kg/ha respectively) were comparable with the P doses reported in literature.

Key words: Castor, Phosphate fertilization, Statistical models

Phosphate fertilization recommendations must optimize crop yield and quality, maximize profitability, and reduce the risk of environmental pollution. Nutrient recommendations are usually based on field experiments that determine the crop response to various rates of fertilizer application. Data from fertilizer studies can be fitted to several statistical models to determine optimum nutrient rates. The selection of the most appropriate model for a particular cropping situation is not obvious (Bock and Sikora 1990, Bullock and Bullock 1994). In addition model selection has considerable effects on estimating optimal fertilizer rates. For example, different models fitting one data set can give comparable coefficients of determination (R^2) but different optimal fertilizer rates (Cerrato and Blackmer 1990, Isfan *et al*. 1995). Although several statistical models are commonly used to describe the crop yield response to fertilizer rates, the choice of one model over another is rarely explained.

Quadratic and square root models have been very popular in India for describing the crop response to fertilization (Cooke 1990), but quadratic model tend to overestimate the response if the maximum point on the curves is taken as the best fertilization rate (Colwell 1994). Linear response and plateau (LRP), quadratic response and plateau (QRP) and logistic model have also been used to describe the crop response to fertilizer (Cerrato and Blackmer 1990, Colwell 1994, Overman *et al*. 1994).

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Using of statistical models for predicting optimal P rates with castor is scanty. The objective was i) to compare five statistical models [linear response and plateau (LRP), quadratic response and plateau (QRP), logistic, quadratic and square root] describing the response of castor to P fertilizer application, (ii) computation of optimum P rates (P_{op}), coefficients of determination (R^2) and standard errors of the estimate.

MATERIALS AND METHODS

Primary experimental data of all India co-ordinated project reports on castor (1999 to 2004) which has shown the response to P fertilization by randomized block design (RBD) analysis was considered and compared. Brief experimental details are under irrigated conditions of Junagadh and S K Nagar castor crop received five and four levels of P_2O_5, viz. 0, 20, 40, 60 and 80 kg/ha and 0, 25, 50 and 75 kg/ha respectively, as basal dose while nitrogen and potassium were applied as per recommended doses in vogue. All other recommended cultural practices were followed to grow the castor crop.

To describe the castor seed yield response to P fertilizer, five statistical models (linear response and plateau, quadratic response and plateau, logistic, quadratic and square root) were fitted to the data using the NLIN procedure of the SPSS software. Optimum P fertilizer rates for total castor seed yield under irrigated conditions of Junagadh and S K Nagar regions were computed year-wise and pooled over years. In theses, models Y is the total castor seed yield in kg/ha, P is

Table 1 Total castor seed yield as influenced by phosphorus fertilization under irrigated conditions.

P_2O_5 doses (kg/ha)	1999–2000	2001–02	2002–03	2003–04	Mean
			Junagadh		
0		2 159	3 647	1 870	2 558.7
20		2 511	3 657	2 004	2 724.0
40		2 705	4 400	2 208	3 104.3
60		2 950	4 163	2 014	3 042.3
80		2 628	4 030	2 115	2 924.3
			S K Nagar		
0	1 657	1 275	3 289	712	1 387
25	2 472	2 023	4 906	1 211	2 122
50	2 496	2 228	4 439	1 368	2 106
75	2 436	2 146	4 874	1 575	2 206

the P_2O_5 fertilizer rate in kg/ha, 'A', 'B' and 'C' are parameter estimates using the NLIN procedure of SPSS.

Linear-plus-plateau (LP): $Y = a+bP$ if $P < C$
$Y =$ Plateau yield if $P \geq C$

Quadratic-plus-plateau (QP): $Y = a + bX + cX^2$ if $P < C$
$Y =$ Plateau yield if $P \leq C$

Logistic: $Y = A/[(1 + \exp (b - cP)]$

Quadratic: $Y = a + bP + cP^2$ and

Square root (SR): $Y = a+bP^{1/2} + cP$

The Pop values were not estimated when 'C' had a positive value for a quadratic model because the Pop estimation is based on relationships with a continuous diminishing form, and when 'B' had a negative value for the square root model, to avoid having Pop estimates as the square of a negative value. To calculate nitrogen optimum doses similar procedure was adopted earlier by Colwell (1994).

RESULTS AND DISCUSSION

The response of P fertilizer to castor was reported by several researchers (Sutaria *et al.* 1998, Anonymous 2001). Phosphate fertilization significantly increased total castor seed yields at all sites in all the years (Table 1).

Data in Tables 2 and 3 show that the models predicted nearly similar maximum yields except logistic at each site-year. The validity of the SR model for predicting maximum yields, however, is uncertain because seemingly absurd rates of fertilization were indicated as necessary to achieve maximum yields. Similarly, the predicted maximum yields provide little basis for selection of one model over others.

A commonly used measure of the goodness of fit for linear regression models is R^2, the amount of the total proportion of variability explained by the regression model. It is defined as the ratio of the regression sum of squares to the corrected total sum of squares. However, for non-linear models this ratio may exceed 1.0 since non-linear models usually do not contain a parameter for the population mean. In an attempt to report a measure of the goodness of fit for the studied models, R^2 will be defined as 1.0 minus the ratio of the residual sum of squares to the corrected total sum of squares. The degree of fit for the various models based on this R^2 measure is presented in Tables 2 and 3. Although this ratio underestimates the degree of fit for non-linear models containing asymptotic parameters such as B, it can be used as a basis for comparing the degree of fit for different non-linear models. Data in Tables 2 and 3 indicate that, when evaluated by using the adjusted R^2 statistic, the five models seem to fit the yield data equally well with some exceptions. Because there is little biological basis for selecting one model over others (Mead and Pike 1975, Nelson *et al.* 1985), the R^2 statistic usually is used to justify the use of a particular model. Judge *et al.* (1982) suggested that by adjusting the R^2 values for degrees of freedom, fertilizer responses with less bias could be estimated. The ranking of models based on R^2 statistic were as follows:

Junagadh:	
2001–02	QRP>Q>LRP>L>SR
2002–03	QRP>L>SR>LRP>Q
2003–04	QRP=LRP>L>SR>Q
Mean	QRP>LRP>L>Q>SR
S K Nagar:	
1999–2000	SR>QRP=LRP=L>Q
2001–02	QRP=LRP>L>SR>Q
2002–03	L>LRP>SR>QRP>Q
2003–04	SR>L>QRP=LRP>Q
Mean	QRP=LRP=L>SR>Q

Under irrigated conditions of Junagadh QRP model has constantly ranked first in all the years and adjusted R^2 values varied from 0.708 to 0.921 (Table 2). Both QRP and LRP models showed a higher R^2 values (>0.900) for the mean castor seed yield (over the years) response to P fertilization. Although similar trend was not observed under irrigated conditions of S K Nagar in individual years the overall mean castor seed yield data fitted well to QRP and LRP models with less bias and a logical and optimum P dose could be discernible (Table 3). Optimum P rates (P_{op}) varied greatly among the tested models and years (Table 2 and 3). The P_{op} values calculated by the quadratic and square root models were greater than that of the other models in all the years. The higher P_{op} obtained with quadratic models is in agreement with results reported in the literature (Bock and Sikora 1990, Murthy and Alivelu 2004). The quadratic curve must be symmetrical around its maximum, which may lead to higher optima. Further the quadratic and square root models tended to overestimate yield at the rates of fertilization it identified as optimum and underestimate yield at the rates of fertilization greater than P_{op}. Although the trend is more difficult to quantify it seems that the LRP model has a tendency to overestimate yields in the portion of the response curve close to where optimum rates of fertilization are indicated. This

Table 2 Estimated values of regression coefficients, optimal phosphorus doses and R^2 for linear response plateau (LRP), quadratic response plateau (QRP), logistic, quadratic and square root models for the total castor seed yield under irrigated conditions of Junagadh

Model	A	B	B_{SE}	C	C_{SE}	$Y_{MAXIMUM}$	P_{OP}	R^2
				2001–02				
LRP	2 159.0	17.60	11.88			2 761.0	34.2	0.831
QRP	2 129.9	25.41	9.07	– 0.23	153.1	2 933.8	-	0.921
Quadratic	2 129.9	25.41	6.41	– 0.23	0.07		55.2	0.842
Square root	2 139.1	139.17	90.33	– 7.84	9.98		48.6	0.800
Logistic	2 789.5	-1.20	0.40	0.05	0.50		24.0*	0.822
				2002–03				
LRP	1 725.0	6.36	25.36			3 979.4	40.0	0.398
QRP	3 647.0	– 17.82	0.00	0.91	0.93	4 197.6	36.2	0.835
Quadratic	3 544.0	24.44	15.33	– 0.22	0.18		55.5	0.292
Square root	3 601.5	97.63	166.06	– 3.97	18.36		151.2	0.469
Logistic	4 197.2	– 1.77	0.78	0.04	0.08		44.3*	0.535
				2003-04				
LRP	1 870.0	6.70	6.86			2 112.3	36.2	0.708
QRP	1 870.0	4.95	0.00	0.08	0.48	2 112.3	32.2	0.708
Quadratic	1 875.9	9.12	6.14	– 0.08	0.07		57.0	0.248
Square root	1 864.5	61.41	54.26	– 3.99	5.99		59.2	0.625
Logistic	2 106.3	-2.04	0.58	0.07	0.11		29.1*	0.641
				Mean				
LRP	2 532.3	12.02	7.09			3 013.2	40.0	0.902
QRP	2 558.7	2.89	0.00	0.26	0.45	3 023.6	37.1	0.918
Quadratic	2 517.0	19.64	5.96	– 0.18	0.07		54.5	0.777
Square root	2 535.4	99.29	81.76	– 5.26	9.04		89.0	0.732
Logistic	3 030.9	– 1.63	0.43	0.05	0.05		32.6*	0.781

SE, Asymptotic standard error estimate

*P dose required for half maximum yield

Table 3 Estimated values of regression coefficients, optimal phosphorus doses and R^2 for linear response plateau (LRP), quadratic response plateau (QRP), logistic, quadratic and square root models for the total castor seed yield under irrigated conditions of S K Nagar

Model	A	B	B_{SE}	C	C_{SE}	$Y_{MAXIMUM}$	P_{OP}	R^2
				1999–2000				
LRP	1 657.0	32.59	24.65			2 468.0	24.8	0.996
QRP	1 657.0	1.00		1.26		2 468.0	24.9	0.996
Quadratic	1 692.3	35.70	9.89	– 0.35	0.12		51.0	0.848
Square root	1 658.0	259.55	9.42	– 19.68	1.09		43.5	0.999
Logistic	2 468.0	-0.71	0.08	0.99	0.00			0.996
				2001–02				
LRP	1 275.0	29.92	3.28			2 187.0	30.5	0.994
QRP	1 275.0	1.00		1.15		2 187.0	27.7	0.994
Quadratic	1287.8	36.17	3.58	– 0.33	0.04		54.8	0.982
Square root	1 270.6	230.39	40.35	– 14.58	4.70		62.3	0.990
Logistic	2 191.8	– 0.32	0.14	0.08	0.02			0.991
				2002–03				
LRP	3 367.8	53.83	256.78			4 713.6	25.0	0.915
QRP	3 438.2	52.61		– 0.47		4 906.0	53.0	0.739
Quadratic	3 438.3	52.6	41.8	– 0.47	0.53		56.0	0.219
Square root	3 316.3	428.09	254.16	– 30.88	29.65		48.0	0.879
Logistic	4 739.6	– 0.81	0.39	1.18	0.00			0.920

Table Contd.

Table 3 (*Concluded*)

Model	A	B	B_{SE}	C	C_{SE}	$Y_{MAXIMUM}$	P_{OP}	R^2
				2003–04				
LRP	712.0	19.96	8.27			1 471.5	38.1	0.947
QRP	712.0	1.00		0.75		1 471.5	31.2	0.947
Quadratic	731.6	19.74	5.48	– 0.11	0.07		89.7	0.943
Square root	714.3	92.57	21.69	0.59	2.53			0.996
Logistic	1 591.6	0.17	0.23	0.04	0.01			0.981
				Mean				
LRP	1 387.0	29.40	3.99			2 156.0	26.2	0.988
QRP	1 387.0	1.00		1.13		2 156.0	25.6	0.988
Quadratic	1 430.3	28.81	12.13	– 0.25	0.15		57.6	0.741
Square root	1 392.7	201.86	48.90	– 12.88	5.70		61.4	0.982
Logistic	2 158.3	– 0.58	0.15	0.13	0.08			0.988

SE, Asymptotic standard error estimate

overestimation is consistent with the nature of this model which has an abrupt discontinuity at the point identified as optimum. Such abrupt discontinuities are difficult to accept from a biological point of view. The overestimation of yields at this discontinuity, of course results in identification of optimum rates of fertilization that are too low. Irrespective of years the P response coefficient (C) of logistic model showed a negligible variation under Junagadh conditions where as similar trend was not observed under S K Nagar conditions. However, the P doses calculated for even half maximum yield (i e B/C) of castor using logistic model was found to be unrealistic and the model did not fit well for the data obtained from both Junagadh and S K Nagar. The P_{op} values computed by QRP and LRP are comparable with the routine RBD analysis inference.

The five statistical models explained the variability in castor seed yield response to P fertilization, but the Pop values calculated by these models varied greatly. Overall the QRP followed by LRP model fitted the data with less bias than the other models, and calculated P_{op} values that minimize the risks of potential economic and environmental pollution problems.

ACKNOWLEDGMENT

Authors are thankful to the research workers at S K Nagar and Junagadh for conducting the experiments under All India Co-ordinated Research Project on castor and to the Project Director, Directorate of Oilseeds Research, Rajendranagar, Hyderabad for his keen interest in the study.

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