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A hybrid wavelet based neural networks model for predicting monthly WPI of pulses in India

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ABSTRACT

The high prices of pulses continue to be the pain point for both consumers and policymakers. In India, the wholesale price index (WPI) is the main measure of inflation. WPI measures the price of a representative basket of wholesale goods.Therefore, accurate forecasting of WPI is necessary by using some advanced statistical techniques. In the present investigation, Wavelet and artificial neural network (Wavelet-ANN) hybrid models are used for multi-step-ahead forecasting of monthly WPI of pulses.The original series is decomposed into the low frequency and high frequency components using Maximal Overlap Discrete Wavelet Transform (MODWT) based on Haar wavelet filter. Subsequently, suitable artificial neural network (ANN) model was fitted to decomposed series before they are combined and predicted using Inverse Wavelet Transform (IWT). A comparative assessment of hybrid models as well as individual counterpart revealed that the hybrid models give significantly better results than the classical artificial neural network (ANN) model for all tested situations.

Key words: ANN, ARIMA, Hybrid model, Pulses, WPI, Wavelet

Time-series analysis deals with observations that are sequentially collected over time. One distinguishing feature in this sequence is that values in the future depend on the past records in a quite stochastic manner, which is mandatory for depicting the future, employing suitable underlying model dynamics. Both linear and nonlinear model hence can be implemented to understand the stochastic process and predicting the future. Objective of the time-series analysis is to find out that or those kinds of models which would better reflect the underlying dynamics and the same can be used for forecasting the series phenomenon. Undoubtedly, the most widely used technique for analysis of time-series data in linear domain is, the Box Jenkins Autoregressive Integrated Moving Average (ARIMA) methodology. There are many applications of this models for forecasting agricultural commodity prices or yield of crop (Paul *et al*. 2013a, 2015; Paul 2014). Recently Artificial neural network (ANN) has been proved to be much promising as an alternative technique for forecasting economic series. Non-parametric, data-driven and self-adaptive nature of ANN has made it more easily approachable method as opposed to other model based nonlinear alternatives. Potential application of ANN can be found in fields like biology, engineering, economics etc. and its use in economics has been assessed by Kuan and White (1994). Very few studies have been undertaken on agricultural price system forecasting using ANN models. Paul and Sinha (2016) demonstrated superiority of ANN over linear ARIMA model using crop yield data.But Agricultural price system modeling is different from modeling of non-farm goods and services due to certain features of agricultural product markets. The characteristic features of agricultural crops include high degree of seasonality, derived nature of their demand and price-inelastic demand and supply functions. In this context modeling and forecasting of agricultural price system requires a strategy that will render efficient prediction performances for both short as well as long term forecast horizons and this goes beyond the capability of single linear ARIMA or nonlinear ANN methodology.

To fill the gap, the application of methods to pre-process input data has been highlighted as an efficient alternative to improve the performance of ANN models. An example of such a method is wavelet analysis, which has recently received much attention. Wavelet analysis provides decomposition of original time-series into high and low frequency components, so that wavelet transformed data can improve the ability of a forecasting model by capturing useful information at multi-resolution levels (Vidakovic 1999, Percival and Walden 2000).Farda *et al*. (2014) applied a hybrid method based on Wavelet-ANN-ARIMA for short term electricity load forecasting and the proposed method was better than ARIMA and ANN. Anjoy and Paul (2017) have proposed a hybrid approach

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for combination of Wavelet-GARCH model and applied in forecasting the potato prices in different markets of India. In the present investigation, an attempt has been made to examine the forecasting performance of Wavelet-ANN hybrid for wholesale price index (WPI) series of Pulse in India. A feed-forward time-delay neural network prediction has been considered for initially wavelet denoised series for 1, 3, 6 and 12 step ahead forecast horizons.

MATERIALS AND METHODS

There are infinitely many stochastic processes that can generate the same observed data, as the number of observations is always finite. However, some of these processes are more plausible and admit better interpretation than others. In literature, time-series models are generally classified into two broad categories namely linear and nonlinear models.

The most pioneered class of models that are frequently used to model linear dynamics structure is the Autoregressive integrated moving average (ARIMA) models which render the linear forecasting.

A univariate process $\{x_t\}$ is said to follow ARIMA (*p, d, q*) model, if it can be represented as.

$$\phi_p(B)\nabla^d y_t = \theta_q(B)\varepsilon_t \text{ or}$$

$$(1-\phi_1 B-\phi_2 B^2-\ldots-\phi_p B^p)\nabla^d x_t = (1-\theta_1 B-\theta_2 B^2-\ldots$$
$$-\theta_q B^q)\varepsilon_t$$

where, p=Autoregressive order, q= Moving average order, d= differencing order, ∇^d (1-B)^d. ϕ (B) and $\{\varepsilon_t\}$ are respectively AR and MA polynomials of degree p and q, $\{\varepsilon_t\}$ is assumed to be standard white noise process following normal distribution with mean zero and variance σ^2.

Artificial neural network (ANN) is a data driven, self-adaptive, nonlinear non-parametric statistical method. There are various forms of the widely applied ANN model.Single hidden layer feed-forward network is the most popular for time-series modeling and forecasting. This model is characterized by a network of three layers of simple processing units, and thus termed as multilayer ANNs. The first layer is input layer; the middle layer is the hidden layer and the last layer is output layer. The time-series data can be modeled using ANN by providing implicit functional representation of time, whereby a static neural network like multilayer perceptron is bestowed with dynamic properties (Haykin 1999). A neural network is made dynamic by embedding either long term or short term memory depending on the retention time, into the structure of a static network. One simple way of building short term memory into the structure of a neural network may be through the use of time delay, which can be implemented at the input layer of the neural network. An example of such architecture is a Time-delay neural network (TDNN).The neural network structure for a particular problem in time-series prediction requires determination of number of layers and total number of nodes in each layer. This is generally done through experimentation of the given data as there is no theoretical basis for determining these parameters. In the present study neural network with single hidden layer has been used and output node is also one. Logistic function has been used as activation function in the hidden layer with the form,

$$f(y) = \frac{1}{1+e^{-y}}$$

For p input lag, q hidden nodes in the hidden layer and one output node, the total number of parameters in a three layer feed-forward neural network is $q(p+2)+1$.

The term wavelet is used to refer to a set of basic functions with a very special structure which is the key to the main fundamental properties of wavelets and their usefulness in statistics. Wavelets are fundamental building block functions, analogous to the trigonometric sine and cosine functions. As with a sine or cosine wave, a wavelet function oscillates about zero. This oscillating property makes the function a wave. Wavelets method can be effectively used in time-series analysis where original signal is transformed into and represented in a different domain which is more amenable to exploration and processing. There are mainly two types of wavelet transform, namely, Continuous wavelet transform (CWT) and Discrete wavelet transform (DWT). CWT consider time-series which is defined over the real axis and DWT deals with time-series that are defined over a range of integers (usually $t = 0, 1, \ldots, N-1$, where N denotes the number of values in the time-series). In case of DWT, dilation and translation parameters are discretized such that the resulting set of wavelets constitutes a frame. But, here the transform is translation variant and require the fixed series $\boldsymbol{X}$ of length 2^j Therefore Maximal overlap discrete wavelet transform (MODWT) is considered which is well defined for all sample sizes N and shift-invariant too. This is also called as nondecimated wavelet transform, as there is redundancy of wavelet and scaling coefficients at each decomposition level of original series following a particular pattern. The usual DWT pyramid algorithm is applied to time-series X, whereas MODWT coefficients are obtained by applying DWT pyramid algorithm once to X and another to the circularly shifted vector TX.Hence, the first application yields the usual DWT (W) of the time-series vector X computed as $W = P X$ and the second application consists of substituting TX for X obtained as, $W = PTX$. Where, W and P can be written as

$$W = [W_1\ W_2 \ldots W_J\ W_J]'\ \text{P} = [P_1\ P_2 \ldots P_J\ Q_J]$$

For a time-series X with arbitrary sample size N, the j^{th} level MODWT wavelet (W_j) and scaling (V_j) coefficients are defined as

$$\tilde{W}_{j,t} \equiv \sum_{l=0}^{L_j-1} \tilde{h}_{j,l} X_{t-l \bmod N} \text{ and } \tilde{V}_{j,t} \equiv \sum_{l=0}^{L_j-1} \tilde{g}_{j,l} X_{t-l \bmod N}$$

where $h_{j,l}$ is the j^{th} level MODWT wavelet filter and $g_{j,l}$ is the j^{th} level MODWT scaling filter. For a time-series X with N samples, MODWT yields an additive decomposition or Multi resolution analysis (MRA), given as

$$X = \sum_{j=1}^{J_0} \tilde{D}_j + \tilde{S}_{J_0}$$

where $\tilde{D}_{j,t} = \sum_{l=0}^{N-1} \tilde{u}_{j,l} \; \tilde{W}_{j,t+l \bmod N}$ and $\tilde{S}_{j,t} = \sum_{l=0}^{N-1} \tilde{v}_{j,l} \; \tilde{V}_{j,t+l \bmod N}$

At a scale j, a set of coefficients $\overline{D}_j$ are called wavelet "details" and capture local fluctuations over whole period of a time-series at each scale. Set of values $\overline{S}_{j0}$ provide a "smooth" or overall "trend" of the original signal. The decomposition process can be iterated, with successive approximations being decomposed in turn, so that the original signal is broken down into many lower resolution components. This is referred to as the wavelet decomposition tree. There are many basic wavelet filter also that can be used in this wavelet transformation, namely, Haar, Daubechis, La8 etc. Some applications of wavelet analysis in agriculture domain may be found in Paul *et al*. (2011) and Paul *et al*. (2013b).

The schematic representation of Wavelet-ANN hybrid is portrayed in Fig 1. A perusal of the Fig 1 illustrates the procedure to obtain the forecasts employing Wavelet-ANN, where TDNN has been employed to fit individual sub series for the proposed method. Based on multi-time scale and observed highly nonlinear pattern of the transformed series led to application of ANN for prediction purpose. When the original series has much nonlinearity as its property, the MODWT has simplified it by breaking it into its sub-frequencies. Therefore, the ANN can now model the details and approximate components sufficiently so that the accuracy of the forecasting process is improved upto a marked extent. Wavelet analysis can effectively diagnose signal's main frequency component and abstract local information of the time-series. Here, at first, original time-series is decomposed into a certain number of sub time-series $\{W_1, W_2, ..., W_J, V_J\}$ by nondecimated wavelet transform. $W_1, W_2, \ldots, W_J$ are wavelet detail component, and V_J is smooth component. These play different role in the original time-series and the behaviour of each sub-time-series is distinct from other. So the contribution to original time-series varies from each other. Then, TDNN is constructed in which the sub time-series at t time are input of TDNN and the original time-series at $t+T$ time are output of TDNN, where T is the time length of forecast. Finally, original series is formed through inverse wavelet transform from detail and smooth part for time T. The key of Wavelet-ANN hybrid model is wavelet decomposition of time-series and the construction of ANN.

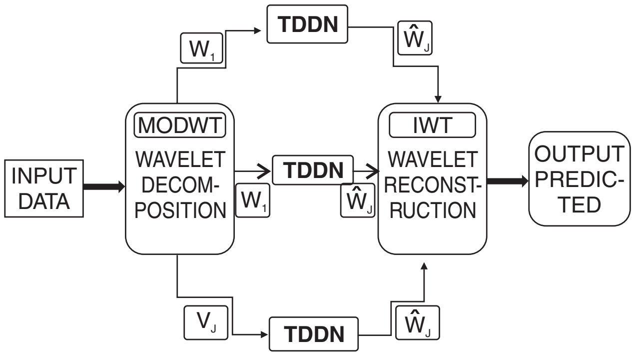

Fig 1 Schematic representation of Wavelet-ANN hybrid model

RESULTS AND DISCUSSION

Data set and basic features

Monthly Wholesale price index (WPI) data series of Pulse in India from January 2005 to March 2016 has been used for the present investigation. Data set has been collected from the office of Economic Adviser, Government of India, Ministry of Commerce and Industry. Under the ministry, the price index data is maintained by Department of Industrial Policy and Promotion (DIPP). Sample data constitute total 135 observations. Last 12 data set was retained for testing purpose and post-sample prediction and 123 observations were used for model building.

Basic characteristics of the data series used in the study are portrayed in Table 1. Minimum value of series was found in March, 2005 and the maximum on November, 2015. Fig 2 shows the time plot of the pulse WPI series for the investigation period. Perusal of Fig 1 indicates the possibility of nonstationarity pattern in the original series. Augmented Dickey Fuller (ADF) test has been employed to test for the presence of unit root for each level and first differenced series. It is found that ADF test statistic values are 1.86 (0.598) and 5.76 (0.018) respectively for level and 1st differenced series. The values in the bracket implies corresponding p-value. It implies that the original series is nonstationary and it has become stationary after 1st differencing.

For choosing appropriate modeling technique and prediction of data, it is necessary to find whether the given time-series is nonlinear or not. Presence of nonlinear pattern provides a useful guide to the application of nonlinear structures for describing the series phenomenon. For testing nonlinearity several tests can be performed including BDS (Brock-Dechert-Scheinkman) test, McLeod and Li test etc. In this study BDS test has been implemented for testing the nonlinearity developed by Brock *et al*. (1996). BDS test utilizes the concept of spatial correlation from chaos theory. Here the null hypothesis is that data series are independently and identically distributed (i.i.d) against the alternative hypothesis that data are not i.i.d.; this implies the presence of nonlinear dependence pattern in the time-series. BDS test is a two-tailed test; the null hypothesis should be rejected if the BDS test statistic is greater than or less than the critical values. As reported in Table 2, for all the

Table 1 Descriptive statistics of pulse WPI

Statistics	Pulse WPI	Statistics	Pulse WPI
Mean	197.43	Standard Deviation	62.06
Minimum	97.10	Skewness	0.82
Maximum	380.20	Kurtosis	0.83

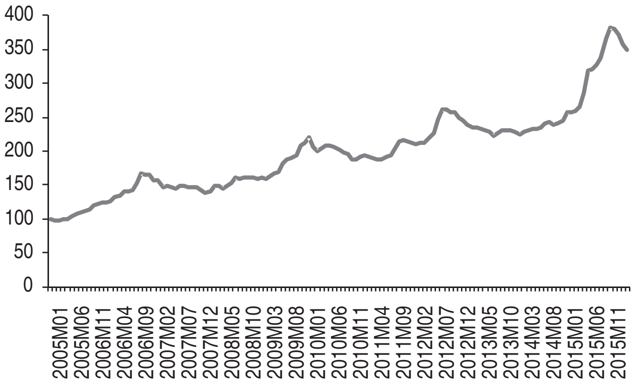

Fig 2 Pulse WPI original series

three selected dimensions BDS test result are found to be significant. This implies that ANN model can be a useful alternative for modeling the index series.

Implementation of forecasting models

Three forecasting models has been employed for the present study, namely ARIMA, TDNN and Wavelet-ANN. Firstly, considering ARIMA model, as the index series was nonstationary, hence series was first differenced based on autocorrelation function (ACF) and partial autocorrelation function (PACF). Best model was selected on the basis of minimum Akaike information criteria (AIC).

The estimated ARIMA (1,1,0) model is represented in the following equation:

$$\nabla y_t = \underset{(0.705)}{1.28} + \underset{(0.083)}{0.41^{**}}\, y_{t-1}$$

The values in the parenthesis denotes corresponding standard error of estimate. ** denotes significant at 1% level.

Best TDNN model with single hidden layer was selected for the study. Logistic and identity activation functions were used as activation function for the hidden node and output node respectively. The number of lagged observations as input and hidden nodes in the hidden layer was selected through experimentation. Number of input nodes was varied from 1 to 7 and number of hidden nodes from 2 to 10. For the present study Levenberg-Marquardt backpropagation algorithm (Hagan and Menhaj 1994) has been employed for training feed-forward networks. Based on training and testing Root mean square error (RMSE) and Mean absolute deviation (MAD) measure, a neural network model with three input nodes and two hidden nodes was found to perform better among all other competing models.

For Wavelet-ANN hybrid original series was first decomposed into multi-resolution level implementing nondecimated Haar wavelet transform. Maximum decomposition level was considered 4 here. Nonlinearity test in the detail and approximation series revealed the presence of nonlinear structure in each sub series. Then each wavelet transformed series was considered for modeling with TDNN. After data preprocessing, including checking for nonstationarity and possible transformation of the series, for fitting TDNN model to each sub frequency level, number of input lag were varied from 1 to 7 and number of hidden nodes were varied from 2 to 10. Levenberg-Marquardt backpropagation algorithm was used at each case for the purpose of training the networks. For different frequency level appropriate TDNN model with single hidden layer were chosen based on training and testing RMSE and MAD measures. Logistic activation function was used as an activation function for the hidden node and for the output node identity activation function was used. Haar filter reconstruction for the prediction of original series was mediated by combining the forecast from individual series. As single output node is considered here, hence multi-step ahead forecasting proceeds in an iterative way. If different forecast horizons are to be considered, then appropriate TDNN model has to be selected for each forecast horizon.

Prediction performances

Validation reports of the discussed models have been provided for 1,3,6 and 12 months ahead forecast. Table 3 presents the comparative results of ARIMA, TDNN and Wavelet-ANN model for various forecast periods in terms of Relative mean absolute prediction error (RMAPE), Relative mean square prediction error (RMSPE) and Mean absolute error (MAE). Here, ARIMA model performs better over TDNN for 1 month and 3 month ahead forecasting, whereas TDNN model perform better over ARIMA in 6 month and 12 month ahead forecasting cases. Comparing only ARIMA and TDNN, for this empirical presentation ARIMA is rendering better validation results for short term forecasting and TDNN is providing better forecasting performance in long term case. When considering Wavelet-ANN combinatory, it can be seen that the model is outperforming the linear ARIMA for short term forecasting and nonlinear TDNN for long term forecasting and throughout better than both the individual forecasting models. Relative performance of ARIMA, TDNN and Wavelet-ANN models for different forecast horizons are also presented in Table 4 on the basis of RMSE ratio. Mathematically, the ratio value <1 indicates the better performance of the numerator model over the denominator model. For 1 month and 3 month ahead forecast horizon,

Table 2 BDS test result for pulse WPI

Dimensions	Epsilon		Statistic	Prob.
2	eps(1)	4.87	7.15	<0.0001
	eps(2)	9.75	7.13	<0.0001
	eps(3)	14.62	7.25	<0.0001
	eps(4)	19.50	7.71	<0.0001
3	eps(1)	4.87	8.25	<0.0001
	eps(2)	9.75	7.68	<0.0001
	eps(3)	14.62	7.59	<0.0001
	eps(4)	19.50	8.16	<0.0001
4	eps(1)	4.87	9.48	<0.0001
	eps(2)	9.75	8.13	<0.0001
	eps(3)	14.62	7.64	<0.0001
	eps(4)	19.50	8.23	<0.0001

Table 3 Predictive performance of different models for various forecast horizons

Forecast horizon	Validation criterions	ARIMA	TDNN	Wavelet-ANN
	RMAPE	1.07	1.48	0.76
1 month ahead	RMSPE	0.04	0.76	0.02
	MAE	3.73	5.13	2.64
	RMAPE	6.24	11.03	2.68
3 months ahead	RMSPE	1.77	6.29	0.50
	MAE	22.38	40.01	9.83
	RMAPE	6.29	5.11	4.06
6 months ahead	RMSPE	1.64	1.23	0.72
	MAE	21.66	19.00	15.00
	RMAPE	20.20	10.94	7.57
12 months ahead	RMSPE	16.78	4.43	2.43
	MAE	70.78	37.26	26.21

Table 5 Actual vs. predicted price series for the 12 months ahead validation period

Actual	ARIMA	TDNN	Wavelet-ANN
264.3	259.0	302.0	266.7
284.6	260.3	263.6	269.8
316.1	261.5	281.6	279.2
318.8	262.8	303.6	295.7
326.2	264.0	297.6	303.5
334.7	265.4	302.8	308.2
364.6	266.6	307.5	317.0
380.2	267.9	325.1	332.3
378.5	269.2	328.3	343.9
370.4	270.5	324.6	345.7
356.4	271.8	320.1	337.8
346.6	273.0	312.7	331.8

RMSE ratio of TDNN/ARIMA is 1.37 and 1.93, respectively. This reflects the case of relative inability of TDNN to handle the short term situation as compared to traditional ARIMA methodology. For the present investigation,Wavelet based hybrid network render outperformance over both the linear and nonlinear forecasting models over different forecast horizons. Wavelet based hybrid model surpasses the relative inability of TDNN for short term forecasting. With the increment of forecast periods up to 1 year, decreasing RMSE ratio value in comparing Wavelet-ANN vs. ARIMA indicates the good performance of Wavelet-ANN as well for long term forecasting. RMSE ratio of Wavelet-ANN/TDNN is less than one for all the post-sample validation periods and least for 3 month ahead forecast.Table 5 displays the actual vs. predicted series for 1 year ahead post-sample case.

Through in wavelet decomposition, generally the original series is transformed through using two complementary filters, that are low pass and high pass filters and we obtain the wavelet detail part and approximation part in subsequent level following decomposition algorithm tree. This is basically the transformation of raw signal into useful component signals, each of which having distinctive features. In general approximation part is the low frequency or high resolution component and transformed details are the high frequency or low scale component. Each of this low frequency and high frequency components has different role in predicting the series phenomenon. Wavelet approximate part tend to capture the global feature or trend or the series which are much identical with the original series, hence imparts an important consideration when we look out for long term forecasting. But higher levels of approximations may reduce the information in raw signal. On the other side, detail parts provide a check in localized information loss. Detail part may be useful for representing the information that is lost. Moreover, choosing of appropriate decomposition level also plays a significant role in short term and long term forecasting due to varied role of low and high scale components in predicting underlying series structures. In the present investigation, decomposition level is considered as 4, which can be minimum up to 1 and maximum up to 7 for the mentioned series following maximum possible level of decomposition into a number $J_0 \leq log_2(N)$. Hence level of decomposition is optimal in this case.

Table 4 Comparative results of the models in terms post-sample RMSE ratio

RMSE ratio	1 month ahead	3 months ahead	6 months ahead	12 months ahead
TDNN/ARIMA	1.37	1.93	0.87	0.50
Wavelet-ANN/ARIMA	0.71	0.54	0.66	0.38
Wavelet-ANN/TDNN	0.52	0.28	0.76	0.75

The most evident logic in using Wavelet-ANN combinatory rather than single ANN is that wavelet decomposition makes it possible to transform the original nonlinear pattern into several sub frequencies and further capture the overall as well as partial features of the series which would not be possible in an otherwise simple ANN methodology. This can also be termed as wavelet denoising, as the approach provides a check in localized information loss.

Wavelet based mathematical transformation has been found to be useful in many areas of mathematics, physics and engineering. Potential utilization of this approach in agricultural system modeling is still to be ushered. In general, application of TDNN for short term forecasting is very limited. Furthermore, despite the popularity and the sheer power of neural networks, the empirical forecasting performance of these models has not been found satisfactory in all cases. In this aspect, the study reveals the relative ability of wavelet denoising approach for modeling and forecasting of temporal price index series of pulse in India. But very limited application of Wavelet based feed-forward neural network structure can be found in price trading and commodity analysis. Application of Wavelet based ANN approach is need of the hour to be emerged with a dynamic utilization in agricultural field. The investigation, however, can be taken forward to examine the performance of Support vector machine (SVM) in place of TDNN, which may open a new era in efficient modeling and prediction of macroeconomic and financial series.

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