



Comparison of lactation curve models for fortnightly test day milk yield

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ABSTRACT

The study of lactation curve helps to predict lactation milk production at a certain point of time and provides useful information for taking desirable decisions on preliminary sire evaluation, early culling of poor producers, breeding and feeding management and comparison of lactation records of unequal length. The test day milk yield data on 511 crossbred cattle were obtained and utilized according to fortnightly interval upto 305th day maintained at Instructional Dairy Farm, G B Pant University of Agriculture and Technology, Pantnagar (Uttarakhand). The different statistical models predicting the shape of lactation curve considered were exponential function, parabolic exponential function, inverse polynomial function and gamma type function. For fortnightly test day milk yield, the inverse polynomial function exhibited the highest coefficient of determination ($R^2=99.97\%$) followed by parabolic exponential (89.32%), gamma type (87.26%) and exponential function (35.70). Inverse polynomial function is the best fitted function for explaining the first lactation curve by utilizing the records of fortnightly test day milk yields.

Keywords: Exponential function, Gamma type function, Inverse polynomial function, Lactation curve, Parabolic exponential function, Test day milk yield

Lactation curve explains the changes in the rate of milk secretion with the advancement of lactation. The entire period of lactation in dairy animals can be partitioned into two phases, i.e. incline and decline phase. Incline phase extends upto 4–8 weeks and thereafter, decline phase follows till the milk secretion comes to an end. Various attempts have been made to describe this course of lactation. In the present study, the lactation curves for crossbred cattle (CB) were obtained using fortnightly test day milk yields (FTDMY) by exponential, parabolic exponential, inverse polynomial and gamma type functions. Prediction of lactation yield at any point of time of lactation with least error is the main aim of lactation curve study in case of irregular milk recordings. In the past, various models (linear and non-linear) have been used to explain the shape of lactation curves in cattle and buffalo. Among all models, the model of best fit is yet to achieve, because of the impact of several non-genetic factors on lactation.

MATERIALS AND METHODS

The test day (TD) milk yield data on 511 crossbred cattle were obtained and utilized according to fortnightly time interval upto 305th day distributed over a period of 28 years starting from 1990 to 2017 maintained at Instructional Dairy Farm, of G B Pant University of Agriculture and Technology, Pantnagar (Uttarakhand). The data for first lactation traits were analysed using Mixed Model Least

Squares and Maximum Likelihood Programme (Model 1 and Model 2) of Harvey (1990) to estimate the genetic parameters for studying the effects of different genetic and non-genetic factors affecting the traits under present study. The standard procedure of Harvey (1990) was used to study the effect of genetic as well as non-genetic factors on various first lactation traits as per the model given below:

$$Y_{ijklm} = \mu + S_i + Y_j + G_k + P_l + e_{ijklm}$$

where, Y_{ijklm} , the observation on m^{th} progeny (1–700) of i^{th} sire of j^{th} genetic group in k^{th} season and l^{th} period; μ , population mean; S_i , random effect of i^{th} sire ($i=1, 2, 3, \dots, 68$); Y_j , fixed effect of j^{th} genetic group ($j=1, 2, \dots, 7$); G_k , fixed effect of k^{th} season of calving ($k=1, 2, 3$); P_l , fixed effect of l^{th} period of calving ($l=1, 2, \dots, 7$) and e_{ijklm} , random error assumed to be normally and independently distributed with mean zero and constant variance, i.e. NID ($0, \sigma_s^2$).

Different statistical models predicting the shape of lactation curve considered in this study were proposed by various workers such as exponential function (Broody *et al.* 1923), parabolic exponential function (Sikka 1950), inverse polynomial function (Nelder 1966), and gamma type function (Wood 1967). The functions were analyzed by multiple regression method to estimate the model parameters (A, b, c, b_0, b_1 , and b_2). The test day milk yield data on 511 crossbred cattle were obtained and utilized according to fortnightly interval upto 305th day. The models are as detailed below:

Exponential function (Broody *et al.* 1923)

$$Y_t = A e^{-ct}$$

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The equation was modified by taking natural logarithms of Y_t thus

$\log_e Y_t = \log_e A + \log_e e^{-ct}$; or $\log_e Y_t = \log_e A - ct$; or $z = A + bt$ where $z = \log_e Y_t$; $A = \log_e A$ and $b = -c$, the constant.

Parabolic exponential function (Sikka 1950)

$$Y_t = A \cdot e^{bt+ct^2}$$

After taking natural logarithm and linear regression model the equation becomes:

$\log_e Y_t = \log_e A + bt + ct^2$; or $z = A + bt + ct^2$; or $z = A + b_1 t_1 + b_2 t_2$ where $z, \log_e Y_t; A, \log_e A; b_1, b; b_2, c; t_1, t$ and t_2, t^2 .

Inverse polynomial function (Nelder 1966)

$$Y_t = t(b_0 + b_1 t + b_2 t^2)^{-1}$$

After dividing both the sides by t and taking reciprocal the above equation can be written as:

$$t/Y_t = b_0 + b_1 t + b_2 t^2$$

Gamma Type Function (Wood 1967)

$$Y_t = A t^b e^{-ct}$$

By taking natural logarithm on both sides the equation becomes:

$$\log_e Y_t = \log_e A + \log_e t^b - ct; \text{ or}$$

$$\log_e Y_t = \log_e A + b \log_e t - ct; \text{ or}$$

$$z = A + b_1 X_1 + b_2 X_2$$

where, $z, \log_e Y_t; A', \log_e A; b', b; X_1, \log_e t; b_2, -c$ and X_2, t

where, Y_t , yield in the t^{th} month; A , constant, the value of initial theoretical rate of milk flow at the time of parturition; e , exponential constant (2.71828); b , ascending slope parameter upto the peak yield; c , descending slope parameter (persistence measure); b_0 , theoretical value at the time of parturition; b_1 , rising extremes of the curve; b_2 , declining extremes of the curve and t , length of time since calving.

The estimated values of coefficient of determination (R^2) and the Root mean square error (RMSE) were computed as:

1. The coefficient of determination (R^2) was calculated by following formula as:

$$R^2 = (\Sigma(\bar{Y})^2 \Sigma_i(Y_i - \hat{Y}_i)^2) / \Sigma_i(Y_i - \hat{Y}_i)^2$$

where, Y_i , observed value; \bar{Y} , mean of observed values and \hat{Y}_i , estimated value

2. The Root mean square error (RMSE) was computed using the following formula:

$$RMSE = \left[\frac{\sum_{i=1}^n (Y_i - \hat{Y}_i)^2}{n} \right]^{0.5}$$

where, n , number of observations; Y_i , observed values and \hat{Y}_i , predicted value.

The adequacy of the goodness of fit of these different functions, used to explain the course of lactation curve, was judged by R^2 values, RMSE and absolute mean deviations (AMD) of predicted milk yield computed according to these functions from observed milk yield.

The efficacy of all four functions was judged by the estimated values of coefficient of determination (R^2) and the root mean square error (RMSE). The adequacy of the

goodness of fit of these different functions, used to explain the course of lactation curve, was judged by R^2 values, RMSE and absolute mean deviations (AMD) of predicted milk yield computed according to these functions from observed milk yield.

RESULTS AND DISCUSSION

The initial milk yield in CB cattle was observed as 7.35 kg (TD-1). The peak milk yield of 10.71 kg approached in the test day 4th (TD-4) and subsequently declined to 7.94 kg in TD-20. Similar findings were reported by Dongre *et al.* (2012) in Sahiwal cattle and Sohal (2016) in Rathi. However, peak yield at third fortnight test day was reported by Dongre *et al.* (2013) in Sahiwal, and by Girimal (2017) in crossbred cattle. The predicted fortnightly test day milk yields (FTDMY) using Exponential function are presented in Table 1. Peak yield was obtained on first test day indicated that the milk production was linearly but inversely related to the advancement of lactation. Exponential function described the descending phase of lactation only. The function fitted best from 10th to 17th FTDMY and under estimated the milk yield for FTDMY-3 to FTDMY-9. The predicted fortnightly test day milk yields (FTDMY) using Parabolic Exponential function are presented in Table 1. The peak yield was obtained on 5th and 6th Test Day (Fortnightly). The Parabolic function indicated lower milk yields for 2nd to 5th Test Day and higher values of the milk yield for 1st and 9th to 15th test day. The predicted fortnightly test day milk yields using Inverse Polynomial function are presented in Table 1. The initial milk yield was 8.11 kg which then increased to a peak yield of 10.40 kg on 4th test day and subsequently declined to 8.03 kg on the last test day.

Thus, it indicated that Inverse Polynomial function estimated an initial ascending phase followed by peak and descending phase with the advancement of lactation. The predicted values of FTDMY by this function is little lower for TD-4 and TD-5, however, it over estimated the value for TD-1. The predicted fortnightly test day milk yields using Gamma Type function are presented in Table 1. The initial milk yield was 7.97 kg which then increased to a peak yield of 10.33 kg on 6th test day and subsequently declined to 7.72 kg on the last test day. It showed little higher values of TD-1, TD-8 to TD-14 and lower values for the milk of TD-2 to TD-5 and TD-18, TD-19 and TD-20.

The predicted FTDMYs for different functions are given in Table 2. The R^2 values, root mean square error (RMSE) and absolute mean deviation (AMD) for different functions are detailed in Table 3 and lactation curves for all prediction functions along with observed FTDMYs are presented in Fig. 5. The appraisal of Table 2 and lactation curves as evidenced by R^2 value, RMSE and AMD revealed that on overall basis, Inverse Polynomial curve ran close to observed lactation curve as compared to Parabolic Exponential and Gamma Type functions. Almost similar trend was observed for Parabolic Exponential and Gamma

Table 1. Predicted fortnightly test day milk yields (FTDMY) and errors of different lactation curve functions

FTDMY	Observed value {LS mean (kg)}	Exponential function		Parabolic exponential function		Inverse polynomial function		Gamma type function	
		Predicted value (kg)	Error (kg)	Predicted value (kg)	Error (kg)	Predicted value (kg)	Error (kg)	Predicted value (kg)	Error (kg)
TD1	7.35	10.17	-0.06	7.77	0.42	8.11	0.76	7.97	0.62
TD2	9.64	10.08	-0.17	9.21	-0.44	9.72	0.08	9.18	-0.46
TD3	10.39	9.98	-0.27	9.89	-0.50	10.24	-0.15	9.79	-0.60
TD4	10.71	9.90	-0.38	10.20	-0.51	10.40	-0.31	10.12	-0.59
TD5	10.59	9.81	-0.46	10.31	-0.29	10.39	-0.20	10.28	-0.32
TD6	10.38	9.72	-0.55	10.31	-0.06	10.30	-0.08	10.33	-0.05
TD7	10.33	9.63	-0.63	10.26	-0.07	10.17	-0.16	10.30	-0.03
TD8	10.02	9.55	-0.69	10.17	0.14	10.01	-0.01	10.22	0.19
TD9	9.82	9.46	-0.76	10.05	0.23	9.84	0.02	10.09	0.27
TD10	9.54	9.38	-0.82	9.91	0.37	9.67	0.13	9.94	0.40
TD11	9.34	9.29	-0.88	9.75	0.41	9.49	0.15	9.76	0.41
TD12	9.24	9.21	-0.94	9.58	0.34	9.32	0.07	9.56	0.32
TD13	9.11	9.13	-1.01	9.39	0.28	9.14	0.03	9.35	0.24
TD14	8.95	9.04	-1.06	9.19	0.24	8.97	0.02	9.13	0.17
TD15	8.82	8.96	-1.11	8.97	0.15	8.80	-0.02	8.90	0.08
TD16	8.66	8.88	-1.16	8.74	0.07	8.64	-0.02	8.67	0.00
TD17	8.57	8.80	-1.22	8.49	-0.08	8.48	-0.09	8.43	-0.14
TD18	8.39	8.72	-1.26	8.22	-0.16	8.32	-0.06	8.20	-0.19
TD19	8.20	8.65	-1.29	7.95	-0.26	8.17	-0.03	7.96	-0.25
TD20	7.94	8.57	-1.31	7.65	-0.29	8.03	0.09	7.72	-0.22

Table 2. Estimated lactation curve parameters of different functions for prediction of Fortnightly Test Day Milk Yields (FTDMYs)

Lactation curve functions	Equations	Parameter					
		A	b	c	b ₀	b ₁	b ₂
Exponential	$y_t = A \cdot e^{-ct}$	10.2580	–	0.0090	–	–	–
Parabolic exponential	$y_t = A \cdot e^{bt+ct^2}$	10.6714	-5.8224	-0.0076	–	–	–
Inverse polynomial	$y_t = t \cdot (b_0 + b_1 \cdot t + b_2 \cdot t^2)^{-1}$	–	–	–	0.0455	0.0754	0.0023
Gamma type	$y_t = A \cdot t^b \cdot e^{-ct}$	8.3289	0.2663	0.0437	–	–	–

Table 3. Different lactation curve functions with their prediction equations, coefficient of determination (R²), root mean square error (RMSE) and absolute mean deviation (AMD) value for fortnightly test day milk yields (FTDMY)

Lactation curve functions	Equations	R ² (%)	RMSE (kg)	AMD (kg)
Exponential	$Y_t = 10.2580 \cdot e^{0.0090t}$	35.70	0.20	0.80
Parabolic exponential	$Y_t = (10.6714) \cdot e^{[5.8224t + (-0.0076)t^2]}$	89.32	0.07	0.26
Inverse polynomial	$Y_t = t \cdot (0.0455 + 0.0754 \cdot t + 0.0023 \cdot t^2)^{-1}$	99.97	0.05	0.12
Gamma type	$Y_t = (8.3289) \cdot t^{(0.2663)} \cdot e^{-(0.0437)t}$	87.26	0.07	0.28

Type function in explaining the lactation curve. The R² values were sufficiently high for all these functions wherein the values for Parabolic Exponential and Gamma Type functions were almost equal, however, the R² value for Exponential function was observed very low. The Inverse Polynomial function explained the maximum variation for first lactation curve in crossbred cattle. As detailed in Table 3, the higher R² value as 99.97% was observed for Inverse Polynomial function followed by Parabolic Exponential function (R²=89.32%), Gamma Type (R²=87.26%) and Exponential function (R²=35.70). The root mean square error (RMSE) was least for Inverse Polynomial function (0.05 kg), followed by Parabolic Exponential function (0.07 kg), Gamma Type function (0.07 kg) and maximum for Exponential function (0.20 kg). Likewise, the least absolute mean deviation (AMD) was shown in Inverse Polynomial function (0.12 kg), followed by Parabolic Exponential function (0.26 kg), Gamma Type function (0.28 kg) and maximum for Exponential function (0.80 kg). Hence, it could be opined that Inverse Polynomial function is the best mathematical model for prediction of fortnightly test day milk yields (FTDMYs) followed by Parabolic Exponential, Gamma Type and Exponential function in crossbred Cattle. The Inverse Polynomial function was found to be the best function for explaining the lactation curve as also reported by Singh (1973) for explaining individual lactation curve, Roy (1983) in Sahiwal-Jersey cattle using individual milk records, Gahlot (1988 and 1989) in Rathi and Rathi-Red Dane crosses, Roy and Katpal (1993) in Jersey cattle, Kumar (1994) in CB cattle, Kumar *et al.* (1996) in Jersey- Sahiwal half-bred cows, Kumar *et al.* (1997) in HF-S half-bred cows, Singh *et al.* (1998) in Jersey-Sahiwal F₁ cows, Gandhi and Dongre (2013) in

Sahiwal cattle and Sohal (2016) in Rathi cattle. However, Singh (1973) reported Gamma Type function to be superior in explaining first lactation records, Kumar (1994) concluded Gamma Type and Parabolic Exponential function followed by Inverse Polynomial and Exponential fitting best to average monthly/ weekly milk records, Singh *et al.* (1996) reported best fitted model as the Gamma Type function followed by Inverse Polynomial, Parabolic Exponential and Exponential function and Singh *et al.* (1997) concluded Gamma Stochastic and Gamma Type function to be fitted best followed by Parabolic Exponential, Inverse Polynomial and Exponential function. The other studies and results of this study indicated that the Inverse Polynomial function was best fit function in explaining the FTDMY recording during first lactation in case of crossbred cattle.

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