



Comparative study of three prediction models in predicting the milk yields of Yunnan Holstein cows

ZHIYONG CAO, XIUJUAN YANG*, XIZHANG, WEIHUANG, LINLI TAO, BIN DENG, CHENCHEN and ZHAOCHENG SUN

Yunnan Agricultural University, Kunming 650 201 China

Received: 31 August 2019; Accepted: 26 November 2019

ABSTRACT

The polynomial model and wood model have been extensively applied to predict the milk yield of cows, which aims to measure and partially explain the uncertainty under single factor condition. However, different sample data would affect the goodness of fit of the models. To investigate the milk yield regularities of the Chinese Holstein cows in Yunnan Standardized Pasture (YSP), data of cows with 1–3 parities using different observation periods were derived from the 401,497 records of 1,826 cows collected from YSP, and fitted in the BP neural network (BPNN) model, the Quadrinomial model and the Wood model. Prediction results of three models were compared. The results show that, for group data (mean), the Quadrinomial model was likely to over-fit under the single factor condition with small sample data, while the Wood model had a better goodness of fit; the BPNN model was more suitable for multi-factor and large sample data analysis.

Keywords: BPNN model, Milk yield prediction, Quadrinomial model, Wood model

The milk yield prediction model based on the cow feeding condition and production management level could be used to predict the overall milk yield of different parities. Due to the closely related relationship between cow milk yield and dry material intake (Sánchez-Pérez *et al.* 2018), the nutrition requirement of lactating cows could be predicted which provides a reliable basis for the lactating cow management, nutrition control and stable milk production (Deen *et al.* 2019, Singh *et al.* 2019). Therefore, milk yield prediction for lactating cows is essential for fine raising of cows. Many studies on mathematical model for milk yield prediction have been carried out (Luo *et al.* 2010, Xiong *et al.* 2011, J Johnson *et al.* 2015, Sharma *et al.* 2019), among which the Wood model (Wood *et al.* 1967) and polynomial model have attained promising predicting results. Different regions and environments might affect the milk yield (S. Mote *et al.* 2016), therefore, there would be significant differences among parameters used in the model.

The current study selected data on the production of Holstein cows in the largest pasture in Yunnan (mountain plateau, tropical monsoon climate), explored the variation rules of parity, lactation time and milk yield of Holstein cows, and compared multiple milk yield sequential variation models.

MATERIALS AND METHODS

Data sources and processing: The data were derived

*Corresponding author e-mail: 179166561@qq.com

from the largest Yunnan standardized pasture in 2017 and 2018, and the milk yield data records (mean daily yield milk data) were exported by the MPC automatic milking system in the pasture. To guarantee the reliability of the data, the milk yield data used for modeling were the FCM (fat corrected milk) data from the 1st to 305th days after delivery, at least 240 records were collected from each lactation period, the lactation parities ranged from 1 to 3, and the milk yield was converted to 4% FCM (Gaines *et al.* 1928). Data satisfying the above-mentioned conditions were finally used for modeling and analysis was derived from 1,826 Chinese Holstein cows, as shown in Table 1.

Table 1. Milk yield

Parity	Cows	Record	Mean daily milk yield $\pm$ SD (kg)
1	433	125049	28.24 $\pm$ 11.97
2	685	198766	31.75 $\pm$ 10.20
3	708	202731	32.97 $\pm$ 9.26

Classification of different lactation periods: The milk yield observation period was classified as 7, 10 and 30 days, respectively, and the mean daily milk yields (kg/d) in different lactation periods were calculated. Then, the Excel2016, SPSS and Python software were employed for sorting out the experimental data and plotting for subsequent model fitting.

Milk yield prediction model and evaluation: The mean

daily milk yields were calculated based on different observation periods (7, 10 and 30 days) for model fitting. The Wood model, Quadrinomial model and Back-propagation Neural Network (BPNN) model were applied, among which, the first two models were used to construct the single-factor prediction models, and observation time as the inputs to output the predicted value for milk yield. BPNN used the current observation period and observed milk yield as the inputs to output the predicted value for the next period. Adjusted R^2 (R_a^2) as the suitable measures of the goodness of fit when the numbers of independent variables are different in different models. The determination coefficient R_a^2 , which ranged from 0 to 1, was adopted to evaluate the model, and a R_a^2 value that was closer to 1 indicated a higher model fitting degree.

$$R_a^2 = 1 - \frac{(1 - R^2)(n-1)}{n-p-1} \quad \dots (1)$$

n, number of samples; p, number of variables.

Wood model:

$$Y = at^be^{-ct} \quad \dots (2)$$

t, time; a, b, c, model parameters; e, natural base; Y, predicted value.

Quadrinomial model:

$$Y = a + bX + cX^2 + dX^3 + eX^4 \quad \dots (3)$$

X, time; a, b, c, d, e, model parameters; Y, predicted value.

BPNN model:

$$Y = f\left(\sum_{i=1}^n w_i x_i - \theta\right) \quad \dots (4)$$

x_i , observation period and current observed milk yield; w_i , weight; θ , deviation value; Y, predicted value.

There were 2 input nodes and 1 hidden layer of the BPNN model framework; as for the hidden node, the Hecht-Nielsen method (Hecht-Nielsen *et al.* 1992) was used to obtain 4 neurons in the hidden layer. BPNN used the current observation period and milk yield in the current period as the input nodes to output the predicted value in the next period.

Feeding management: The cows were fed twice a day with the full-hybrid daily ration in enclosure, and they had free access to food and water, and were milked for three times a day.

RESULTS AND DISCUSSION

Comparative analysis of the observed values of milk yields among different periods and different parities: The mean milk yields of cows at different parities during various lactation periods were analyzed. Then, the lactation period was used as the horizontal coordinate, while the mean daily milk yield at different observation periods was used as the vertical coordinate, to plot the mean daily milk yield variation trend within different lactation periods, as shown in Figs 1–3.

It could be seen from Figs 1–3 that, the milk yield variation trend was as follows: the mean daily milk yield

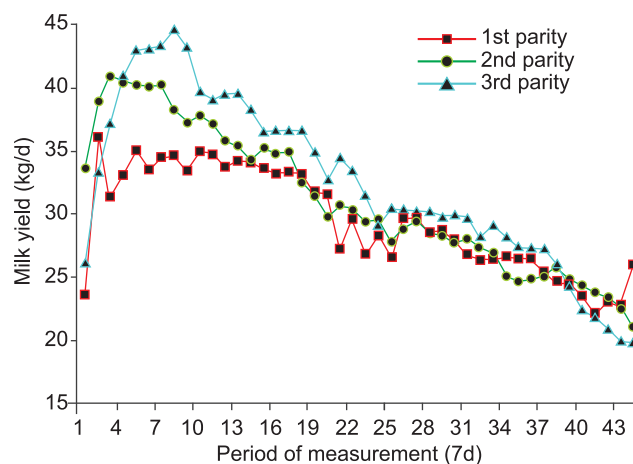


Fig. 1. 7-day period milk yield variation trend.

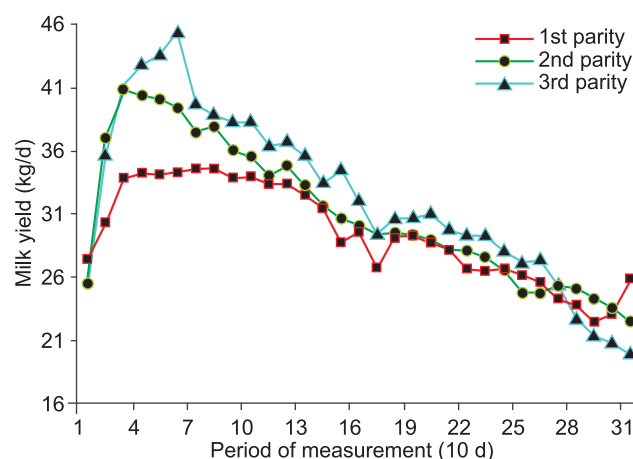


Fig. 2. 10-day period milk yield variation trend.

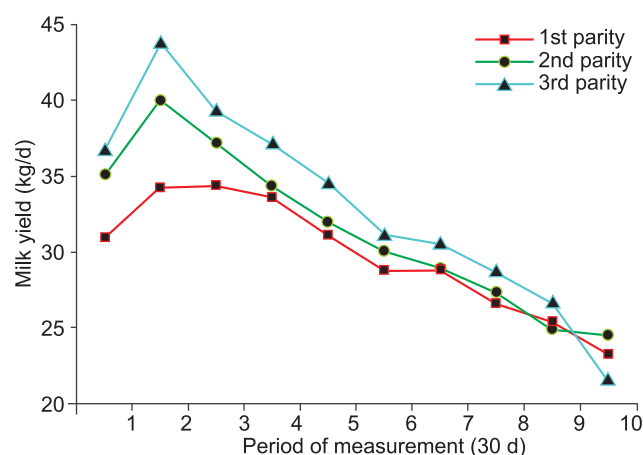


Fig. 3. 30-day period milk yield variation trend.

of cows at parity 1 was slowly increased, the highest daily milk yield was low, the lactation peak appeared late, and the milk yield downtrend was slow at the late lactation period; the milk yields of cows at parities 2 and 3 were rapidly increased, the highest daily milk yield was high, the lactation peak appeared early, and the milk yield was decreased at a relatively fast rate at late lactation period;

such results suggested that cows at parity 1 had strong lactation duration, while cows at parities 2 and 3 had weak lactation duration.

The mean daily milk yield during the 1st lactation period was 28.24 kg/d, close to that (28.5 kg/d) reported by Penasa *et al.* (2016), higher than that (26.02 kg/d) reported by Bilal *et al.* (2016), higher than that (27.10 kg/d) reported by Cinar *et al.* (2015), lower than that (35.93 kg/d) measured by Heins *et al.* (2016). The mean daily milk yield during the 2nd lactation period was 31.75 kg/d, lower than that (34.10 kg/d) reported by Xiong *et al.* (2011). The mean daily milk yield during the 3rd lactation period was 32.97 kg/d, close to that (33.65 kg/d) reported by Xiong *et al.* (2011). The mean daily milk yields for parities 2 and 3 within the lactation period were slightly higher than the mean daily milk yield (30.4 kg/d) of the mixed cows for multiple parities reported by Kristensen *et al.* (2015).

Observations indicate that the peak day of 1st lactation period was 60th day. The peak day of 2nd lactation period was 18th day. The peak day of 3rd lactation period was 53rd day. The lactation peak day for parity 1 had lagged behind those for parities 2 and 3, and the durability for cows at parities 1 and 2 was better than that for cows at parity 3; but the peak milk yield and mean daily milk yield for cows at parities 2 and 3 were higher than those at parity 1. Such results indicated that, parity would affect the lactation peak day and the peak milk yield.

Different fitting model parameters: The models of observed values and parities at different periods were obtained through fitting. The major model parameters are shown in Table 2.

Comparison of the fitting effects of mean daily milk yields predicted by different models: Cow lactation gradually rises to peak stage, and then slowly declines. The peak of lactation is the turning point of the increasing prolactin trend into the decreasing trend, and it is an important feature of

Table 3. BPNN prediction model parameters

Parity (Period)	BPNN model parameters
1(30d)	H1_1=TANH (−0.9000−0.0822×[POM(d)]+0.0470×[MY]) H1_2=TANH (−8.0368+0.0757×[POM(d)]+0.2285×[MY]) H1_3=TANH (−5.6709−0.0743×[POM(d)]+0.2028×[MY]) H1_4=TANH (6.4011−0.1238×[POM(d)] −0.1934×[MY])
2(30d)	H1_1=TANH (−2.8474−0.2041×[POM(d)]+0.1287×[MY]) H1_2=TANH (2.8157+0.0377×[POM(d)] −0.0830×[MY]) H1_3=TANH (0.9679−0.0791×[POM(d)] −0.0197×[MY]) H1_4=TANH (−2.4082+0.1326×[POM(d)]+0.0493×[MY])
3(30d)	H1_1=TANH (−2.6832−0.0997×[POM(d)]+0.0839×[MY]) H1_2=TANH (2.3101+0.0352×[POM(d)]−0.0850×[MY]) H1_3=TANH (0.7216+0.3356×[POM(d)] −0.0877×[MY]) H1_4=TANH (0.9317−0.1260×[POM(d)] −0.0210×[MY])

TANH is the hyperbolic tangent function, POM represents the count value of different observation periods, and MY is the observed value of milk yield at the current period. The parameters of the best fitted BPNN model are provided for each Parity (Period) according to Table 4.

Table 2. Major parameters of three prediction models

Parity (Period)	Major coefficients of the Wood model	Major coefficients of the Quadrinomial model	Major coefficients of the BPNN model
1(7d)	a=28.0079; b=0.1792; c=0.0205	a=−4.2E−5; b=0.0045; c=−0.1639; d=2.0196; e=26.9927	33.8416−2.4349×H1_1−2.4273×H1_2+15.2838×H1_3+17.1467×H1_4
2(7d)	a=37.0073; b=0.0726; c=0.0179	a=−5E−5; b=0.005; c=−0.1577; d=1.3338; e=35.988	50.2949+26.5536×H1_1+4.7122×H1_2+11.1274×H1_3−0.2950×H1_4
3(7d)	a=29.8939; b=0.2563; c=0.0307	a=−0.0001; b=0.012; c=−0.393; d=4.4348; e=26.31	28.9052+13.2876×H1_1−1.0622×H1_2−42.8434×H1_3−30.3915×H1_4
1(10d)	a=29.9290; b=0.1124; c=0.0202	a=−0.0001; b=0.0116; c=−0.3194; d=2.9911; e=25.845	41.2402−2.7705×H1_1+14.3087×H1_2+9.7563×H1_3−0.8954×H1_4
2(10d)	a=29.8338; b=0.2869; c=0.04307	a=−0.0004; b=0.026; c=−0.6212; d=4.9945; e=26.622	−6.9807−2.7780×H1_1−2.0537×H1_2+134.9287×H1_3−7.7116×H1_4
3(10d)	a=29.9619; b=0.3243; c=0.0489	a=−0.0005; b=0.0359; c=−0.8405; d=6.8899; e=23.994	39.8863−1.0367×H1_1+5.6591×H1_2−9.0564×H1_3−48.9970×H1_4
1(30d)	a=34.8963; b=0.2071; c=0.0889	a=−0.0158; b=0.3955; c=−3.4976; d=11.2576; e=22.8455	30.1785+28.5855×H1_1+4.7809×H1_2−10.6070×H1_3+2.1326×H1_4
2(30d)	a=40.6938; b=0.1865; c=0.0983	a=−0.016; b=0.4174; c=−3.7069; d=10.952; e=28.183	29.3606−3.3745×H1_1−14.7044×H1_2−18.6525×H1_3−33.2201×H1_4
3(30d)	a=44.0492; b=0.2525; c=0.1254	a=−0.0211; b=0.5323; c=−4.6371; d=13.826; e=28.142	38.0169−6.6115×H1_1−22.0317×H1_2+6.9277×H1_3+31.8162×H1_4

The major coefficients of the Wood model are a, b, c (1); the major coefficients of the quadrinomial model are a, b, c, d (2). H1_1, H1_2, H1_3 and H1_4 are the major parameters of the BPNN model, and their calculations are displayed in Table 3.

the whole lactation period (Macciotta *et al.* 2005). In Table 4, the milk yield of maximum value (peak of the observational value) was calculated according to different observing periods of the 7 days, 10 days and 30 days, the BPNN model performed best in the prediction of observing periods of the 7 days, 10 days of 1st lactation period, and the quadrinomial model performed best in the prediction of observing periods of the 7 days of 1st lactation period. The BPNN model performs best in all predictions for of 2nd lactation period. The Wood model performed best in the prediction of observing periods of the 7 days of 3rd lactation period, and the Quadrinomial model performed best in the prediction of observing periods of the 10 days and 30 days of 3rd lactation period.

The goodness of fit of the BPNN model was higher than those of the Wood model and the Quadrinomial model, while that of the Wood model was close to that of the Quadrinomial model. For the Wood model, the mean degrees of fitting of the predicted milk yields at various observation periods (7, 10 and 30 days) for cows at parity 1 were 0.8373, 0.8977 and 0.9638; while those at parity 2 were 0.9541, 0.8542, and 0.9548; and those at parity 3 were 0.9254, 0.8964 and 0.9489. For the Quadrinomial model, the mean degrees of fitting of the predicted milk yields at various observation periods (7, 10 and 30 days) for cows at parity 1 were 0.8338, 0.9236 and 0.9890; those at parity 2 were 0.9665, 0.9042 and 0.9757; and those at parity 3 were 0.9480, 0.9218 and 0.9647. For the BPNN model, the mean degrees of fitting of the predicted milk yields at various observation periods (7, 10 and 30 days) for cows at parity 1 were 0.8905, 0.9828 and 0.9821; those at parity 2 were 0.9222, 0.9894 and 0.9736; and those at parity 3 were 0.9950, 0.9977 and 0.9752.

The Wood model and the Quadrinomial model can well predict in the presence of few sample data (namely, the 30-day data for each parity), which can realize the data smoothness through the means of the cows, and can partially counteract the errors (noise, observation and measurement). The Wood model and the quadrinomial model can measure and partially explain for the uncertainty under single factor condition, which have favorable self-adaptive control capacity and can predict new data based on the given knowledge. Different from the statistics-based prediction model, the BPNN model is completely data-driven, which suggests that the complex data pattern in the time sequence can be naturally captured through the learning mechanism of the network itself. The BPNN model performs well in the presence of multiple sample data, particularly; it can use the multi-factor as the input. In this study, lactation time and historical milk yield were used as the inputs.

To sum up, when the cow milk yield sample size was small and simple (single factor), the Wood model and Quadrinomial model could be utilized to maximally extract the cows mean information from the limited data. In addition, the BPNN model could be employed if many individual cow data were available, and the data dimensionality was high (multi-factor), with high

requirement on the predicting ability but no requirement to explain the model itself.

Effect of parity and time interval on the goodness of fit of the milk yield model: In this study, parity had significant influence on the goodness of fit of the predicted milk yield, while observation period partially affected the goodness of fit of the predicted milk yield. The Wood model had the highest prediction goodness of fit (0.9638) of the mean daily milk yield for cows at parity 1 within 30 days of lactation period, and the mean goodness of fit of the model was 0.9147. Like the Wood model, the Quadrinomial model also had the highest prediction goodness of fit (0.9890) of the mean daily milk yield for cows at parity 1 within 30 days of lactation period, and the mean goodness of fit of that model was 0.9364. In the BPNN model, the prediction goodness of fit of the mean daily milk yield for cows at parity 2 within 30 days of lactation period was the highest (0.9977), and the mean goodness of fit of that model was 0.9676. At the lactation period of 30 days, the degrees of fitting for the Wood model and Quadrinomial model in predicting the milk yield were dramatically improved compared with those at the lactation period of 10 days. The mean degrees of fitting of the three models followed the order of BPNN model > Quadrinomial model > Wood model. Thus, it could be seen that, even the same model had different degrees of fitting among different parities, which also demonstrated the unique features of milk yields for cows of different species at different parities. For cows at the same parity, the same model was used, and the variation range of the goodness of fit was small even though the different periods were used to predict the milk yield, and the means were almost the same.

In Table 4, the degrees of fitting of the three models for the mean daily milk yields at different parities and periods were over 0.83. The degrees of fitting of the three models had displayed the same trend; to be specific; the goodness of fit of the mean daily milk yield at 7 days of lactation period for parity 1 was the lowest, followed by that at 10 days of lactation period for parity 2. Such results might be related to the unstable milk yield of cows after delivery, as well as the great developmental difference among different cow individuals, especially at lactation periods 1 and 2 (Fox *et al.* 1999). For immature cow individuals, the negative balance of energy at the initial lactation period would inevitably result in significant fluctuations in milk yield, and such condition would be alleviated with the increase in parity.

The Wood model and the Quadrinomial model were likely to over-fit due to the small number of parity 1 samples. In addition, the great number of parity samples, together with the small amount of outlier observed values with unknown causes would also partially affect the final predicted results. At the observation period of 30 days, the Wood model and the Quadrinomial model had evidently improved degrees of fitting of the predicted milk yield compared with those at the interval of 10 days. Such results were relatively consistent with the fitted results of cow's

Table 4. Model performance

Parity (Period)	T_m	Y_m	Y_{avrm}	Wood model				Quadrinomial model				BPNN			
				T_p	Y_p	Y_{avrp}	R_a^2	T_p	Y_p	Y_{avrp}	R_a^2	T_p	Y_p	Y_{avrp}	R_a^2
1(7d)	2	36.05	29.52	9	34.60	29.73	0.8373	9	34.91	31.55	0.8338	2	36.15	29.64	0.8905
2(7d)	3	40.86	31.09	4	38.10	30.89	0.9541	6	39.33	32.47	0.9665	4	40.96	31.09	0.9828
3(7d)	8	44.56	32.43	8	39.86	31.77	0.9254	9	42.30	32.10	0.9480	6	43.27	32.53	0.9821
1(10d)	8	34.64	29.49	6	32.43	28.90	0.8977	7	34.95	29.33	0.9236	8	34.60	29.57	0.9222
2(10d)	3	40.83	31.08	7	38.57	27.02	0.8542	6	39.37	30.98	0.9042	4	41.01	31.23	0.9894
3(10d)	6	45.40	32.31	7	39.99	31.71	0.8964	6	42.15	32.27	0.9218	5	43.48	32.55	0.9736
1(30d)	3	34.38	29.73	2	33.72	29.48	0.9638	3	34.54	29.73	0.9890	2	34.82	29.67	0.9950
2(30d)	2	40.04	31.46	2	38.04	30.89	0.9548	2	38.34	30.60	0.9757	2	40.06	30.15	0.9977
3(30d)	2	43.91	33.03	2	40.83	31.84	0.9489	2	41.17	31.76	0.9647	1	44.76	31.03	0.9752

T_m stand for occurrence time of lactation peak (observed value); T_p stand for occurrence time of lactation peak (predicted value); Y_m stand for maximum milk yield (observed value); Y_p stand for maximum milk yield (predicted value).

milk yields (3 varieties) from a Xinjiang Cattle Farm (Wang *et al.* 1999) using 7 models. Gupta *et al.* (2016) also reported that monthly milk yield data was suitable for gamma type functions and mixed log function.

To sum up, it is feasible to observe the cows at the period of 30 days when the models were used to predict the milk yield. The current research would contribute to save the labor force and detection cost, while enhance the production efficiency. By contrast, the BPNN model could well utilize the observed data at all periods.

It could be observed that, for cows at the same species, the milk yield prediction model variables could be different under different factors such as production environment and feeding management, although the basic trend was consistent. To accurately construct the milk yield prediction model of the pasture and to predict the lactation level, the production data in the pasture should be used for fitting, and the related factors that affected the milk yield prediction model parameters should be carefully analyzed, to attain better effect. The variation range of the goodness of fit for the three models in predicting the milk yields of Chinese Holstein cows in a Yunnan standardized pasture was 0.7039–0.9982. Different models and parities would affect the goodness of fit for the predicted milk yield, and different milk yield prediction models and parities had significant influence on the peak milk yield and lactation peak day, while time interval had no obvious influence on the goodness of fit of the milk yield. For group data (mean), the Quadrinomial model was likely to over-fit under the single factor condition with a small sample size, while the Wood model had a good goodness of fit, and the BPNN model was more suitable for multi-factor and large sample data analysis.

ACKNOWLEDGEMENTS

This research was funded by the project “Major science and technology in Yunnan Province/2018ZF012”.

REFERENCES

- Bilal G, Cue R I and Hayes J F. 2016. Genetic and phenotypic associations of type traits and body condition score with dry

matter intake, milk yield, and number of breedings in first lactation Canadian Holstein cows. *Canadian Journal of Animal Science* **96**(3): 434–47.

Cinar M, Serbest U, Ceyhan A and Gorgulu M. 2015. Effect of somatic cell count on milk yield and composition of first and second lactation dairy cows. *Italian Journal of Animal Science* **14**(1): 3646.

Deen A U, Tyagi N, Yadav R D, Kumar S, Tyagi A K and Singh S K. 2019. Feeding balanced ration can improve the productivity and economics of milk production in dairy cattle: a comprehensive field study. *Tropical Animal Health and Production* **51**(4): 737–44.

Fox D G, Van Amburgh M E and Tylutki T P. 1999. Predicting requirements for growth, maturity, and body reserves in dairy cattle. *Journal of Dairy Science* **82**(9): 1968–77.

Gaines W L. 1928. *The energy basis of measuring milk yield in dairy cows*. Bulletin (University of Illinois (Urbana-Champaign campus). Agricultural Experiment Station no. 308.

Gupta A, Singh M, Gandhi R, Singh A, Dash S and Dash S. 2016. Comparison of different lactation curve models in Sahiwal cattle up to fourth parity using monthly test day milk yields. *Indian Journal of Dairy Science* **69**(4): 460–66.

Hecht-Nielsen R. 1992. Theory of the backpropagation neural network, pp. 65–93. *Neural Networks for Perception*. Academic Press.

Heins B J, Chester-Jones H, Ziegler D, De Ondarza M B, Schuling S E, Ziegler B and Sniffen C J. 2016. 1243 Relationships between early life growth and first lactation performance of Holstein dairy cows. *Journal of Animal Science* **94**(suppl_5): 598–99.

Johnson I R, France J and Cullen B R. 2016. A model of milk production in lactating dairy cows in relation to energy and nitrogen dynamics. *Journal of Dairy Science* **99**(2): 1605–18.

Kristensen T, Jensen C, Østergaard S, Weisbjerg M R, Aaes O and Nielsen N I. 2015. Feeding, production, and efficiency of Holstein-Friesian, Jersey, and mixed-breed lactating dairy cows in commercial Danish herds. *Journal of Dairy Science* **98**(1): 263–74.

Xiong Benhai, Ma Yi and Luo Qing-yao. 2010. Study on lactation curve models of chinese holstein for the second parity. *Scientia Agricultura Sinica* **43**(23): 4910–16. (in Chinese).

Macciotta N P P, Vicario D and Cappio-Borlino A. 2005. Detection of different shapes of lactation curve for milk yield in dairy cattle by empirical mathematical models. *Journal of Dairy Science* **88**(3): 1178–91.

- Penasa M, De Marchi M and Cassandro M. 2016. Effects of pregnancy on milk yield, composition traits, and coagulation properties of Holstein cows. *Journal of Dairy Science* **99**(6): 4864–69.
- Sánchez-Pérez R, Berasain M D M, Elghandour M M M Y, Mellado M, Diaz L G C, Cipriano M and Salem A Z M. 2018. Mathematical model to predict the dry matter intake of dairy cows on pasture. *Indian Journal of Animal Sciences* **88**: 598–601.
- Mote S, Chauhan D and Ghosh N. 2016. Effect of environment factors on milk production and lactation length under different seasons in crossbred cattle. *Indian Journal of Animal Research*. 50. 10.18805/ijar.9496.
- Sharma N, Narang R, Ratwan P, Kashyap N, Kumari S, Kaur S and Raina V. 2019. Prediction of first lactation 305-days lactation milk yield from peak yield and test day milk yields in crossbred cattle. *Indian Journal Of Animal Sciences* **89**(2): 200–03.
- Singh D K, Sahu S P and Teufel N. 2019. Increasing dairy cows productivity through new balanced concentrate feed: A Study in Bihar, India. *Indian Journal of Animal Nutrition* **36**(1): 11–16.
- Wang Ya-chun, *et al.* 1999. Dairy genetic analysis of curve fitting and model parameters. *Acta Veterinaria et Zootechnica Sinica* **30**(5): 399–404. (in Chinese)
- Wood P D P. 1967. Algebraic model of the lactation curve in cattle. *Nature* **216**(5111): 164–65.
- XIONG Ben-hai, *et al.* 2011. Study on lactation curve models of chinese holstein for the third parity. *Scientia Agricultura Sinica* **44**(2): 402–08. (in Chinese)