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Modeling and evaluation of lactation curve functions in Gir cattle

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ABSTRACT

The aim of the study was to model and evaluate five different non-linear lactation curve functions for their efficiency of explaining the variations in first lactation milk yield of Gir cows maintained in the farmers herds. Information on 4,334 fortnightly test day yields of 223 cows calved during the period from 2013-2017 were used for the study. Twenty fortnightly yields starting from the day 15 of lactation were used for fitting the five different non-linear mathematical models, viz. Exponential decline function (EDF), Gamma function (GF), Inverse polynomial function (IPF), Mixed log function (MLF) and Parabolic exponential function (PEF). The curve functions (a, b and c) with standard errors and different evaluation parameters, viz. adjusted R^2-value, Akaike information criterion (AIC), Bayesian information criterion (BIC), Durbin watson (DW) statistic and root mean square error (RMSE) were estimated by non-linear regression analysis using PROC NLIN procedure Newton method of SAS (SAS Institute, 2010). The adjusted R^2 value of the models ranged from 69.28 (exponential decline function) to 99.36% (parabolic exponential function). All the DW estimates were positive ranging from 0.1573 for exponential decline function to 0.7707 for parabolic exponential function. The RMSE (0.1453) and AIC (4.9152) estimates were also lowest for parabolic exponential function while highest for exponential decline function. Based on the results, it may be concluded that among five functions, parabolic exponential function is the best fitted lactation curve model followed by mixed log function, gamma function, inverse polynomial function while exponential decline function was the least efficient in explaining the variations in first lactation daily yield in Gir cattle.

Keywords: Fortnightly milk yields, Gir cattle, Lactation curve, Non-linear mathematical models

Gir breed is one among the few best dairy cattle of the country known for higher milk production. The home tract of the breed covers the Gir hills and forests of Kathiawar including Bhavnagar, Junagadh, Amerli and Rajkot districts of Gujarat state. Farmers maintain the Gir cattle mainly for milk production and these animals contribute significantly to the milk pool of the Gujarat state and country (Gaur *et al.* 2003).

Milk production is a physiological function which mainly depends on the genetic potential and body conformation and health of a cow. Generally, the daily milk yield of a cow does not follow consistent trend throughout the lactation as with the advancement of stage of lactation, the rate of milk secretion also changes. During the initial stage of lactation, the daily milk yield rapidly increases to reach the peak yield and thereafter gradually decreases till drying (Leon-Velard *et al.* 1995). The biometrical properties of lactation milk yield shows individual variation and knowledge on this will help to understand the persistency of the animal and formulate the managemental strategies to retain the optimum production potential without much affecting its health and body condition. Lactation curve is defined as the graphical representation of the variation in milk production of a cow over the time or day in yield. A typical lactation curve represents the ascending, persistent and descending stages of lactation. During the ascending phase milk yield increases steeply towards the peak yield followed by the persistent phase in which the higher milk yield achieved is sustained and the descending phase indicating the rate of decrease in the milk production till drying off of the animal. Thus, fitting the lactation curve of cattle helps to understand the persistency of individual animals, predicting the lactation yield, genetic evaluation of dairy cattle as well as designing suitable management and breeding strategies (Macciotta *et al.* 2005).

Different linear and non-linear mathematic models have been suggested by various workers for fitting the lactation curve of dairy cattle. Studies conducted by various workers suggest that the models differ in their efficiency of fitting the lactation curve in different cattle breeds as one model suited to one particular breed may not fit to other breed perfectly. Hence, it becomes imperative to find the best suitable model which can explain the variation in lactation milk yield effectively. Moreover, perusal of available literatures also revealed very scanty work on the lactation curve modeling of Gir cattle maintained under field conditions. In view of the above facts, present study was undertaken to develop and evaluate five different lactation curve functions to find the best suitable model for fitting

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the lactation curve in Gir cattle.

MATERIALS AND METHODS

Data for the present study was collected from 223 Gir cows calved at the farmers' herds during the period from 2013–2017. A total of 4,334 fortnightly test day yields starting from the day 15 of the lactation were used for fitting five different non-linear mathematical models as detailed below:

1. Exponential decline function (Brody *et al.* 1923):

$$Y_t = ae^{-ct}$$

2. Parabolic exponential function (Sikka 1950):

$$Y_t = a \exp (bt-ct^2)$$

3. Inverse polynomial function (Nelder 1966):

$$Y_t = t(a + bt + ct^2)^{-1}$$

4. Gamma function (Wood, 1967):

$$Y_t = at^b e^{-ct}$$

The constants can be derived by solving the above equation after transformation on the log scale

$$\text{In}(Y_t) = \text{In}(a) + b\text{In}(t)-ct$$

The milk yields up to week t is given by

$$Y_t = a \int_{0t01} t^b \exp(-ct)dt$$

Thus, the total 305-days milk yield as the integral of the average fortnightly milk yields.

5. Mixed log function (Guo and Swalve 1995):

$$Y_t = a + bt^{½} + c\log t + e_t$$

where, Y_t, Average daily yield in the t^{th} fortnight of lactation; a, initial milk yield after calving; b, ascending slope parameter up to the peak yield; c, descending slop parameter; t, length of time since calving; e_t, residual error.

The curve functions (a, b and c) with standard errors and different evaluation parameters for identification of the most suitable model, viz. adjusted R^2-value, Akaike information criterion (AIC), Bayesian information criterion (BIC), Durbin Watson (DW) statistic and root mean square error (RMSE) were estimated by non-linear regression analysis using PROC NLIN procedure Newton method of iteration SAS (SAS Institute, 2010). The estimation of different parameters for evaluation of the functions was done using the following formulae:

$$\text{RMSE} = \sqrt{\text{RSS}/(\text{n-p-1})}$$

$$R^2 \text{ value} = 1-(\text{RSS}/\text{TSS})$$

$$R^2_{adj} = 1-(n-1)/(n-p)\times(1-R^2) \text{ or } 1-MS_E/MS_T$$

$$\text{AIC} = n \times \log(\text{RSS}/n) + 2p$$

$$\text{BIC} = n \times \log(\text{RSS}/n) + p \times (\log(n))$$

$$\text{DW Statistic} = \frac{\sum_t^n (e_t - e_{t-1})^2}{\sum_{t=1}^n e_t^2}$$

where, RSS, Residual sum of squares; TSS, Total sum of squares; n, number of test days; p, number of parameters estimated in the model MS_E, Error mean square; MS_T,Total mean square; e_t is residual at time t; and $e_{t-1,}$ residual at time t–1.

The accuracy of the models was adjudged by the adjusted R^2 value, RMSE, AIC and BIC estimates. Higher the R^2 value and lower the RMSE, AIC and BIC values indicate better model fitting of the lactation curve. The autocorrelation among the residual estimates obtained in the analysis was decided by the DW statistic estimates which range from 0 to 4. Value around 0 indicates positive autocorrelation and value around 4 indicates negative autocorrelation while value around two indicates absence of correlation among the residual estimates. Further, the correlation coefficients between actual observed fortnightly test day yields and the yields predicted by different functions were also estimated.

RESULTS AND DISCUSSION

In the study, five non-linear lactation curve functions viz., Exponential decline function (EDF), Gamma function (GF), Inverse polynomial function (IPF), Mixed log function (MLF) and Parabolic exponential function (PEF) were fitted to derive the lactation curve parameters. The estimated lactation curve parameters or curve functions (a, b and c) along with standard errors are given in Table 1. Except the "b" parameter of MLF, all the other parameters of all the five functions were positive. Similar to the present findings, Bangar and Verma (2017) also reported a negative

Table 1. Lactation curve parameters and goodness of fit estimated by different functions for prediction of first lactation fortnightly test day yields in Gir cattle

Functions	Parameters of functions			Goodness of fit				
	a	b	c	Adjusted R^2 (%)	RMSE	AIC	BIC	DW
EDF	11.4440 (0.4968)	–	0.0282 (0.0042)	69.28	1.0062	8.7850	4.7734	0.1573
PEF	9.0687 (0.1031)	0.0428 (0.0027)	0.0037 (0.0001)	99.36	0.1453	4.9152	–72.6237	0.7707
IPF	0.0523 (0.0154)	0.0604 (0.0076)	0.0047 (0.0006)	81.31	0.7849	8.2881	–5.1648	0.2743
GF	9.3031 (0.3717)	0.3004 (0.0426)	0.0696 (0.0061)	92.91	0.4836	7.3195	–24.5374	0.2686
MLF	14.4779 (0.3842)	–5.8408 (0.5089)	5.7370 (0.6403)	92.97	0.4812	7.3094	–24.7387	0.3474

EDF, Exponential decline function; PEF, Parabolic exponential function; GF, Gamma function; MLF, Mixed log function; IPF, Inverse polynomial function; a, Beginning yield; b, Coefficient of increase; c, Coefficient of decrease; RMSE, Root mean square error; AIC, Akaike's information criteria; BIC, Bayesian Information Criteria and DW, Durbin-Watson statistic.

"b" parameter of –9.69 for MLF in Gir crossbred cattle. Positive parameters for EDF, PEF, IPF and GF in Holstein cows reported by Zadeh (2019) also support the results obtained in the present study.

The goodness of fit estimates viz. adjusted R^2 per cent, RMSE, AIC, BIC and DW statistic obtained for different lactation curve functions are given in Table 1. These estimates are useful to evaluate and identify the best suited lactation curve function which describes the salient features of the variations in milk yield during different stages of lactation. The adjusted R^2 per cent of the functions fitted in the study ranged from 69.28 for EPF to 99.36 for PEF. The GF (92.91%) and MLF (92.97%) had more or less similar accuracy of fitting while the IPF had the fitting accuracy of 81.31%. Contrary to present findings, Zadesh (2019) reported more or less similar accuracy estimates of 93 and 94% for all the four models, viz. EDF, PEF, IPF and GF in single and twin calved Holstein cows. Bangar and Verma (2017) reported adjusted R^2 estimates of 67.9 to 89.3% in primiparous Gir crossbred cows which was also supported by Boujenane (2013). Perusal of RMSE estimates of the functions revealed lowest value for PEF (0.1453) and highest for EDF (1.0062) and as expected models with higher R^2 value had the lowest RMSE value. Similar trends were noticed for the other two estimates, viz. AIC and BIC. The AIC ranged from 4.9152 for PEF and 8.7850 for EDF while the BIC ranged from –72.6237 for PEF and 4.7734 for EDF indicating the superiority of PEF in fitting the first lactation curve in Gir cattle. All the five DW statistic estimates obtained in the study were positive and ranged from 0.1573 for EDF to 0.7707 for PEF indicating positive autocorrelation among the residual estimates which may increase the chances of positive error in the predictions.

The fortnightly average daily milk yields recorded during the first lactation of Gir cows and the yields predicted by different functions are presented in Table 2 and depicted in Fig. 1. The actual average milk yield in first fortnight was 9.265 kg and average peak yield of 10.228 kg was observed in 4th fortnight. Similar to present finding, Bangar and Verma (2017) also reported an average peak yield of 10.08 kg in the second month of lactation in Gir crossbred cattle. Bahashwan (2018) also reported a peak yield of 9.72 kg of milk in 60 days of lactation in Dhofari cattle. The lactation curve of actual observed test day yields showed gradual increase in the initial phase of lactation till the peak period and thereafter declined gradually till drying. Similar to the present findings, Farhangfar and Rowlinson (2007) also observed a persistent lactation curve in Iranian Holstein heifers. On the contrary, Bahashwan (2018) observed an inconsistent lactation curve in Dhofari cattle as the peak yield of 9.72 kg observed in 60 days has reduced to 5.11 kg in 120 days which reduced further to 2.69 kg in 210 days and increased to 3.91 kg in 270 days which again reduced 0.50 kg in 305 days of lactation. Perusal of the lactation curve observed in the present study showed a fairly low rate of decrease from the achieved peak yield till the 9th fortnight followed by a slow and gradual decrease in milk yield which indicated that the Gir animals are more persistent in their first lactation milk production. The variations in shape of lactation curves obtained in different studies (Tekerli *et al.* 2000, Fathi Nasri *et al.* 2008, Farhangfar and Rowlinson 2007, Rashia Banu 2010, Zadeh,

Table 2. Estimated daily milk yield and prediction errors for fortnightly test day milk yields using different functions

Test day	Actual	EDF		PEF		IPF		GF		MLF	
		Predicted	Error	Predicted	Error	Predicted	Error	Predicted	Error	Predicted	Error
1	9.265	11.126	1.861	9.431	0.166	8.516	–0.749	8.677	–0.588	8.637	–0.628
2	9.957	10.816	0.859	9.736	–0.221	10.417	0.460	9.967	0.010	10.194	0.237
3	10.159	10.515	0.356	9.979	–0.180	10.869	0.710	10.500	0.341	10.664	0.505
4	10.228	10.223	–0.005	10.152	–0.076	10.826	0.598	10.678	0.450	10.750	0.522
5	10.177	9.939	–0.238	10.254	0.077	10.585	0.408	10.650	0.473	10.651	0.474
6	10.197	9.662	–0.535	10.281	0.084	10.261	0.064	10.493	0.296	10.450	0.253
7	10.057	9.394	–0.663	10.234	0.177	9.908	–0.149	10.251	0.194	10.188	0.131
8	9.952	9.132	–0.820	10.112	0.160	9.549	–0.403	9.952	0.000	9.887	–0.065
9	9.824	8.878	–0.946	9.920	0.096	9.199	–0.625	9.617	–0.207	9.561	–0.263
10	9.642	8.631	–1.011	9.660	0.018	8.861	–0.781	9.259	–0.383	9.218	–0.424
11	9.348	8.391	–0.957	9.338	–0.010	8.540	–0.808	8.886	–0.462	8.863	–0.485
12	9.032	8.158	–0.874	8.961	–0.071	8.236	–0.796	8.508	–0.524	8.501	–0.531
13	8.641	7.931	–0.710	8.538	–0.103	7.949	–0.692	8.129	–0.512	8.134	–0.507
14	8.189	7.710	–0.479	8.074	–0.115	7.678	–0.511	7.753	–0.436	7.764	–0.425
15	7.741	7.496	–0.245	7.581	–0.160	7.423	–0.318	7.382	–0.359	7.393	–0.348
16	7.200	7.287	0.087	7.066	–0.134	7.183	–0.017	7.021	–0.179	7.021	–0.179
17	6.635	7.085	0.450	6.538	–0.097	6.957	0.322	6.669	0.034	6.650	0.015
18	5.895	6.888	0.993	6.005	0.110	6.743	0.848	6.328	0.433	6.280	0.385
19	5.347	6.696	1.349	5.476	0.129	6.542	1.195	5.999	0.652	5.911	0.564
20	4.774	6.510	1.736	4.957	0.183	6.351	1.577	5.682	0.908	5.544	0.770

Actual, Actual observed test day yields; EDF, Exponential decline function; PEF, Parabolic exponential function; IPF, Inverse polynomial function; GF, Gamma function; MLF, Mixed log function.

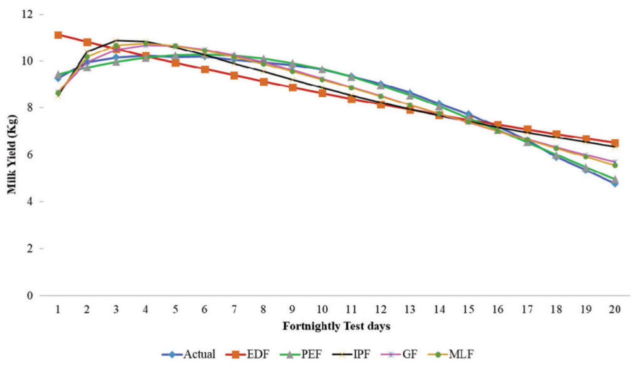

Fig. 1. Actual and predicted fortnightly test day milk yields or different lactation curve functions. EDF, Exponential decline function; PEF, Parabolic exponential function; GF, Gamma function; MLF, Mixed log function; IPF, Inverse polynomial function.

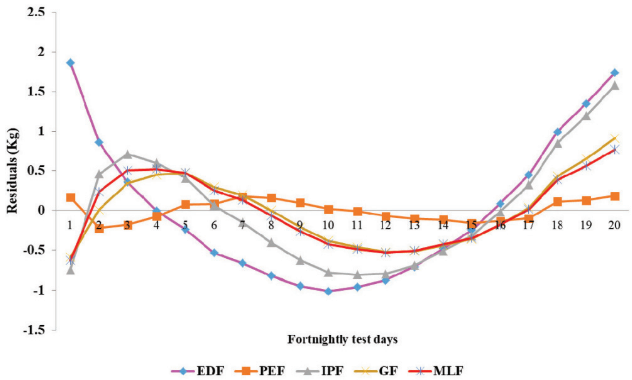

Fig. 2. Residuals (kg) of predicted fortnightly test day milk yields for different functions. EDF, Exponential decline function; PEF, Parabolic exponential function; GF, Gamma function; MLF, Mixed log function; IPF, Inverse polynomial function.

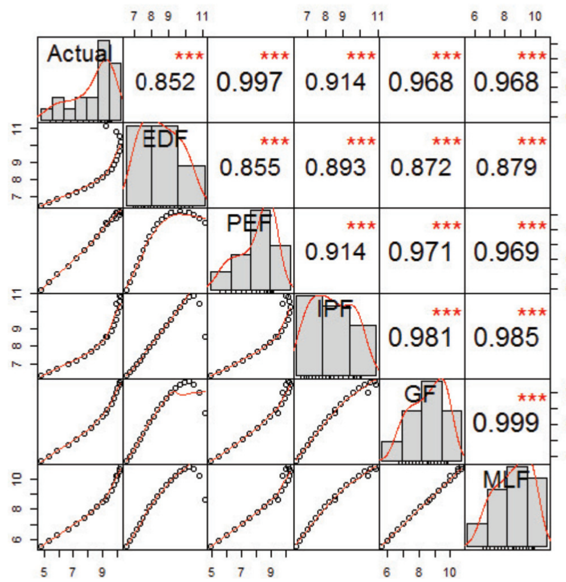

Fig. 3. Correlations coefficients between actual observed daily milk yields and milk yields predicted by different functions. Actual, Actual observed test day yields; EDF, Exponential decline function; PEF, Parabolic exponential function; IPF, Inverse polynomial function; GF, Gamma function; MLF, Mixed log function; ***Significant (P<0.001).

2014; Bangar and Verma 2017, Zadeh, 2017, Bahashwan 2018) may be attributed to various factors like herd, breed, parity, health status of animal, calving season, management etc. The results of present study revealed that the first lactation milk yield of Gir cows followed the trend of a defined standard lactation curve.

Fig. 1 depicts the lactation curves for actual and predicted fortnightly yields of different non-linear mathematical functions. The lactation curve of EDF showed a typical straight line as the predicted milk yield has decreased gradually from first (11.126 kg) to 20th fortnight (6.510 kg) indicating its poor prediction efficiency. Gahlot *et al.* (1988) also found that the EDF could not explain the variations in lactation daily yield in Rathi cattle which might be due to the exclusion of inclining function (b) in the EDF model. However, the PEF model explained the variations effectively as shown in Fig. 2 which was contradicted by the findings of Singh and Bhat (1978) and Gahlot *et al.* (1988) who observed poor fitting of PEF during the initial stage of lactation till reaching the peak yield. The poor fitting of lactation curve in Hariana halfbreds by PEF function was also reported by Yadav and Sharma (1985).

Five different lactation curve functions were used to predict the daily milk yields of Gir cows and the prediction errors calculated as the difference between actual and predicted yields (Table 2). The EDF overestimated the predicted yields during the first (ascending) and third (declining) stage of lactation. Since the EDF was proposed mainly to describe the third declining phase of lactation and not the second phase, i.e. the stagnation phase, this model could not able to explain the variations effectively as it produces a consistently declining lactation curve. Perusal of Table 2 shows that the prediction errors were lower for PEF as compared to other four functions indicating that this model has best explained the variations present in the fortnightly milk yields of Gir cows. However, Gahlot *et al.* (1988) reported that the PEF proposed by Sikka provided a good fit for curves of first lactation cows, but it was less efficient for subsequent lactations, being symmetric around the peak of lactation. In the present study, the Sikka model produced better fits than the Nelder model of inverse polynomial function which is supported by the results of Olori *et al.* (1999) who opined that the Nelder model under-predicts the milk production around the peak and over-predicts it immediately afterwards. The results also revealed that the gamma function and mixed log function had similar prediction efficiencies.

Fig. 2 depicts the residual errors (kg) of different fortnightly milk yields predicted by different functions. The predicted errors were highest for the EDF followed by IPF while GF and MLF had more or less similar and lower prediction errors. EDF model predicted the peak yield of 11.126 kg at the first fortnight itself while the PEF (10.281 kg) in 6th fortnight, IPF (10.869 kg) in 3rd fortnight, GF (10.678 kg) in 4th fortnight and MLF (10.750 kg) in 4th fortnight. The peak yield of 10.281 kg predicted by PEF was closest to the actual peak of 10.228 kg observed in the present study. Perusal of Fig. 2 revealed that the PEF had the lowest prediction errors compared to other models as the data points plotted were almost closure to the X-axis while GF and MLF had more or less similar data points indicating the similarity in their prediction efficiency. The Pearson's correlation coefficients between actual and daily milk yields predicted by different functions are presented in Fig.3 which ranged between 0.852 and 0.999. In general, all the estimates were significantly correlated ($P<0.001$) however, EDF had lowest correlation with actual and yields predicted by different methods reflecting its lower efficiency in explaining the variations in the first lactation milk yield of Gir cattle.

The results of the present study revealed that the parabolic exponential function (PEF) was the best non-linear mathematical function for explaining the variations in the first lactation milk yield of Gir cattle followed by mixed log function (MLF) and gamma function (GF). The exponential decline function (EDF) was the least efficient in fitting the lactation curve in Gir cattle followed by inverse polynomial function (IPF).

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