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Using mathematical models for lactation curve of milk and fat production in Iranian Holstein cows

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ABSTRACT

An appropriate mathematical model is required to fit into lactation curves to study the environmental and biological variables that affect milk production. This study examined the suitability of 5 mathematical models (linear and nonlinear form of Wood, Wilmink, Guo and Swalve and inverse polynomial function of Nelder) for lactation curve traits (milk, fat yield and fat content) of 305-d milk yield of Iranian Holsteins. Models were compared based on R^2, RMSE, and MSPE criteria by using NLIN and REG procedures in SAS. Using GLM procedure in SAS, factors of herd, calving year, calving season, season of production, age at calving, age at test and days in milk were significant on daily milk yield and the fat content during production period ($P < 0.01$). The study revealed that the linear form of Wood model and Wilmink function were the best for describing milk, fat yield and fat content during 305-d of lactation.

Key words: Environmental factors, Fat concentration, Lactation curve, Mathematical function

The lactation curves provide useful information for breeding programs and also for developing suitable management decisions and production strategies at the farm level (Ruiz *et al.* 2000). Linear and nonlinear methods were used to estimate the parameters of production peak, and inclining and declining phases of milk and fat production over the course of lactation (Farhangfar *et al.* 2000). The incomplete gamma function or Wood function proved relatively powerful in fitting the observed daily milk yield (Rekik *et al.* 2003). Val-Arreola *et al*. (2004) found that the Dijkstra equation was the best option for description of lactation curve in comparison with Wood, Rook, Gaines and Pollot functions. Wilmink model was found significantly more effective than other functions for fitting of fat per cent curve (Quinn *et al.,* 2006). Guo and Swalve model tend to underestimate peak yield, while overestimating the post–peak yield (Olori *et al.* 1999). Fat yield reached an early peak and then declined but more slowly than did the milk (Schutz *et al.* 1990). Herd, calving year, calving season, calving age, and parity influence milk and fat production (Ruiz *et al.* 2000, Tekerli *et al.* 2000). Changes in management, feeding regime, and other environmental factors are responsible for some of milk yield variations through years (Hatungumukama *et al.* 2008). This study seeks to firstly evaluate the 5 mathematical functions for describing milk yield, fat content, and fat yield of Iran primiparous Holsteins; and secondly to asses some of the environmental factors affecting daily milk yield (DMY) and fat content (FC).

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MATERIALS AND METHODS

Data: First lactation records (214159) of Iranian Holstein cows between 1999 and 2008 were used as data. All cows underwent milking 3 times a day. The following editions were applied to the data set: (a) age at the first calving was between 18.3 to 30.5 months, (b) the number of test day records for individual cows was between 5 and 10, (c) all lactations were > 200 d < 305, (d) minimum and maximum of milk yield and fat percentage in the data set were respectively 2.1–64.6 and 2 – 5 kg. After editing, the dataset consisted 111, 059 monthly test days. The summary of descriptive statistics of the final data is presented in Table 1.

Mathematical models

Five different functions were selected and applied in this study for fitting milk yield and fat concentration (Table 2). These models provided appropriate description and prediction of milk yield and fat concentration (Ruiz *et al.* 2000, Farhangfar *et al.* 2000, Quinn *et al.* 2006). Gamma function (Wood 1967) is one of the most popular models

Table 1. Descriptive statistics of the data set

Variable		Min	Max	CV(%)
Number of monthly records	111059			
Number of herds	264			
Number of cow	14901			
Mean of days in milk (d)	271.04±24.15	200	305	8.91
Mean of milk yield (kg)	28.87±6.82	2.1	64.6	23.64
Mean of fat percent (%)	3.35±0.68	2	5	20.57
Mean of fat yield (kg)	0.949±0.0033	0.78	1.251	5.99
Mean of age at first calving (month)	24.76±1.42	18.38	30.49	5.75

Min, minimum; Max, maximum, CV, coefficient of variation.

Table 2. Equations used to describe the lactation curve of dairy cows

Equation	functional form
Wood (nlin)	$Y_t = at^b e^{-ct}$
Wood (lin)	$\log Y_t = \log a + b \log t - ct$
Wilmink	$Y_t = a + be^{-kt} + ct$
Guo and Swalve	$Y_t = a + bt^{1/2} + c \log t$
Nelder	$Y_t = a + bt^{-1} + ct$

Y is milk yield, *t* is time of lactation and *a, b, c, d* are parameters of the models.

used to describe the lactation curve. In this three-parameter model, *a* is linked to milk yield at the beginning of lactation, *b* to ascending phase before peak yield, and *c* to the decreasing phase after peak yield. This model can be linearized by a simple logarithmic transformation and easily solved by ordinary least square analysis by multiple regressions (Tekerli *et al.* 2000).

Wilmink (Wilmink 1987) introduced 2 lactation models. In the first model, a quadratic term was added and an adjustment was made to the exponential term. This model (WL) was then adjusted to obtain the second model by dropping the quadratic term from the initial function. In this model, *a* is the level of production, *b* is the initial raise to peak, and *c* is the decrease rate after peak. Factor K was set equal to 0.05 and was associated to the time of yield peak. Guo and Swalve (Guo *et al.* 1995) introduced a model, referred to as the mixed log model. The other model was the inverse polynomial model (IQP) of Nelder which was applied to dairy cattle by Yadav *et al.* (1977). Fitting curves were carried out with marquardt algorithm in the NLIN procedure in SAS 9.1. Criteria for the selection of lactation models were coefficient of determination (R^2), root mean square error (RMSE) and mean square prediction error (MSPE) (Val-Arreola *et al.* 2004).

Analysis of the environmental effects

The effects of the environmental factors and covariates on milk yield (model 1) and fat content (model 2) were investigated by analysis of variance (PROC GLM) and the results were given as a least-square means (LSM) of daily milk yield (DMY) with standard errors. The models are as follows:

Model 1:

$$Y_{ijk} = \mu + H_i + CY_j + CS_k + b_m (AGC_{ijk} - \ddot{AGC}) + \sum_{R=1}^{2} b_R (AGE_{ijk} - \bar{AGE}) + b_n (DIM_{ijk} - \ddot{DIM}) + e_{ijk}$$

where Y_{ijk} is the daily milk yield of nth animal affected by ith herd (264 herd), j^{th} calving year (10 years from 1999 to 2008), K^{th} season of calving (four seasons: spring, summer, autumn and winter), b_R is the linear and quadratic coefficients regression of age at test (covariate), b_m and b_n are the linear regression coefficients of age at calving and days in milk (covariates), and e_{ijk} is the random effect of residual with expectation and variance equal to 0 and δ_e^2 respectively. μ is the overall mean and H_i, CY_j, CS_k, *AGC*, *AGE* and *DIM* are effects of herd, calving year, season of calving, age at calving, age at test day and days in milk, respectively.

Model 2:

$$Y_{ijkm} = \mu + H_i + CY_j + CS_k + TS_m + \sum_{R=1}^{2} b_R (AGE_{ijk} - \ddot{AGE}) + e_{ijkm}$$

where and TS_m are the daily fat content and effect of season of milk production respectively and the other factors are similar to the model 1. Least square means of factors affecting milk trait and fat concentration were obtained along with the analysis of variance using LSMEANS option.

RESULT AND DISCUSSION

Model selection for milk yield, fat content and fat yield

Estimated values of different model parameters and adequacy are shown in Table 3. In these models, the range of R^2 for milk yield was between 0.81 and 0.97. The highest and lowest R^2 of milk yield were related to inverse polynomial and linear form of Wood functions, the best RMSE value was found using linear form of Wood, and inverse polynomial functions provided the worst RMSE value. MSPE value of Wilmink, Guo and Swalve, Nelder, and the linear form of Wood functions are significantly close together and are dramatically more effective than that of the nonlinear Wood function. Nonlinear form of Wood overestimates milk yield, especially in late lactation. The best value of MSPE is related to the linear form of Wood model. Days and the peak yield in all models were close to 31 kg except for linear form of Wood which is 30.72 kg. The

Nelder, Wilmink and Guo and Swalve functions estimated the peak day earlier than the other models. Actual peak yield and day were 32.10 kg and 70 days respectively. The widely used nonlinear form of Wood model had its peak 10 days later than the actual peak. Regarding R^2, RMSE, and MSPE, the results showed that the linear form of Wood produces the best fit for milk yield in primiparous Iranian Holstein. Farhangfar *et al.* (2000) found that the variation of monthly milk yield during the first lactation of dairy cows would be more accurately explained by the linearity of the incomplete gamma function. Actual average and estimated average milk yield of the functions are presented in Fig. 1.

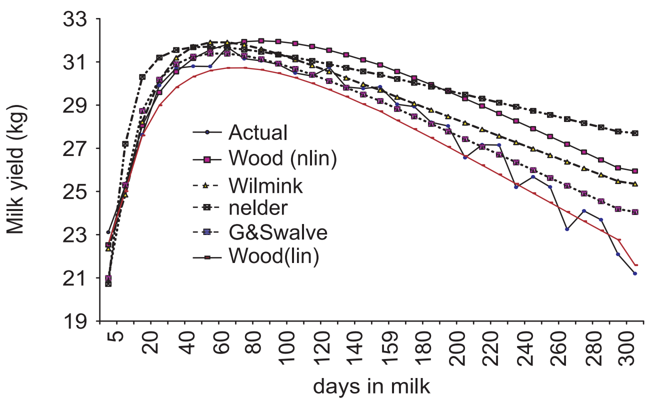

Fig. 1. Actual and estimated milk yield using different models.

Most models overestimate milk yield in mid and end lactation. Nelder, Wilmink, and especially nonlinear form of Wood functions tend to overestimate milk yield during peak yield and late lactation. This is in agreement with Scott *et al.* (1996). Nelder function started at low yield and peaked earlier than usual (Fig. 1). Kumar and Bhat (1976) found that this model gave good fit of lactations that started at low yield and peaked earlier than other models. Batra *et al.* (1986) observed that the Nelder function provided a better fit than the gamma function based on R^2, when weekly milk data was utilized. These results were not among the ones obtained in this study. The reason might be the fact of using monthly milk data in this study instead of weekly recording.

Linear form of Wood compared to the nonlinear form, under-predicted milk yield during the peak. This model, however, controls the variation of milk yield in late lactation. Ruiz *et al.* (2000) showed that in Wood function if linear regressions were made by least square, the transformed milk would yield proportionally to the square of untransformed data and improve the fit as judged by a decrease in the MSPE. If functions for different transformations of data are used, the R^2 is not a valid criterion for comparing functions (Grossman *et al.* 1988, Gipson *et al.*1990). But it seems that RMSE is not a good criterion for comparing functions in linear and non linear functions because the parameters are basically affected by transformation.

Like other studies (Wood 1967, Schutz *et al.* 1990, Stanton *et al.* 1992) curves for fat percentage were characterized by an early decline, near 70- day post calving followed by a steady increase to the end of lactation. Advantages of fit statistics in fat concentration using R^2, RMSE, and MSPE are presented in Table 3. R^2 varies between 0.61 and 0.78, and the best co-efficiency of determination is related to that of Wilmink function. Amount of R^2 in fat concentration is lower than milk yield. Considerable day-to-day variation was noted in the fat content of milk (Whittemore *et al.* 1987).

The RMSE for nonlinear Wood, Wilmink, Guo and Swalve and Nelder functions are closely related but this value for linear form of Wood is 0.207 because of linear transformation. Therefore, the best criteria for comparison of functions for milk yield and fat concentration is R^2 and MSPE. The minimum value of MSPE for fat concentration was shown in Wilmink model, too. The actual average and the estimated average fat concentration by the functions are presented in Fig. 2. The percentage of milk fat is typically a convex function overtime which means that it counteracts inversely to yield. Deviation of nonlinear and linear forms of Wood as well as inverse polynomial function is higher than the actual fat concentration curve but Guo and Swalve and Wilmink functions produce reasonable fit. The linear form of Wood is the worst model for fitting fat concentration in milk during days.

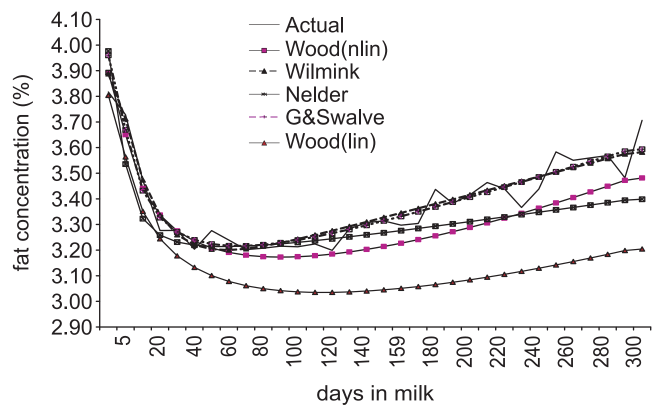

Fig. 2. Actual and estimated fat concentration using different models.

Finally, the result of fitting functions for evaluation of traits showed that linear form of Wood for milk yield is accurate in describing milk yield but, but the recommended model which is satisfactory in describing fat yield and concentration is the Wilmink equation. Quinn *et al.* (2006) demonstrated that the Wilmink model would be the best for fitting fat concentration in lactation. Wood model also showed that in linear form were the poorest when estimating fat concentration. Fig. 3 showed the actual and the estimated fat yield in milk during days-fat yield peaked at approximately 60 days in milk in the first parity. Fat yield peaked earlier than milk (Table 3) that peaked by day 10 in

Table 3. Estimated parameters (mean ± SE) for milk yield, fat concentration, and fat yield in different functions

	Function				
	Wood(nlin)	Wilmink	Guo andSwalve	Nelder	Wood (lin)
			Milk yield		
A	17.03 (0.12)	34.47 (0.05)	11.17 (0.21)	34 (0.05)	2.78 (0.007)
B	0.18 (0.002)	–15.37 (0.16)	–2.31 (0.02)	–65.91 (0.81)	0.19 (0.002)
C	0.002 (2E–5)	–0.03 (3E–4)	9.31 (0.09)	–0.02 (3E–4)	–0.002(2E–5)
Peak yield	31.97	31.92	31.34	31.70	30.72
Peak day	90	65	75	58	80
R^2	0.95	0.93	0.92	0.81	0.97
RMSE	6.460	6.466	6.469	6.520	0.260
MSPE	3.107	1.343	1.310	1.343	1.249
			Fat concentration		
A	4.55 (0.02)	3.04 (0.005)	4.62 (0.02)	3.08 (0.005)	1.493 (0.005)
B	–0.1 (0.001)	1.07 (0.01)	0.13 (0.002)	4.46 (0.08)	–0.100 (0.001)
C	–0.001(18E-5)	0.001 (3E-5)	–0.60 (0.009)	0.001 (3E-5)	0.001 (19E-5)
R^2	0.76	0.78	0.76	0.61	0.73
RMSE	0.677	0.676	0.677	0.680	0.207
MSPE	0.0183	0.0070	0.0074	0.022	0.089
			Fat yield		
A	0.778 (0.007)	1.066 (0.002)	0.703 (0.01)	1.633 (0.002)	–0.328 (0.01)
B	0.081 (0.002)	–0.0232 (0.008)	–0.04 (9E–4)	–1.02 (0.037)	–0.091 (0.002)
C	0.0012 (2E-5)	-0.00066 (14E-5)	0.152 (0.0042)	–0.0006 (1E-5)	–0.001 (3E-5)
Peak yield	1.0087	1.0152	1.0117	1.0130	1.860
Peak day	64	59	54	41	41
R2	0.96	0.97	0.64	0.61	0.90
RMSE	0.277	0.276	0.277	0.277	0.31
MSPE	0.001192	0.001192	0.001245	0.001253	0.06410

a, b and c are parameters that define the scale and shape of the curve in the model; R^2, coefficient of determination; RMSE , root mean square error; MSPE, mean square of prediction error.

dairy cows. Peak fat day for Nelder and linear form of Wood models are the same and the nonlinear form of Wood showed the latest peak fat yield (Table 3).

Evaluation of factors affecting daily milk yield (DMY)

The overall mean of daily milk yield was 28.87 kg/d. The effects of herd, calving year, calving season, age at calving, age at test (linear and quadratic), and milk days were significant on daily milk yield. This model explained 48% of the total variability observed in DYM.

Similar factors are reported in the literature to affect milk production (Jamrozik *et al.* 1997, Tekerli *et al.* 2000, Farhangfar *et al.* 2007). The minimum least square means (LSM) for DMY were observed in cows calved in 1999 with 24.71 ± 0.11 kg/d and increased gradually and reached the maximum (31.34 ± 0.10 kg/d) in 2008.

About the calving year, a slow increase from 1999 to 2008 showed improvement in knowledge of feeding, breeding, and economic management of dairy herd. The maximum LSM of milk yield was in autumn and winter (28.31 ± 0.08 and 28 ± 0.079) and in summer and spring milk yield decreased (27.61 ± 0.078 and 26.9 ± 0.079 respectively). Tekerli *et al.* (2000) found fall and winter to have higher total milk yield. Syrstad (1965) declared that both age and season of calving would influence milk yield and fall and near-winter calving appeared to be the optimal for calving. The results showed that calving seasons significantly influenced lactation, which is in agreement with the results given by Stanton *et al.* (1992) and Tekerli *et al.* (2000).

The result of phenotypic study of lactation curve in Iranian Holsteins by Farhangfar *et al.* (2007) indicated that cows calving in spring and summer would produce lower total lactation milk yield than those calving during autumn and winter. Reduction of age at the first calving appears to have a negative effect on the percentage of the first lactation milk yield and fat (Pirlo *et al.* 2000). Age at calving is more important in earlier lactations (Khan and Shook 1996). Cows calving younger have lower production, especially in the first lactation (Mostert *et al.* 2001). Koncagul and Yazgan (2008) reported that cows calving in summers would produce lower milk yield during the peak of lactation and so there would appear a total milk decrease in this season. Age at first calving (covariate) showed a significant impact on milk production (Ojango *et al.* 2009). Gara *et al.* (2009) also reported that age at first calving of 22 to 24 months is optimal for milk yield.

Evaluation of factors affecting fat concentration (FC)

Environmental factors influencing fat percentage are similar to those of milk production. However, little information is available for factors affecting fat percentage curve during lactation; model 2 showed these factors (herd, calving year and season, season of production and age at test in linear and quadratic forms).

Difference in fat concentration among herds is associated to management practice which in turn influences milk production and fat percentage. Like milk yield, fat percentage would change dramatically from calving year of 1999 to 2008 and this increase is reported to be significant ($P < 0.01$). The minimum fat percentage (3.14 ± 0.0130) is associated to calving year of 1999 and the maximum (3.42 ± 0.0120) is in 2008.

LSM of fat percentage were the highest for cows calving in summer and the lowest in winter. Fat percentages are relatively close together throughout lactation in different seasons of calving (3.32, 3.33, 3.32 and 3.30 respectively). Stanton *et al.* (1992) pointed out that the influence of different calving season variations is relatively consistent on fat percentage.

LSM of fat concentration was significantly different between seasons of production ($P< 0.001$). Seasonal variations in milk compositions were commonly observed among dairy cattle in dry regions. In general, fat concentration was the highest in winter (3.44 ± 0.0092) and the lowest in summer (3.21 ± 0.0091). Seasons with high temperatures have negative influence on milk traits, mainly fat content.

The TD records are effective in generating lactation curve estimates for milk and fat yield and fat concentration. Five standard lactation curve functions were used for prediction of milk and fat yield and fat concentration during days in milk in Iranian primiparous Holsteins. Nonlinear form of Wood equation seems to be able to make the best description of milk yield. The results of fitting these functions for fat yield and fat concentration showed that Wilmink model is appropriate in describing these traits during days in milk. The effects of environmental variables, herd, calving year, calving season, season of production, age at calving, and age at test (covariates) on the main lactation curve traits (milk yield and fat concentration) were significant. The highest milk yield and fat concentration were associated with cows that calved autumns and summers respectively. The effect of calving year was significant on both DMY and FC.

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