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Fitting of fox model with autoregressive of order one using expected value parameters

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Fishing plans can be expressed in terms of surplus production. Although Schaefer model is most widely used surplus production model, it provides a curve of symmetrical parabola, which need not necessarily be always true to the natural phenomena since maximum sustainable yield (MSY) may vary considerably depending on temporal and spatial changes in the ecosystem. However, in Fox model, catch increases asymmetrically towards the total biomass with increasing effort, which is more realistic than Schaefer model, and hence biologists prefer to Fox model. Further, when we are dealing with catch-effort fisheries data, which are observed over continuous time periods, the data points are generally correlated among themselves. Prajneshu and Ravichandran (2003) emphasized the importance of reparameterization of the parameters for fitting of nonlinear Fox model in fisheries to reduce large correlations among the parameter estimates. Thus, by examining the presence of autocorrelation in the observations, in the present study a method of fitting a Fox model with serially correlated error structure using expected value parameters is proposed which is illustrated with an example, considering the catch-effort data observed from Gobindsagar reservoir, India.

The Fox model (1970) facilitates estimation of MSY and the optimum fishing effort for harvesting the MSY (E_{MSY}). The equilibrium Fox model is given by

$$C_t = KE_t \exp(-E_t/r) \qquad (1)$$

where, C_t and E_t are catch and effort at time t; r is the intrinsic growth rate; K is the carrying capacity and correspondingly, MSY = rK/e; (e=2.71828) and E_{MSY}=r.

As we are dealing with time-series data, it is, therefore, required to check for the validity of the above model by examining the independency assumption of error term. The Durbin-Watson test can be employed for the said purpose and is based on the assumption that the errors (ε_t's) follow autoregressive of order one. The corresponding test statistic 'd' is defined as

$$d = \frac{\sum_{t=2}^{n} \left(\varepsilon_t - \varepsilon_{t-1}\right)^2}{\sum_{t=1}^{n} \varepsilon_t^2}, \; 0 \le d \le 4. \qquad (2)$$

A statistic 'd' value ranges between 0 and 4. A value of 'd' near 2 indicates little autocorrelation; a value toward 0 indicates positive autocorrelation while a value toward 4 indicates negative autocorrelation.

To handle a situation when there is an evidence for the presence of autocorrelation, an autoregressive (AR) error term ε_t of order one is added to the right hand side of above equation (1):

$$\varepsilon_t = \Phi\varepsilon_{t-1} + u_t; \; |\Phi| < 1 \qquad (3)$$

where, u_t is independently and normally distributed with zero mean and constant variance and Φ denotes the autoregressive parameter. Incorporating an AR (1) additive error structure, the Fox model becomes:

$$C_t = KE_t \exp(-E_t/r) + \Phi\varepsilon_{t-1} + u_t \qquad (4)$$

Partial reparameterization of fox model with AR (1)

When we deal with nonlinear estimation procedures for estimating the parameters, high parameter correlation may be detected which is indeed undesirable. Ratkowsky (1990) suggested to use expected-value parameters for choosing the values of the explanatory variable in such a way to make the parameter correlations low.

To be convenient for mathematical notations, the above equation (4) is rewritten in the following form:

$$C = KE \exp(-E/r) + \Phi\varepsilon \qquad (5)$$

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It may not always be necessary to replace all the parameters of a model by expected-value parameters, however; only the offending parameters can be replaced. Thus, to obtain expected-value parameters for r and K of the above equation (5), we need to choose values E_1 and E_2 of the explanatory variable E, within the observed range of E and correspondingly the values of ε_1 and ε_2 of ε are to be selected. Then, we get the expected values from equation (5) as follows:

$$C_1 = KE_1 \exp(-E_1/r) + \Phi'\varepsilon_1 , \tag{6}$$

and

$$C_2 = KE_2 \exp(-E_2/r) + \Phi'\varepsilon_2 . \tag{7}$$

Solving the equations (6) and (7), we get

$$r = \frac{(E_2 - E_1)}{\left[(E_2\ (C_1 - \Phi'\varepsilon_1)) / \{E_1\ (C_1 - \Phi'\varepsilon_2)\} \right]} ,$$

and

$$K = \frac{(C_1 - \Phi'\varepsilon_1)}{E_1 \exp\left[-E_1 \log\{E_2\ (C_1 - \Phi'\varepsilon_1)/(E_1(C_2 - \Phi'\varepsilon_2))\}/(E_2 - E_1) \right]} .$$

Provided $E_2 > E_1$, $C1 > \Phi'\ \varepsilon_1$ and $E_2\ (C1 - \Phi'\ \varepsilon_1) > E_1\ (C1 - \Phi'\ \varepsilon_2)$, since the parameters ‘r’ and ‘K’ should be strictly positive.

Substituting back into the original equation (5), we get

$$C = \{(C_1 - \Phi'\ \varepsilon_1)/E_1\}.E.\exp[\log\{E_2\ (C_1 - \Phi'\ \varepsilon_1)/ (E_1(C_2 - \Phi'\ \varepsilon_2))\}(E_1 - E)/(E_2 - E_1)] + \Phi\ \varepsilon \tag{8}$$

where Φ' is the estimated value of Φ obtained by fitting the above equation (4).

Now, this process has eliminated the original parameters r and K, replaced by new parameters which by virtue of being expected-value parameters. Different parameterizations of the same basic model will produce the same goodness-of-fit and the same fitted values; however, they may differ greatly in their estimation behavior (Ratkowsky 1990). The unknown parameters and autoregressive parameter in the above nonlinear models are estimated using Levenberg-Marquardt method (Seber and Wild 1989).

Measures of model adequacy

It is generally assessed by the following statistics:

(i) Root mean square error,

$$\text{RMSE} = \left[\frac{1}{n} \sum_{t=1}^{n} \left(C_t - \hat{C}_t \right)^2 \right]^{\frac{1}{2}} ,$$

(ii) Mean absolute error,

$$\text{MAE} = \frac{1}{n} \sum_{t=1}^{n} \left| C_t - \hat{C}_t \right| , \ t=1,2,...,\ n,$$

where, n is the number of observations; $\hat{C}_t$ is the predicted fish catch at time t.

The better fitted-model will have the lower value of the above statistics. Further, error analysis is required to check the assumptions made for the model to be developed. Thus, independence assumption of the errors needs to be tested. To test the independence assumption of errors run test procedure is available in the literature (Ratkowsky 1990). However, the normality assumption is not so stringent for selecting nonlinear models because their errors may not follow normal distribution.

To illustrate the above methodology, data on fishing effort and its corresponding yield or catch in Gobindsagar reservoir during 1974–75 to 1989–90 observed by Kaushal *et al.* (2006) is considered. The statistical package – SPSS 17.0 version was extensively used for various data analyses. Different sets of initial parameter values were tried to meet the global convergence criterion for best fitting of the nonlinear models. The MSY and its corresponding optimum efforts of different forms of Fox model are computed and the results of the fitted models are presented in Table 1. From the Table 1, it is seen that randomness assumption does not follow for Fox model since the corresponding run test $|Z|$ value (2.303) is greater than 1.96 of normal distribution at 5% level of significance. Further, Durbin-Watson statistic has been calculated and the statistic value is toward zero and hence positive autocorrelation is suspected. The Fox model is refitted incorporating the AR (1) error structure and the results are imparted in Table 1. Further, run test and Shapiro-Wilk test are employed to the errors of catch. Here, the run test $|Z|$ value and Shapiro-Wilk test p-value of the refitted model indicated that both the randomness and normality assumptions are satisfied. The Durbin-Watson statistic values calculated from the refitted model is very close to 2 i.e. presence of autocorrelation is negligible. Also, a significant improvement in RMSE and MAE values is seen in the refitted model as compared to the original Fox model. However, the correlation matrix of the estimated parameters shows that the extreme value of correlation coefficient between parameters ‘r’ and ‘K’ i.e. $\rho(r,K)$ indicates that the two parameters are not estimated independently, while the values of $\rho(r,\Phi)$ and $\rho(K,\Phi)$ are acceptable. For getting a possible solution to this, it has been attempted to fit equation (8), which is derived from equation (5) with expected-value parameters for ‘r’ and ‘K’, since they are the possible offending parameters of the model, while the parameter Φ, is being kept unchanged. A pair E_1=594 and E_2=813 correspondingly, $\varepsilon_1 = -62.0549$ and $\varepsilon_2 = 162.0726$, gives the best result in terms of least correlation coefficient and the required conditions for ‘r’ and ‘K’ to be positive are also

Table 1. Summary statistics for fitting of different forms of Fox model to catch-effort data observed from Gobindsagar reservoir

	Fox Model	Fox Model with AR (1)	Reparameterization of Fox Model with AR (1)
Parameters			
K	1.176 *	1.544	-
	(0.556)	(0.616)	
r	2252.098	1216.731	-
	(3374.256)	(837.606)	
C_1	-	-	525.568
			(45.127)
C_2	-	-	740.577
			(53.964)
Φ	-	0.614	0.614
		(0.245)	(0.245)
Statistics			
MSY (in tonnes)	974	691	691
(no. of gill nets)			
2252	1217	1217	
Durbin-Watson statistic	0.847	1.735	1.735
Model adequacy			
RMSE	140.412	115.264	115.264
MAE	123.696	97.795	97.795
Residual analysis			
Run test $\|z\|$	2.303	0.259	0.259
Shapiro-Wilk test			
p-value	0.125	0.231	0.231

*The values given in parentheses are the corresponding asymptotic standard errors.

satisfied. In practice, E_1 and E_2 are being chosen in such a way that these are not close to each other. The corresponding values of C i.e., C_1=735 and C_2=811 are taken as initial values for computation of the final estimates of the parameters C_1 and C_2. The parameter estimates are again presented in Table 1. Now, the correlation coefficients are well acceptable as the correlation coefficients between the parameters are reasonably low (absolute values < 0.25). Thus, we can say that the parameters are estimated nearly independently. A perusal of the estimates of MSY for different forms of Fox model revealed that a simple Fox model have over-estimated the MSY and optimum effort values as compared to the values of MSY (691 tones) and optimum effort (1217 no. of gill nets) estimated by reparameterization of the Fox model with AR (1).

SUMMARY

The present paper embodies to fit the Fox model using expected-value parameters by incorporating error term with autoregressive of order one. This is illustrated with an example, considering the serially correlated catch-effort data observed from Gobindsagar reservoir, leading to obtain the estimates of MSY value along with the optimum effort to achieve it.

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