

Fitting of allometric model with expected-value parameters for different species of snow trout from Jhelum River, Kashmir

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ABSTRACT

In the present investigation, an attempt was made to develop length-weight relationship of snow trout considering its 6 different species from Jhelum River, Kashmir. Analysis of covariance (ANCOVA) results showed that the existing 2 distinct groups of snow trout species regarding its length-weight relationship do not follow isometric growth. Further, the data sets of 2 different groups are meaningfully fitted to allometric models with expected-value parameters.

Key words: Allometric model, Expected-value parameters, Length-weight

Length-weight relationship of fish allows the estimation of the average weight of the fish of a given length group by establishing a mathematical relationship between them (Beyer 1987) and thus, most of the fishery biologists are interested to work out the above relationship. It is also useful for assessing the relative well-being of the fish population (Bolger and Connolly 1989). Like any other morphometric character, the length-weight relationship can be used as a character for the differentiation of taxonomic units and this relationship is seen to change with various developmental events in life such as metamorphosis, growth and the onset of maturity. Besides this, the length-weight relationship can also be used in setting yield equations for estimating the number of fish landed and comparing the population in space and time (Beverton and Holt 1957).

Snow trout belongs to sub-family Schizothoracinae and an important group among coldwater fishery is a major constituent of the endemic carp fishery of River Jhelum, Kashmir. Snow trouts are represented by 8 genera and 17 species. Of these, six species namely, *Schizothorax plagiostomus*, *S. curvifrons*, *S. nige*, *S. esocinus*, *Diptychus maculatus* and *S. labiatus* are considered in the present study. Snow trouts are highly preferred to the local masses because of its food value and taste that fetches high price in the market. However, in detailed analysis on length-weight relationship of this important fish group is not yet on record. Thus, the present study aims to develop a length-weight relationship

of snow trout considering its different species for direct use in fishery assessment.

MATERIAL AND METHODS

If 'L' and 'W' represent the length and weight of the fish respectively, then the well-known allometric model is defined by:

$$W = aL^b \quad (1)$$

where 'a' and 'b' are parameters. As one of the parameters 'b' appears nonlinearly, equation (1) depicts a nonlinear type of relationship. The above parameters 'a' and 'b' are estimated preferably by 'nonlinear estimation procedures' assuming an additive error term (Venugopalan and Prajneshu 1998). Since the linearization of the above equation needs multiplicative error assumption that tends to be valid when the variability of dependent variable weight increases with increasing values of weight (Ratkowsky 1990) however, this does not seem logical that as weight of a fish increases, variability of weight also increases. When we deal with nonlinear estimation procedures for estimating the parameters, high parameter correlation may be detected which may indeed be undesirable. Ratkowsky (1990) suggested to use expected-value parameters for choosing the values of the explanatory variable in such a way to make the parameter correlations low. Further, expected-value parameters can be advantageously used, as they are nearly unbiased, normally distributed and minimum variance estimators.

To obtain expected-value parameters of the above equation (1), we need to choose values L_1 and L_2 of the explanatory variable L, within the observed range of L. Then, we get the

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expected values from equation (1) as follows.

$$W_1 = aL_1^b$$

and

Solving these equations, we get

and

Substituting back into the original equation (1), we get

(2)

Now, this process has eliminated the original parameters, replaced by new parameters W_1 and W_2 which by virtue of being expected-value parameters.

Also, the Analysis of Covariance (ANCOVA) is a combination of analysis of variance and regression analysis. Hence, ANCOVA is applied to check if the regression lines of length-weight relationship for different species of snow trout are parallel using SPSS (12.0 Version) Syntax Commands explained by Singh and Nayak (2007). Further, Levenberg-Marquardt method (Seber and Wild 1989) is the most widely used and reliable procedure for computing nonlinear least square estimates and has been applied under the present study.

Moreover, summary statistics like R-square, Root Mean Square Error (RMSE) are also calculated.

$$R^2 = 1 - \frac{\sum (W_t - \hat{W}_t)^2}{\sum (W_t - \bar{W})^2}$$

and

$$RMSE = \left[\frac{\sum_{t=1}^n (W_t - \hat{W}_t)^2}{n} \right]^{1/2},$$

where

W_t , fish weight of tth observation; $\hat{W}_t$, predicted fish weight of tth observation; $\bar{W}$, average weight; n, number of observations, $t = 1, 2, \dots, n$.

The better model will have the least value of RMSE while the maximum value of R-square is expected for the same.

Model Fitting

The combined data for six different species of snow trout say, species – 1: *Schizothorax plagiostomus*; 2: *S. curvifrons*; 3: *S. niger*; 4: *S. esocinus*; 5: *Diptychus maculatus* and 6: *S. labiatus* have been analyzed by ANCOVA (Syntax method of SPSS), Regression (nonlinear) method with the help of

SPSS 12.0 version available at DCFR, Bhimtal. In the present study, 160 specimens have been considered for length-weight data of different species of snow trout collected from Jhelum river, Kashmir (Yousuf, 2005 for details of data collection). The specimens of snow trout presently considered range 205–486 mm in length and 70–1,295 g in weight. At the end, only 2 regression lines for 2 different species groups of snow trout are fitted to the above equations (1) and (2).

RESULT AND DISCUSSION

The slope of the six regression line due to different species are highly significant different (<0.01). The regression lines due to different species are further examined in detail. Since the species 1 and 2 are not significantly different at 5% level of significance, they are combined together and slope of this regression line is further compared with species 6 and the corresponding test is not significant different at the critical level as shown in Table 1. Thus, the species 1,2 and 6 of snow trout can be combined together regarding length-weight relationship of them. In the similar fashion, species 3,4 and

Table 1. User-specified contrasts test results on comparing different species of snow trout by ANCOVA method

Comparison of species	F-value	p-value
1 v/s 2	0.225	0.636
1 v/s 3	9.991	0.002
1 v/s 4	8.320	0.005
1 v/s 5	32.778	0.05
1 v/s 6	1.110	0.294
2 v/s 3	6.442	0.012
2 v/s 4	5.198	0.024
2 v/s 5	1.921	0.168
2 v/s 6	1.608	0.207
3 v/s 4	0.058	0.811
3 v/s 5	1.250	0.265
3 v/s 6	13.320	0.000
4 v/s 5	0.764	0.384
4 v/s 6	11.487	0.001
5 v/s 6	6.368	0.013
1&2 v/s 3	9.069	0.003
1&2 v/s 4	7.461	0.007
1&2 v/s 5	3.138	0.079
1&2 v/s 6	1.842	0.177
1,2&6 v/s 3	11.346	0.001
1,2&6 v/s 4	9.529	0.002
1,2&6 v/s 5	4.565	0.034
3&4 v/s 1	15.932	0.000
3&4 v/s 2	8.865	0.003
3&4 v/s 5	1.370	0.244
3&4 v/s 6	20.321	0.000
3,4&5 v/s 1	16.452	0.000
3,4&5 v/s 2	7.899	0.006
3,4&5 v/s 6	21.059	0.000
1,2&6 v/s 3,4 & 5	20.909	0.000

Dependable variable: logW; Covariate: Species

Table 2. Summary statistics of the models fitted

	Model: $W=aL^b$		Model:	
	Group 1	Group 2	Group 1	Group 2
<i>Parameter Estimates</i>				
a or W1	4.46×10^{-7} (0.000)	5.00×10^{-3} (0.005)	117.507 (8.044)	165.157 (7.949)*
b or W2	3.517 (0.151)	1.905 (0.188)	507.799 (9.837)	260.726 (4.951)
<i>Goodness of fit Statistics</i>				
R ²	0.875	0.616	0.875	0.616
RMSE	77.923	38.946	77.923	38.946

*The corresponding asymptotic standard errors are shown in parentheses.

Group 1: A combination of species 1,2, 6; group 2: A combination of species 3,4,5.

5 are not significant different from one another regarding the length-weight relationship and thus they are combined together. The two groups say, group 1: a combination of species 1,2 and 6 and group 2: a combination of species 3,4 and 5 has been, further, examined and the corresponding test is found to be highly significant different at 5% level of significance (Table 1). Thus we can say that there are only two distinct groups of snow trout species under consideration, regarding its length-weight relationship. Hence, we have to fit only 2 distinct regression lines at the end.

The allometric model given by equation (1) is fitted to the data of snow trout species group 1 and group 2, separately. The estimates of parameters, goodness of fit statistics of the fitted models is presented in Table 2. Sometimes, fishery biologist are interested to know if there is either isometric or allometric growth pattern of fish. To check the isometric growth of the fish, a null hypothesis $H_0: b=3$ is tested against $H_1: b \neq 3$. The corresponding t-test statistics have been calculated as 3.424 and 5.824 for group 1 and 2 respectively whose value are greater than the critical value 1.96 at 5% level of significance for large sample size say, >50 . Thus the fish growth does not follow isometric growth in both groups.

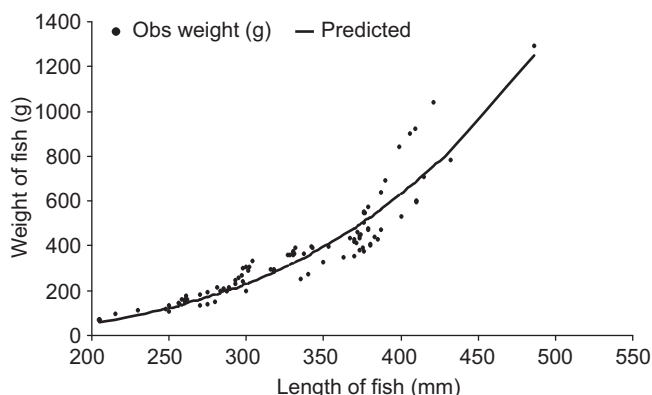


Fig. 1. Fitted allometric model to the dataset of group 1 using expected value parameters.

However, the correlation coefficient between $\hat{a}$ and $\hat{b}$ i.e., $(\hat{a}, \hat{b})$ is nearly (-1) for both the groups-I & II. An extreme value of $(\hat{a}, \hat{b})$ indicates that the two parameters 'a' and 'b' are not estimated independently (Prajneshu and Ravichandran 2003). For getting a possible solution to above, it has been attempted to fit equation (2) using the data of both groups separately. For group-I, a pair $L_1=248, L_2=376$ give the best result's in terms of least correlation coefficient. Similarly, a pair $L_1=240, L_2=305$ has been defined for the group-II. In practice, it is not required to go for all possible pairs of combinations. However, the methodology would work satisfactorily so long the L_1 and L_2 are not close to each other. The corresponding values of W i.e., $W_1=118, W_2=505$ for group 1 and $W_1=96, W_2=312$ for group 2 are taken as initial values for computation of the final estimates of the parameters W_1 and W_2 . The parameter estimates along with asymptotic standard errors are given in Table 2. The correlation coefficient, computed are 0.425 and 0.351 for group 1 and 2 respectively. Now, we can say that 2 parameters W_1 and W_2 are estimated almost independently as the corresponding correlation coefficients are very small for both the expected groups. The RMSE values calculated from the

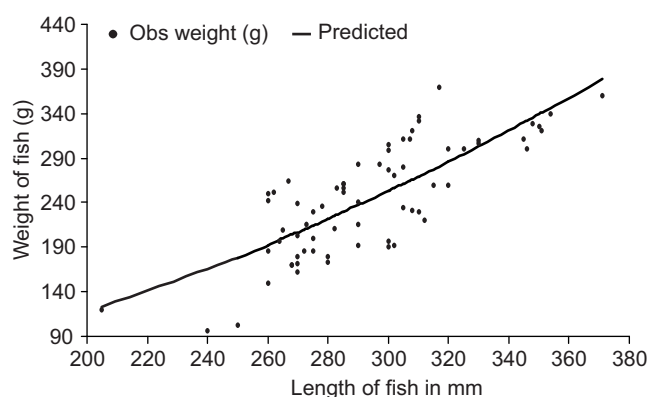


Fig. 2. Fitted allometric model to the dataset of group 2 using expected value parameters.

original model (1) and new model (2) with expected groups which are shown in Table 2. As a consequent of expected-value parameters, the degree of curvature reduces or exhibits close-to-linear behavior. Moreover, the graphs of fitted models along with observed values are shown in Figs 1 and 2 for group 1 and 2 data sets respectively. It is clearly seen from the graphs that a new model with expected-value parameters entails a very close-to-linear model having optimum properties of the estimators for which the degree of linearness is more in the data set of group 2.

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