Prospects of livestock and dairy production in India under time series framework

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ABSTRACT

Share of livestock sector especially dairy production in total gross domestic product (GDP) has shown a continuous rise trend over the last 30 years. Autoregressive integrated moving average (ARIMA) methodology was applied for modeling and forecasting of milk production of India. Auto-correlation (AC) and partial auto-correlation (PAC) functions were estimated, which led to the identification and construction of ARIMA models, suitable in explaining the time series and forecasting the future production. A significant increasing linear trend in the total milk production in India was found. To this end, evaluation of forecasting is carried out with mean absolute prediction error (MAPE), relative mean absolute prediction error (RMAPE) and root mean square error (RMSE). The best identified model for the data under consideration was used for out-of-sample forecasting up to 2015.

Key words: ARIMA Model, Forecasting, Milk production, Stationarity, Trend

Livestock sector is emerging as an important growth leverage of the Indian economy. With an annual production of 74 million tonnes in 1998–99, it is increased to 127.3 million tonnes in 2011. In livestock management, as well as in other cases, forecasting strategies are based on the development of either descriptive or explanatory models. In addition to the forecasting character, the multivariate descriptive models have the advantage that by “stepwise modelling”—namely by adding stepwise predictors and comparing the quality of fit, certain inferences concerning the importance of the predictors can be made. Descriptive models used to predict and analyze time series data attempt to decompose the dependent variable into 4 main components: simple time trends, periodic fluctuations, predictors’ effect and the error component. A common realisation of this approach is the development of the multivariate ARIMA models (Box et al. 2007). Prajneshu and Venugopalan (1998) used this for describing trends in marine fish production data of the country. Singh et al. (2007) applied statistical models for forecasting milk production in India. Paul and Das (2010, 2013) applied ARIMA model for modelling and forecasting of Inland fish production in India as well as fish landing in Ganga basin. Paul et al. (2013) applied Seasonal ARIMA (SARIMA) model for forecasting of total meat export from India. One advantage of the ARIMA approach is that it is able to provide a good understanding of the system. This model has been dominating time series analysis for several decades. In the present work, ARIMA model was used for modelling and forecasting of milk production of India and trend of production over the last three decades has been studied.

MATERIALS AND METHODS

Data description

Data on growth rate of agriculture and allied sectors as well as livestock sector from 1994 to 2004 and time series data of India’s milk production (in million tonnes) from 1979 to 2011 were taken from the report of the working group on animal husbandry and dairying for the XI Five Year Plan (2007–2012) and partly from Department of Animal Husbandry, Dairying and Fisheries, Ministry of Agriculture (http://www.nddb.org/English/Statistics/Pages/Milk-Production.aspx). Milk production data in million tonnes from the year 1979 to 2007 was used for model development, and data from 2008 to 2011 were used for model validation purpose. The SAS 9.3 statistical software package has been used for data analysis.

Autoregressive integrated moving average (ARIMA) model

A generalization of ARMA models which incorporates a wide class of non-stationary time-series is obtained by introducing the differencing into the model. ARIMA econometric modeling takes into account historical data and decomposes it into an autoregressive (AR) process where there is a memory of past events and integrated (I) process which accounts for stabilizing or making the data stationary, making it easier to forecast, and a moving average (MA) of forecast errors, such that the longer the historical data, the more accurate forecast will be as it learns from over time. The simplest example of a non-stationary process which reduces to a stationary one after differencing is Random
Walk. A process \( \{ y_t \} \) is said to follow an Integrated ARMA model, denoted by ARIMA \((p, d, q)\), if \( \forall dy_t = (1-B)^d \varepsilon_t \) is ARMA \((p, q)\). The model is written as
\[
\phi(B)(1-B)^d y_t = \theta(B) \varepsilon_t \tag{1}
\]
where
\[
\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p
\]
\[
\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q
\]
\[\varepsilon_t \sim WN (0, \sigma^2),\]
WN indicating White Noise.

The integration parameter \(d\) is a nonnegative integer. When \(d = 0\), ARIMA \((p, d, q)\) model reduces to ARMA \((p, q)\) model.

The ARIMA methodology is carried out in three stages, viz. identification, estimation and diagnostic checking. Parameters of the tentatively selected ARIMA model at the identification stage are estimated at the estimation stage and adequacy of tentatively selected model is tested at the diagnostic checking stage. If the model is found to be inadequate, the three stages are repeated until satisfactory ARIMA model is selected for the time-series under consideration. An excellent discussion of various aspects of this approach is given in Box et al. (2007). Most of the standard software packages, like SAS, SPSS and EViews contain programs for fitting of ARIMA models.

**Estimation of parameters**

Estimation of parameters for ARIMA model is generally done through Nonlinear least squares method. Fortunately, several software packages are available for fitting of ARIMA models. To this end, in this paper, SAS software package is used. The Akaike information criterion (AIC) and Bayesian information criterion (BIC) values for ARIMA model are computed by:
\[
AIC = T \log (\sigma^2) + 2(p+q+1) \tag{2}
\]
and
\[
BIC = T \log (\sigma^2) + (p+q+1) \log T \tag{3}
\]
where denotes the number of observations used for estimation of parameters and \(\sigma^2\) denotes the Mean square error.

**RESULTS AND DISCUSSION**

**Trend estimation**

Assuming presence of deterministic linear trend in the time series, following model is fitted:
\[
Y_t = \mu + \delta t + \varepsilon_t, \quad t = 1, 2, \ldots, T \tag{4}
\]
where \(Y_t\) is the total milk production in million tones in the \(t\)th year, \(\mu\) is the general mean, \(\delta\) is the coefficient of trend \(\varepsilon_t\)'s are uncorrelated with zero mean and constant variance \(\sigma^2\). Let
\[
\hat{\varepsilon}_t = Y_t - \hat{\mu} - \hat{\delta} t
\]
The fitted trend equation is obtained as:
\[
Y_t = 21.449 + 2.872 t \pm t_{(1.553), (0.079)}
\]
where the values within brackets denote corresponding standard errors of estimates. The trend is found to be significant at 1% level of significance.

It is evident from Fig. 1 that the growth rate (percent share in GDP on the basis of 1993–94 prices) in agriculture sector over the years has been steady and fluctuating significantly depending upon the monsoon and other climatic factors. Of late there has been deceleration of agricultural growth. On the contrary, livestock sector has shown a steady growth and thus providing stability to the overall family income. Fig. 2 depicts% share of agriculture and livestock to total GDP and also share of livestock to agriculture.

**Fitting of ARIMA model**

The detrended residual series \(\hat{\varepsilon}_t\) is found to be nonstationary. In order to attain stationarity, differencing is done. From the estimated autocorrelation function (acf), reported in Table 1, and figure 3a to 3d, it is found that it decays very slowly thereby requires to be differenced so that the resulting series depicts a pattern for a possible ARMA modeling. In order to select the order of the ARIMA model, unit root test proposed by Dickey and Fuller (1979) is applied for parameter \(\rho\) in the auxiliary regression

\[
\Delta Y_t = \rho Y_{t-1} + \alpha_1 \Delta Y_{t-1} + \varepsilon_t
\]
where \(\Delta Y_t = Y_t - Y_{t-1}\). The relevant null hypothesis is \(H_0: \rho = 0\) and the alternative is \(H_1: \rho < 0\). For the given data, the estimate of \(\rho\) is computed as \(-0.64\) with calculated \(t\)-statistic as \(-4.62\), which is less than the critical value of \(t\) at 5% level of significance, i.e. -1.95 (Franses, 1998, Page 82). Therefore, \(H_0\) is not rejected at 5% level and so \(\rho = 0\).
Thus, there is presence of one unit root and so differencing is required. Usually, differencing is applied until the acf shows an interpretable pattern with only a few significant autocorrelations. On taking the first difference of the original series, it is seen that only a few acfs, reported in Table 1, are high making it easier to select the order of the model. On taking the second differencing of the original series it is seen that the sum of the autocorrelations of double differenced series is −0.492 which concludes that the series is over differenced (Franses 1998).

Therefore after differencing once, the resultant series becomes stationary. On examining its autocorrelation functions (acf) and partial autocorrelation functions (pacf) and on the basis of minimum AIC and BIC values, the ARIMA(1, 1, 0) model is selected. Parameter estimates along with corresponding standard errors of fitted ARIMA (1, 1, 0) model are reported in Table 2.

Diagnostic checking
The model verification is concerned with checking the residuals of the model to see if they contained any systematic pattern which still could be removed to improve the chosen ARIMA, which has been done through examining the autocorrelations and partial autocorrelations of the residuals of various orders. For this purpose, various autocorrelations up to 16 lags were computed and the same along with their significance tested by Box-Ljung statistic are provided in Table 3. As the results indicate, none of these autocorrelations was significantly different from zero at any reasonable level. This proved that the selected ARIMA model was an appropriate model for forecasting.
Table 3. ACF and PACF of the residuals of fitted ARIMA model

<table>
<thead>
<tr>
<th>Lag</th>
<th>Autocorrelation</th>
<th>Std. Error</th>
<th>Box-Ljung Statistic</th>
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<tr>
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<td>.120</td>
<td>7.089</td>
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</table>

The ACF and PACF of the residuals are given in Fig. 4, which also indicated the ‘good fit’ of the model.

Validation

One-step ahead forecasts of milk production along with their corresponding upper confidence interval and lower confidence interval for the year, 2008 to 2011 in respect of above fitted model are reported in Table 4.

For measuring the accuracy in fitted time series model, Mean absolute error (MAE), Mean absolute percentage error (MAPE) and Relative mean absolute prediction error (RMAPE) are computed by using the formulae given in eqs. 5, 6 and 7. The MAPE, RMAPE and RMSE values for fitted ARIMA (1,1,0) model are respectively computed as 1.9, 1.5% and 2.68.

\[
\text{MAPE} = \frac{1}{4} \sum_{i=1}^{4} \left| \hat{Y}_{t+i} - \hat{\hat{Y}}_{t+i} \right| \tag{5}
\]

\[
\text{RMAPE} = \frac{1}{4} \sum_{i=1}^{4} \left( \hat{Y}_{t+i} - \hat{\hat{Y}}_{t+i} \right) \left| Y_{t+i} \right| \times 100 \tag{6}
\]

\[
\text{RMSE} = \sqrt{\frac{1}{4} \sum_{i=1}^{4} \left( \hat{Y}_{t+i} - \hat{\hat{Y}}_{t+i} \right)^2} \tag{7}
\]

Forecasting

The best model i.e. ARIMA (1,1,0) model as given in eq. 8, was used for forecasting of total milk production in India for the period 2012–2015 and the same along with the forecast error variance is reported in table 5.

\[
\Delta Y_t = \alpha_0 + \alpha_1 \Delta Y_{t-1} + \epsilon_t \tag{8}
\]

\[
Y_t = \alpha_0 + (1+\alpha_1) Y_{t-1} - \alpha_1 Y_{t-2} + \epsilon_t \tag{9}
\]

where \( \Delta Y_t = Y_t - Y_{t-1} \)

So the forecast for the year 2012 to 2015 can be computed by eq. 9.

It has been found that there is a significantly increasing trend in the total milk production in India. ARIMA (1,1,0) model quite satisfactorily captured the variation present in the data set. The model demonstrated a good performance in terms of explained variability and predicting power. The findings of the present study provided direct support for the potential use of accurate forecasts in decision making and livestock management in India.
REFERENCES


