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Fourier-autoregressive (F-AR) coefficient non-linear time-series model for forecasting asymmetric cyclical data

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The autoregressive (AR) modelling is generally employed for forecasting time-series data and finds excellent applications in describing those situations in which the present value of the variable depends linearly on its past values (Box *et al.* 2009). However, one drawback of this model is that the coefficients are assumed to be constants. Ludlow and Enders (2000) proposed a generalization of AR model of order 1 by allowing autoregressive coefficient to be a time-dependent deterministic function α_t. Although its form is not completely specified, behaviour of α_t can generally be represented by a sufficiently long Fourier series (Bloomfield 2000). For example, if α_t is an absolutely integrable function, for any desired level of accuracy, it is possible to write the Fourier- autoregressive (F-AR) model as

$$y_t = \alpha_t y_{t-1} + \varepsilon_t,\ \alpha_t = A_0 + \sum_{k=1}^{T/4}\left[A_k \sin\frac{2\pi kt}{T} + B_k \cos\frac{2\pi kt}{T}\right], \tag{1}$$

where T/4 is the maximum number of frequencies contained in the process generating α_t for modelling quarterly time-series data and, A_k, B_k, k=1,2,…, T/4 are the Fourier coefficients. Unlike linear AR model, F-AR model is a nonlinear time-series model.

Fun and Yao 2003, Milas *et al.* 2006) and its heartening aspect is that it is capable of describing asymmetric data (Amendola and Storti 2002, Kiani 2009), wherein average number of observations on the up cycle is different from the down cycle. This model can be applied for modelling and forecasting any asymmetric time-series data in Fishery and Animal Sciences pertaining to Export and import quantities/ amount data of animal products, or Milk, meat and wool production data, etc.

Fitting and forecasting of F-AR model: Method of least squares is used to estimate the parameters of F-AR model. Further, H_{0k}:A_k=0, B_k=0 against H_{1k}: at least one of A_k or B_k is nonzero, k = 1, 2, ..., T/4 are tested using t-test with (T-2) degrees of freedom, where single pair of sine-cosine terms is taken in eq. (1). For identification of significant Fourier coefficients, critical values are calculated by performing a Monte Carlo simulation study. For sample size T, each realization of sequence $\{y_t\}$ is assumed to follow independent and identical standard normal distribution. The simulations are repeated a large number of times, say 20000. From each of the simulated samples, t-statistics are calculated and a histogram is drawn by taking ordered t-statistic along X-axis and frequencies along Y-axis. The critical value at 5% level is calculated such that 5% observations lie above that point. If value of t-statistic is greater than this critical value, test is significant at 5% level and the significant variables are then included in the model. Further, goodness-of-fit of the fitted model is examined using Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC).

To obtain size of the test, Double bootstrap technique (Efron and Tibshirani 1993) is employed. To this end, outset t-statistic values are first obtained by parametric bootstrap method using estimated coefficients of F-AR model. Then, in the second-stage, simulated statistic values are computed based on samples drawn from null distribution. For each of the first-stage outset values, t-statistic values corresponding to particular first-stage outset value are computed and replaced by unity if it is greater than outset t-statistic value, otherwise it is replaced by zero. Proportion of t-statistic values greater than outset t-statistic value is also calculated. If this is less than some pre-specified level, say 0.05, H_{0k} is rejected at 5% level for a particular first-stage outset value. Repeating this procedure, say 500 times with respect to first-stage, proportion of 'Rejection of H_{0k} when it is true' is computed which gives size of the test.

For calculating power of the test, the first-stage outset t-statistic values are obtained along similar lines as above. The first-stage samples are considered as populations under alternate hypotheses. In the second-stage, regression is performed on each first-stage sample. Then, simulated statistic values are computed based on samples drawn again using parametric bootstrap approach. The same procedure as mentioned in last paragraph is repeated with 'greater'

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replaced by 'less' and 'true' by 'not true'.

Relevant computer programs for performing all the above tasks were developed in SAS/IML, Ver. 9.2 software package and the same are appended as an Annexure. To compare the performance of F-AR model vis-à-vis AR model, one-step ahead forecasts for hold-out data are obtained from fitted models and Mean square forecast errors (MSFE) are computed.

Statistical analysis: Indian oil sardine (*Sardinella longiceps*) is one of the most important commercial fishes in India and can be found in the northern regions of the Indian Ocean. Twentysix years (1985–2010) quarterly oil sardine fish landings data (in tonnes) recorded at Central Marine Fisheries Research Institute, Kochi, India were used in the study. A perusal showed cyclical behaviour of the data and suggested asymmetricity in the dataset.

In order to fit eq. (1), significant Fourier coefficients were identified by generating 20,000 data sets through Monte Carlo simulation and t- statistics were calculated. Further, histograms are exhibited in Fig. 1. The critical value at 5% level was calculated as 2.00. So, significant Fourier coefficients at 5% level were found as A_1, B_1, A_2, B_3, B_6, A_{15} and B_{16} and the fitted F-AR model was obtained as

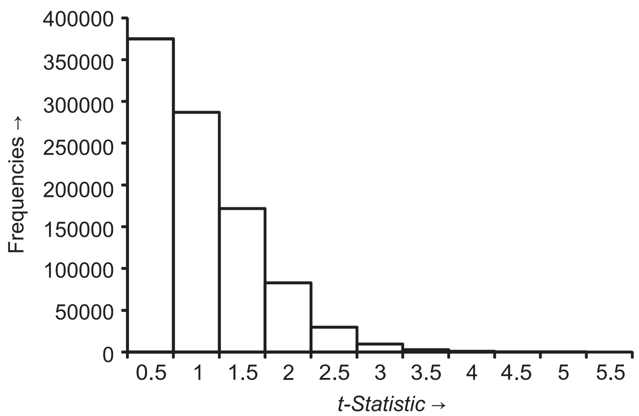

Fig. 1. Histogram for simulated ordered t-statistic.

$$y_t = -0.08+0.13\, y_{t-1} + \left[-0.87 \sin\left(2\,\pi.1\frac{t}{96}\right)\right] - 1.03 \cos\left(2\,\pi.1\frac{t}{96}\right) - 1.79 \sin\left(2\,\pi.2\frac{t}{96}\right) + 1.95 \cos\left(2\,\pi.3\frac{t}{96}\right) - 1.61 \cos\left(2\,\pi.6\frac{t}{96}\right) + 1.19 \sin\left(2\,\pi.15\,\frac{t}{96}\right) + 0.68 \cos\left(2\,\pi.16\,\frac{t}{96}\right) y_t - 1 + \varepsilon_t, \tag{2}$$

and the AIC and BIC values were respectively computed as 21.07 and 25.24. Corresponding values for fitted AR model were computed as 2157.359 and 2159.913. Lower values of both the statistics reflected superiority of F-AR model over AR model for describing the data under consideration. To get a visual idea, the fitted F-AR model along with data is exhibited in Fig. 2. Subsequently, sizes of the tests were computed and the results for significant ones are reported in Table 1. By using double bootstrap methodology, corresponding power of the tests were also calculated and the results are reported in Table 1.

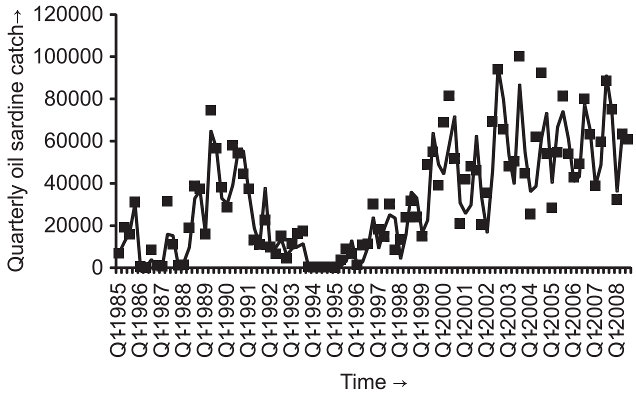

Fig. 2. Fitted F-AR model along with data.

The quarter-wise forecasts for hold-out data for two years were also computed and the same are reported in Table 2. Finally, the MSFE value for F-AR model was computed as 41099124.32, which being less than that for AR model, viz. 995694711.90, showed that the F-AR model was superior to AR model for forecasting purposes also.

Table 1. Computation of sizes (at 5% level) and powers of the tests

Fourier coefficients	Test statistic	Size of the test	Power of the test
A_1	t_{A_1}	0.395	0.75
B_1	t_{B_1}	0.336	0.86
A_2	t_{A_2}	0.413	0.77
B_3	t_{B_3}	0.392	0.81
B_6	t_{B_6}	0.395	0.86
A_{15}	$t_{A_{15}}$	0.338	0.79
B_{16}	$t_{B_{16}}$	0.392	0.91

Table 2. Computation of forecasts using F-AR and AR models

Quarter number/ Year	Actual value (in tonnes)	Forecast by F-AR model	Forecast by AR model
1st / 2009	38635	44065.039	61321.740
2nd / 2009	14855	16168.642	61263.639
3rd /2009	74225	71846.180	61272.648
4th / 2009	39712	43690.751	61271.251
1st / 2010	67470	66030.316	61271.647
2nd / 2010	31303	32517.604	61271.434
3rd / 2010	41523	30713.824	61271.439
4th / 2010	119047	114673.314	61271.438

SUMMARY

The aim of this study was to apply Fourier autoregressive (F-AR) model to describe and forecast asymmetric cyclical data. For carrying out statistical analysis, computer programs were developed using SAS, Ver. 9.2 software package. Twentysix years (1985–2010) quarterly oil sardine fish landings data (in tonnes) recorded at Central Marine Fisheries Research Institute, Kochi, India were used. Superiority of F-AR model over AR model was demonstrated by developing one-step ahead forecasts for two years' hold-out data. Its potential use is to develop optimal import and export policies for Oil sardines. This type of information would also go a long way in enabling the Fishing industry in optimization of its resources. Efficient Oil sardine management strategies need to be evolved in order to allocate optimum number of boats and trawlers, etc.

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ANNEXURE

Code for calculation of Fourier coefficients, Test statistics, p-Values, Size and Power of test

(i) Estimation of Fourier coefficients and calculation of t-statistics:

```
Proc import datafile = "PATH OF DATA FILE.EXTN" out =
oilsardine dbms = EXTN replace;
sheet = 'output'; getnames = yes;
run;
%macro estimation (dataname =, i =);
proc iml;
use &dataname.;
read all into y;
close &dataname.;
x = j(nrow(y)-1,2*(nrow(y)/&period.),0);
y_vec = y[2: nrow(y),1];
do t = 2 to nrow(y);
   do k = 1 to 2*(nrow(y)/&period.);
      if mod(k,2)^= 0 then
      x[t-1,k] = sin(2*&pi.*t*k/nrow(y))*y[t-1];
      else
      x[t-1,k] = cos(2*&pi.*t*k/nrow(y))*y[t-1];
   end;
end;
beta_est = inv(x´*x)*x´*y_vec; residual = y_vec - x*beta_est;
ess = residual´*residual;
p = diag(inv(x´*x)); diag_sum_p = p[+,]; mse = ess/(nrow(x)-
ncol(x));
numerator_t = beta_est‘; denominator_t = sqrt (mse*diag_sum_p);
t_statistics = numerator_t/denominator_t;
create t_value var {t_statistics}; append;
create test_value&I. var {t_ statistics}; append;
quit;
proc transpose data = test_value&i. out = statistics&i.;
var t_ statistics;
run;
%mend estimation;
```

(ii) Monte-Carlo simulation:

```
%macro simulation (i =);
data _null_;
   x = round (ranuni (0)*10000); call symputx ('seed',x);
run;
data simulation&i.;
   do time = 1 to &n.;
      y = rannor (&seed.);
      output;
   end;
   keep y; run;
%mend simulation;
%macro montecarlo_estimation;
%do i = 0%to &no_simulation.;
   %if &i. = 0%then%do;
%estimation (dataname = oilsardine,i=&i.);
      %end;
   %else%do;
      %simulation(i=&i.);%estimation(dataname =
simulation&i.,i=&i.);
   %end;
   %append (base_data=statistics0,append_data=&syslast);
%end;
data mc_statistics (drop = _name_);
set statistics0;
if _n_ = 1 then delete; run;
%mend montecarlo_estimation;
```

(iii) Double bootstrap:

```
%macro single_bootstrap (n_boot=, b=);
data _null_;
x = round (ranuni(0)*10000);
call symputx ('seed',x);
run;
data bootdata0;
set oilsardine; run;
data boot_data;
   do sample = 1 to &n_boot.;
```

```
        do i = 1 to nobs;
                x = round (ranuni(&seed.) * nobs);
                set bootdata&b. nobs = nobs point = x;
                output;
        end;
    end;
stop;
run;
%mend single_bootstrap;
%macro double_bootstrap;
%single_bootstrap (n_boot = &first_bootsample., b = 0);
%do j = 1%to &first_bootsample.;
    data bootdata&j.;
        set boot_data;
        if sample = &j.;        keep y;
    run;
%end;
%do j = 1%to &first_bootsample.;
    %single_bootstrap (n_boot = &second_bootsample., b = &j.);
    %do b = 1%to &second_bootsample.;
        data bootdata&j.&b.;
                set boot_data;
                if sample = &b.;
                keep y;
        run;
    %end;
%end;
%mend double_bootstrap;
%macro bootstrap_estimation;
%double_bootstrap;
%do i = 0%to &first_bootsample.;
    %if &i. = 0%then%do;
    %estimation (dataname = oilsardine,i=&i.);
        %append (base_data=statistics0,append_data=&syslast);
        %end;
    %else%do;
        %estimation (dataname = bootdata&i.,i=&i.);
        %append (base_data=statistics0,append_data=&syslast);
        %do j = 1%to &second_bootsample.;
        %estimation (dataname = bootdata&i.&j.,i=&i.&j.);
    %append (base_data=statistics0,append_data=& syslast);
    %end;
%end;
%end;
data bootstrap_statistics (drop = _name_); set statistics0;
if _n_ = 1 then delete;
run;
%mend bootstrap_estimation;
%macro append (base_data =,append_data =);
    proc append base = &base_data. data = &append_data.;
    run;
%mend append;
%macro bootstrap_data_preparaion;
data boot_final_data1; length id $ 4.;
    set bootstrap_statistics;
    if _n_ = 1 then id = "Fout"; else delete;
run;
data boot_final_data2;
    set bootstrap_statistics; if _n_ = 1 then delete;
run;
data boot_final_data3(drop = i j);
length id $ 4.;
do i = 1 to &first_bootsample.;
    do j = 0 to &second_bootsample.; set boot_final_data2;
        id = compress(i||j);
        output;
    end;
end;
run;
data boot_final_data ;
    set boot_final_data1 boot_final_data3;
run;
%macro first_level_bootsample;
%let j = 0;
data first_boot_pdata1; set boot_final_data; if id = "fout";
run;
%do i = 1%to &first_bootsample.;
    data boot_p_value&i.; set boot_final_data; if id = "&i.&j.";
    run;
    %append (base_data=first_boot_pdata1,append_data=
& syslast);
    %end;
data f_boot_pdata; set first_boot_pdata1; drop id;
run;
%mend first_level_bootsample;
%macro second_level_bootsample;
%let k = 0;
%do i = 1%to &first_bootsample.;
    %do j = 0%to &second_bootsample.;
    data boot_p_value&i.&j.; set boot_final_data; if id = "&i.&j.";
    run;
    %append (base_data=boot_p_value&i.& k., append_data=
& syslast);
    data second_boot_pdata1&i.&k.; set boot_p_value&i.&k.;
        if _n_ = 1 then delete; run;
    data s_boot_pdata&i.&k.;
        set second_boot_pdata1&i.&k.; drop id;
    run;%end;
%end;
%mend second_level_bootsample;
%first_level_bootsample;%second_level_bootsample;
%mend bootstrap_data_preparaion;
```

***(iv) Calculation of p-value and size of test statistics*:**

```
%macro size_statistics;
%do i = 1%to &no_coeff.;
data indiv_coeff_p_interim; set final_mc_pdataset;
keep col&i.;
run;
data size_of_test;
set indiv_coeff_p_interim;
if col&i. < 0.05 then five_percent = 1; else five_percent = 0;
run;
proc means data = size_of_test sum noprint; var five_percent;
output out = sot&i. sum =/autoname ;
run;
%append (base_data=sot1,append_data=sot&i.);
%end;
%mend size_statistics;
%macro p_value (dataname=,num=,q=);
%do x = 1%to &no_coeff.;
    data statistics; set &dataname.;
        keep col&x.;
    run;
    proc transpose data = statistics out = a;
        var _all_;
```

```
    run;
    data b (drop = _name_);
          set a;
    run;
    data c;
          array coeff{*} col1-col%eval(&num. + 1);
          set b;
          count = 0;
          do     i = 2 to &num. + 1;
                    if abs(coeff[1]) < abs(coeff[i]) then do; count =
count + 1;
                    end;
          end;
          empirical_p = count /&num.;
    run;
    data &dataname._pdataset&x.; set c;
          keep empirical_p;
    run;
    %append (base_data=&dataname._pdataset1, append_data=
& dataname._pdataset&x.);
%end;
data &dataname._pdataset;
    set &dataname._pdataset1; if _n_ = 1 then delete;
run;
proc transpose data = &dataname._pdataset out = pdataset_new;
    var _all_;
run;
data final_&dataname._pdataset&q.; set pdataset_new; drop
_name_;
run;
%mend p_value;
%macro size_of_test;
%do j = 1%to &no_sim_size.;
    %montecarlo_estimation;

%p_value(q=&j.,dataname=mc_statistics,num=&no_simulation.);
    %append
(base_data=final_mc_statistics_pdataset1, append_data=final_
mc_statistics_pdataset&j.);
%end;
data final_mc_pdataset; set final_mc_statistics_pdataset1;
    if _n_ = 1 then delete;
run;
%size_statistics;
data size_of_test; set sot1;
    if _n_ = 1 then delete;
    five_percent = five_percent_sum/_freq_; keep five_percent ;
run;
proc transpose data = size_of_test out = sizeoftest; var
five_percent;
run;
data size_of_test;
    set sot1;
    if _n_ = 1 then delete;
    five_percent = five_percent_sum/_freq_; keep five_percent ;
    label five_percent = "five percent";
run;
proc transpose data = size_of_test out = sizeoftest; var
five_percent;
run;
data sizeoftest(drop=_name_); length _label_ $ 15; set sizeoftest;
    rename col1-col48 = t1-t48; rename _level_ = alpha;
run;
%mend size_of_test;
%size_of_test;
```

(v) Calculation of power of the test:

```
%macro power_of_test;
    %bootstrap_estimation;
    %bootstrap_data_preparaion;
%p_value(q=1,dataname=f_boot_pdata,num=&first_bootsample.);
data final_fboot_pdataset;
    set final_f_boot_pdata_pdataset1;
run;
%do q = 1%to &first_bootsample.;
    %let j = 0;
    %p_value (q=&q., dataname=s_boot_pdata& q.&j.,num=
& second_bootsample.);
    %append (base_data=final_s_boot_pdata10_pdataset1,
append_data=&syslast);
%end;
data final_sboot_pdataset; set final_s_boot_pdata10_pdataset1;
    if _n_ = 1 then delete;
run;
data power_dataset; set final_fboot_pdataset final_sboot_pdataset;
run;
%p_value(q=1,dataname=power_dataset,num=&first_bootsample.);

data double_boot_pvalue; set final_power_dataset_pdataset1 ;
run;
proc transpose data = double_boot_pvalue out = power_
calculation;
var _all_;
run;
data power;
    set power_calculation; beta = round((1 - col1),0.001);
    keep beta;
run;
proc transpose data = power out = power_of_test; var _all_;
run;
data power_of_test; set power_of_test;
    rename col1-col48 = t1-t48; rename _name_ = power;
    label _name_ = "power of the test";
run;
%mend power_of_test;
%power_of_test;
```

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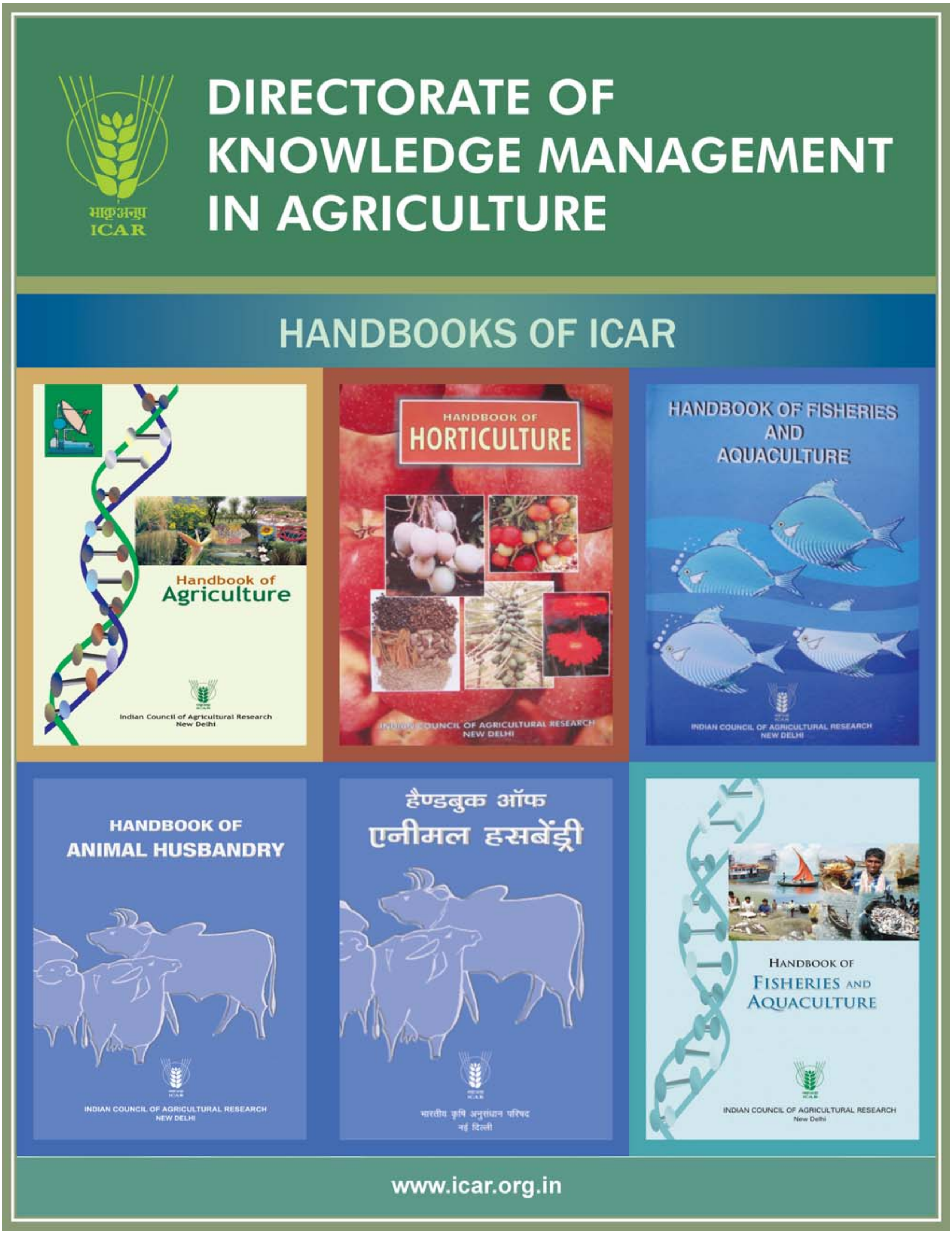

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