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Selecting appropriate nonlinear growth models using Bootstrap technique

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ABSTRACT

Growth is an important phase in the life of animals which influences the different forms of production such as milk, meat etc. The relationship between body weight and age is important particularly in meat producing animals. Many works have already been done for fitting of non linear growth models and choosing best model to describe growth pattern. In the present study, attention is given to the study of statistical properties of goodness of fit criteria for selecting best model to describe the growth pattern by using bootstrap technique. The distributions of the goodness of fit criteria R^2 (determination coefficient), RMSE (root mean square error) and ARR (absolute reduction ratio) are found to be non-normal. Based on these statistical measures the best model is selected to describe growth pattern in given body weight data of goat. On comparison of 3 nonlinear growth models, viz. Logistic, Gompertz and Von bertalanffy model, the third one was found to be the best model.

Key words: Absolute reduction ratio, ARR, Bootstrap technique, Nonlinear growth models

Growth, an important phase in the life of animals, influences the different forms of production such as milk, meat etc. The relationship between body weight and age is important particularly in meat producing cattle, goat, pig, sheep etc. Growth studies are very important for livestock production and are generally done by fitting growth models which summarize information needed to understand the biological phenomenon of growth. Since a series of weight-age data points are analytically difficult to interpret, it is, therefore, desirable to study statistically the growth of animals. Statistical analysis of data on growth of animals involves choosing a suitable growth curve/model and proper method of pooling the results over the group of experiments. Choosing a best model depends on different criteria like highest determination coefficient (R^2), lowest root mean square error (RMSE), lowest absolute reduction ratio (ARR) etc. Kolluru (2000) fitted growth model under heteroscedastic error structure model for studying growth pattern of crossbred cattle. Sarkar (2004) fitted Exponential, Gompertz, logistic and monomolecular model in pig separately for each sex. Wahi *et al.* (2004) studied the growth pattern in crosses and pure Indian breeds of goats. Gompertz model was found to be the best model to describe the growth pattern in goats. Kor *et al.* (2006) aimed to determine the growth of 26 Akkeci (white goat) female kids by using different non linear growth models. Eyduran *et al.* (2008) used the new approaches like absolute reduction ratio (ARR) to determine the best nonlinear growth model.

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The aim of this paper is to fit nonlinear growth model which describes the growth pattern of goats and to study the statistical properties of the different goodness fit criteria's (R^2, RMSE, ARR) using bootstrap resampling technique.

MATERIALS AND METHODS

The body weight data is available at 12 different points the body weight data of 110 goats from goat farm of the institute for the year 2009 was used for study of time namely, W1: body weight at first month,W2: body weight at second month,W3: body weight at third month,W4: body weight at fourth month,W5: body weight at fifth month,W6: body weight at sixth month,W7: body weight at seventh month,W8: body weight at eighth month,W9: body weight at ninth month,10: body weight at tenth month,W11: body weight at 11th month and W12: body weight at 12th month (all the weights available are measured in kg.). In this study following 3 nonlinear growth models were considered. The mathematical forms of the models are

1. Logistic model: $W(t) = \dfrac{a}{1+be^{-ct}} + e$
2. Gompertz model: $W(t) = a\exp\left(-b\exp(-ct)\right)$
3. Von-Bertalanffy model: $W(t) = a\left[1-b\exp(-ct)\right]^3$

where, *W(t)*, body weight at age *t*; *b*, constant coefficient; c, maturity rate; a, mature weight.
In the present study, the estimation of parameters was carried out by Levenberg-Marquardt method using SAS software package version 9.2.

Criteria of model selection: For selection of best fit models, the following criterion was used in this study.

(i) The lowest RMSE (root mean squared error)

The RMSE is defined as

$$\text{RMSE} = \left[\Sigma (y - \hat{y})^2 / n\right]^{1/2}$$

where $\hat{y}$ is the estimated value of y, weight at the time *t*.

ii) The highest value of R^2 (%)

There are different expressions for R^2. The one appropriate for nonlinear model is given by

$$R_1^2 = 1 - \frac{\Sigma (y_i - \hat{y}_i)^2}{\Sigma (y_i - \bar{y})^2}$$

In addition to the above criterion we can use another new approach on evaluating best model derived from asymptotic correlation coefficient (ACC) namely absolute reduction ratio (ARR).

(iii) Absolute reduction ratio (ARR) (Eyduran 2008)

This method is based on the asymptotic correlation coefficient (ACC) between the growth parameters in each nonlinear model. The ratio is performed using maximum ACC and minimum ACC for each model.

$$ARR(\%) = \left| \frac{\min ACC - \max ACC}{\max ACC} \times 100 \right|$$

It is expected that absolute reduction ratio will become equal to a positive value. The best model is a model whose absolute reduction ratio has the least positive value.

Some statistical properties of the above criteria were also studied using bootstrap resampling technique (Efron and Tibshirani 1993) and based on these results, the best nonlinear growth model to explain the growth is also chosen.

RESULTS AND DISCUSSION

There are number of growth models available to describe the relation of body weight with the age of animals. Among the nonlinear growth models the Logistic, Gompertz and Von Bertalanffy models are important. All 3 models namely Logistic, Gompertz and Von Berttallanffy are fitted to the body weight data of 142 goat animals. These models fitted exactly 110 animals each. To study the statistical properties of the criteria of goodness of fit like R^2, RMSE and absolute reduction ratio (ARR) empirically, the most common resampling method, Bootstrap technique was used. For this, bootstrap samples of different sizes (1000, 500 and 200) were created from the master sample (110 animals i.e. no. of animals that fit data well in 3 models) by using sampling with replacement where selection is done on the progenies level. All the three models viz. Logistic, Gompertz and Von Bertalanffy were fitted to the body weight data of all animals of each bootstrap samples. For each animal, its growth parameters were estimated by using Lavenverg-Marquardt algorithm and also for model selection, *RMSE* and *ARR* were computed for each bootstrap sample of each animal. Results obtained through different sizes of bootstrap samples and for actual data are presented in Table 1. The bootstrap estimate values were found biased (Table 1) and had less standard error as compared to actual data sample.

R^2 value was highest with less standard error in Von Bartallanffy models irrespective of sample size. Similarly, *RMSE* and *AAR* values were lowest with less standard error. The bootstrap technique gave similar result as it was observed for actual data. Sample size had very less effect on these values (R^2, *RMSE* and *ARR*). Within sample, similar trends were observed. Depending only on R^2 value, all the models fit well, but combining all the criteria it may be concluded that Von Bertallanffy model is the most suitable for modeling goat growth.

The statistical properties like mean, median, skewness and kurtosis and nature of the curve of the goodness of fit criteria, *RMSE* and *ARR* of Von Bertallanffy model based on the 1000 bootstrap samples are also discussed here. Statistical measures viz. mean, median, standard deviation, skewness, inter quartile range, range, kurtosis, coefficient of variation etc. based on the 1000 bootstrap samples of, *RMSE* and *ARR* were computed and are presented in Table 2.

It was observed that overall mean value for R^2 is 0.9979

Table 1. Overall mean values of goodness of fit criteria along with their standard deviation based on different bootstrap sample sizes (1000, 500 and 200) and estimate using actual data with SD

Model	Sample size	R^2 (mean)	RMSE (mean)	ARR (mean)	R^2 (SD)	RMSE (SD)	ARR (SD)
Logistic model	Actual	0.9866	0.8320	45.5150	0.00016	0.03690	3.45670
	1000	0.9976	0.8550	47.5150	0.00014	0.02690	3.25830
	500	0.9977	0.8532	47.5024	0.00014	0.02653	3.29840
	200	0.9976	0.8541	47.0678	0.00014	0.02730	3.09140
Von Bertalanffy model	Actual	0.9897	0.7768	31.1400	0.00016	0.02770	3.29640
	1000	0.9979	0.7876	32.2060	0.00013	0.02670	3.18330
	500	0.9980	0.7858	32.2342	0.00012	0.02582	3.25870
	200	0.9979	0.7872	31.8464	0.00013	0.02650	2.99630
Gompertz model	Actual	0.9896	0.7934	35.9870	0.00021	0.03680	3.36780
	1000	0.9979	0.8034	36.9320	0.00013	0.02680	3.25720
	500	0.9979	0.8015	36.9771	0.00013	0.02594	3.28030
	200	0.9979	0.8029	36.5410	0.00013	0.02670	3.04155

Table 2. Statistical properties of goodness of fitness statistics for Von Bertallanffy model based on the 1000 bootstrap samples

	R^2	RMSE	ARR
Mean	0.9980	0.7877	32.2067
Std Deviation	0.0001	0.0268	3.1833
Median	0.9980	0.7871	32.1604
Range	0.0009	0.1627	18.9645
Inter quartile range	0.0002	0.0361	4.4744
Skewness	–0.0662	0.1100	0.1638
Coefficient of variation	0.0131	3.3967	9.8841
Kurtosis	0.0721	0.0554	–0.1560

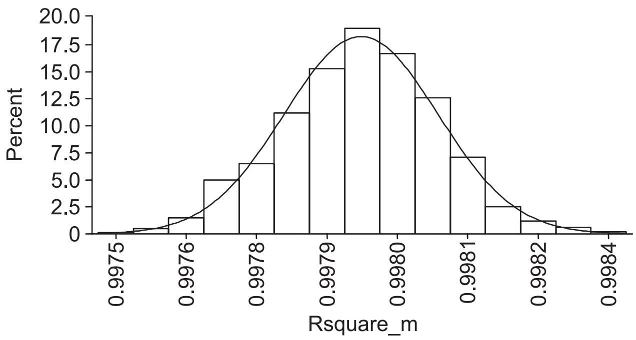

Fig. 1. Histogram of R square.

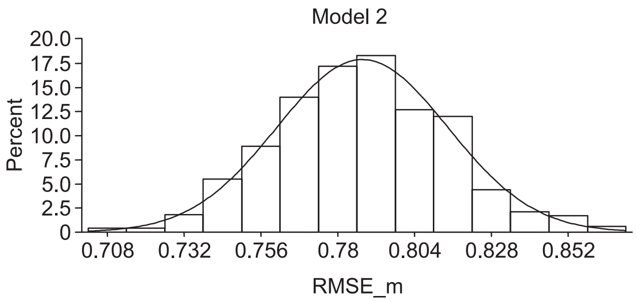

Fig. 2. Histogram RMSE.

with standard deviation as 0.00013. It also has median nearly equivalent to mean value. The coefficient of skewness and the coefficient of Kurtosis were –0.0662 and 0.0721 respectively. The curve of R square was found negatively skewed and more picker than that of normal curve. Histogram (Fig. 1) also indicated the non-normality behavioral of the curve. Similarly in RMSE, it had mean 0.7876 with standard deviation as 0.0267. Median was

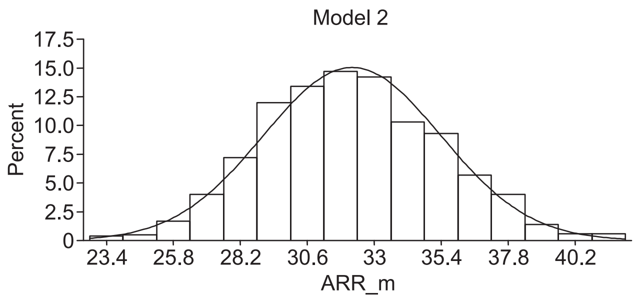

Fig. 3. Histogram of ARR.

0.7870 which showed that the curve is almost symmetry. The coefficient of skewness and the coefficient of Kurtosis were 0.11002 and 0.0554 respectively. It was positively skewed and more picker than that of the normal curve, indicating non-normality nature of the curve. This can also be seen from the histogram (Fig. 2). ARR had mean value of 32.2066 with standard deviation as 3.1833, showing that all the bootstrap samples have almost same ARR means. The median value was 32.1604 which is nearer to the mean value, indicating almost symmetric nature of the curve of ARR means. The coefficient of skewness 0.1637, showing slightly positively skewed. The coefficient of Kurtosis was –0.1560; which showed that the curve is more flat than that of the normal curve (Fig. 3).

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