



Modeling seasonal growth of fish using modified Gompertz model with sine wave function

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Growth of fish is not uniform throughout the year especially in fluctuating or seasonal environments. The need to use a seasonal version of the growth models has been discussed by many (Sparre 1990, Pauly *et al.* 1992, Akamine 2009, Lewis and William 2010) and consequently, seasonal growth models were developed. For example, the VBG model was modified by addition of a sine wave function to adjust for seasonal variations (Pitcher and MacDonald 1973) and seasonally oscillating versions of the VBG model have also been developed by Somers (1988) and Hoenig and Hanumara (1990). Moreover Singh *et al.* (2007) observed that VBG model was appropriate to explain early stages of growth of *Tor putitora* in monoculture and polyculture systems at Pantnagar, Uttarakhand. However, it is not that only the modified version of VBG model will give the best fit either for all kinds of fish species or the same fish species exposed to different aquatic environments when seasonal variation or cyclical fluctuation is involved. In this study, a modified version of Gompertz growth model was considered for the length-at-age dataset of *Tor putitora* from Gobindsagar reservoir.

The following are the most commonly used growth models in fisheries:

Von-Bertalanffy growth model

$$L_t = L_{\infty} \left[1 - e^{-K(t-t_0)} \right], \quad \text{--- (1)}$$

Gompertz growth model

$$L_t = L_{\infty} \exp \left[-\exp \left\{ -K(t-t_0) \right\} \right] \quad \text{--- (2)}$$

where

L_t , fish length at age t ; L_{∞} , maximum fish length; K , growth coefficient (per year); t , age (in years); t_0 , theoretical age (in years) when fish length was zero.

The von-Bertalanffy model with the sine wave function introduced by Pitcher and MacDonald (1973) is as follows:

$$L_t = L_{\infty} \left[1 - e^{-\left[C \sin \left(\frac{2\pi(t-S)}{52} \right) + K(t-t_0) \right]} \right], \quad \text{--- (3)}$$

where, C describes the magnitude of the growth oscillations around a non-seasonal or non-fluctuating growth curve, S is the starting point (relates to phase), and the 52 indicates a time scale in weeks. Using P , rather than a fixed 52 weeks of equation (3), it extends the scope of the model:

$$L_t = L_{\infty} \left[1 - e^{-\left[C \sin \left(\frac{2\pi(t-S)}{P} \right) + K(t-t_0) \right]} \right], \quad \text{--- (4)}$$

where, P is the period of the cycle.

In the similar fashion, when a sine wave is added to the Gompertz model of equation (2), it becomes

$$L_t = L_{\infty} \exp \left\{ -\exp \left[-\left[C \sin \left(\frac{2\pi(t-S)}{P} \right) + K(t-t_0) \right] \right] \right\} \quad \text{--- (5)}$$

Model fitting

The above equations (1) to (5) were fitted to observations of length-at-age for *Tor putitora* collected from Gobindsagar reservoir. Tandon and Johal (1983) studied age and growth of *Tor putitora*, based on a specimen of size 127.5 cm in total length and weighing 24.5 kg caught in gill net from Gobindsagar reservoir.

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The above growth models are nonlinear in nature as at least one derivative with respect to one parameter of different models being a function of the parameter(s). Details of growth models have been given by Thornley and France (2007). There are four main methods available in literature (Seber and Wild 1989) to obtain estimate of the unknown parameters of a nonlinear regression model, namely (i) Gauss-Newton Method, (ii) Steepest-Descent Method, (iii) Levenberg-Marquardt Technique and (iv) Do Not Use Derivative (DUD) Method. The Levenberg-Marquardt method is the most widely used and reliable procedure for computing nonlinear least square estimates and was used in the present study.

To examine model performance, a measure of how the predicted and observed variables covary in time is needed. For nonlinear models, root mean square error (RMSE) and mean absolute error (MAE) are often used for assessment:

$$RMSE = \left[\frac{1}{n} \sum_{t=1}^n (L_t - \hat{L}_t)^2 \right]^{1/2}$$

and

$$MAE = \frac{1}{n} \sum_{t=1}^n |L_t - \hat{L}_t|,$$

where

$\hat{L}_t$, Predicted fish length at age t ; n , number of observations, $t=1,2,\dots$.

The best model will have the lowest numerical values for these statistics. In addition, the randomness assumption of the residuals needs to be tested before taking final decision about the adequacy of the model. To achieve the above task, the run test procedure is available (Ratkowsky 1990), but the normality assumption is not stringent for selection of nonlinear models because their residuals may not follow normal distribution.

The models were fitted using the Nonlinear Regression option on SPSS 12.0 version. Different sets of initial

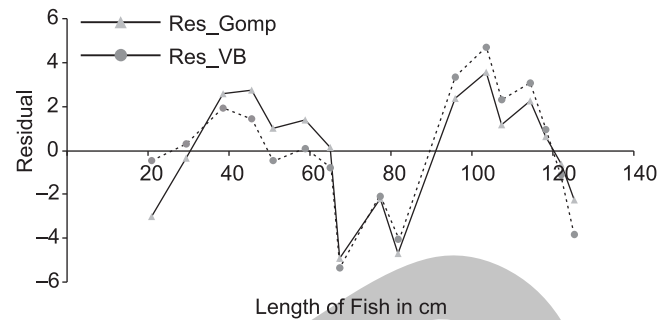


Fig 1. The residuals remaining after fitting of Gompertz and von-Bertalanffy models to the dataset.

parameter values were tried to meet the global convergence criterion for best fitting of the nonlinear models.

The Gompertz and VBG models are fitted to the above length-at-age data of *Tor putitora*. When residuals of the Gompertz and VBG models were fitted against expected length (Fig. 1), a cyclical pattern was seen. It is suggested to add a sine wave function to the model for a possible solution. The original form of VBG model with a sine wave function of equation (3) failed to give optimal solution for the present dataset considered. The modified versions of the VBG and Gompertz models (equations (4) and (5) respectively) were again fitted to the above dataset and the parameter estimates are given in Table 1. The values of RMSE and MAE significantly improved, as compared to the simple Gompertz and VBG models, and the corresponding run test ($|z|$) values are also less than the critical value of 1.96, which means that the randomness assumption is satisfied. The modified version of the Gompertz model describes the *Tor putitora* data of the present aquatic environment better than other popular growth models (Figure 2). Moreover, the asymptotic length of *Tor putitora*, estimated using the modified Gompertz growth model is approximately 169 cm (Table 1), which is quite

Table 1. Summary statistics for model fitting

	Gompertz	Modified Gompertz model of equation (5)	Von-Bertalanffy	Modified VBG model of equation (4)
<i>Parameter estimates</i>				
K	172.928 (11.71)	169.444 (13.941)	423.500 (176.29)	304.893 (156.39)
	0.117 (0.01)	0.121 (0.014)	0.019 (0.01)	0.030 (0.02)
	6.849 (0.66)	6.701 (0.733)	-1.651 (0.45)	-1.032 (0.75)
S	-	1.902 (0.778)	-	0.443 (1.23)
C	-	0.059 (0.015)	-	0.016 (0.014)
P	-	8.992 (1.078)	-	10.169 (1.73)
<i>Model adequacy</i>				
RMSE	2.516	1.290	2.665	1.449
MAE	2.108	1.014	2.126	1.187
<i>Residual analysis</i>				
Run test	1.936	0.518	0.991	1.020
Shapiro-Wilk testp-value	0.219	0.575	0.913	0.605

Bracketed values are the corresponding asymptotic standard errors.

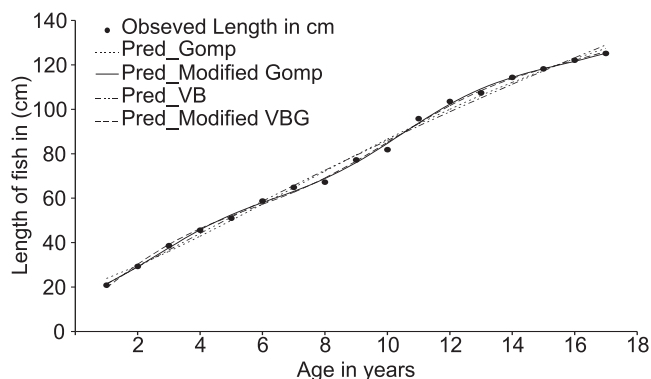


Fig 2. Graphical display of observed and predicted growth in length of *Tor putitora*.

acceptable because the maximum size recorded in India is 275 cm (Jhingran 1975) and in Nepal is 180 cm (Shrestha 1999).

SUMMARY

The paper intended to introduce a modified version of the Gompertz growth model, a sine wave function, which could result in a smooth transition between changes in growth due to seasonal or any other variation. The modified model was considered for the length-at-age dataset of *Tor putitora* from Gobindsagar reservoir, and it showed a significantly improved result as compared to non-seasonal growth models.

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