



Modelling of the first lactation milk yield of Frieswal cows using different mathematical functions

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ABSTRACT

A study was conducted to find the most appropriate mathematical model that describes the first lactation milk yield of Frieswal cattle. Data on 42,368 individual first lactation test day yields of 1,072 Frieswal cows calved during 2005 to 2014 in Ambala and Meerut Military dairy farms were utilized for the study. The first test day milk yield was recorded on 6th day after calving while the subsequent records were collected at seven days interval and so the average 43 test day yields were taken for fitting the lactation curve models. Five different mathematical models, viz. Exponential decline function (EDF), Parabolic exponential function (PEF), Inverse polynomial function (IPF), Gamma function (GF) and Mixed log function (MLF) were fitted. The accuracy of fitting (R^2 value) the models revealed that the MLF (96.14) was more appropriate followed by IPF (95.57), GF (93.85), PEF (83.68) and EDF (69.09). The RMSE estimate of MLF was lowest (0.3483) as expected while the EDF had the highest RMSE value of 0.9858. The AIC criterion was lowest for IPF (5.7175) and highest for GF (8.0212). The BIC values of five functions ranged between -83.6262 for MLF to 3.1809 for EDF. All the DW estimates were positive and ranged between 0.3656 for EDF to 0.7106 for GF indicating positive autocorrelation between the residuals. Based on the results obtained in the present study, it may be inferred that the first lactation yield was explained accurately by the mixed log function (MLF) in Frieswal cattle. As the inverse polynomial (IPF) and gamma function (GF) also had satisfactory results, these two functions can also be used for fitting the lactation curve models in Frieswal cattle. On the other hand, exponential decline function and parabolic exponential functions least explain the first lactation curve in Frieswal cattle.

Key words: Daily milk yield, First lactation, Frieswal cattle, Lactation curve, Mathematic model

Milk yield is an important economic trait in dairy cattle production as it significantly contributes to the profitability. The economics of milk production depends mainly on the quantity of milk produced and the lactation persistency. The milk production by a cow is a continuous physiological function which does not follow a consistent trend throughout the lactation. The rate of milk secretion in a cow varies with the advancement in lactation. Yadav and Sharma (1985) reported that the biometrical properties of lactation are different in different genetic groups, even if the environmental and managerial factors are constant. The graphical representation of milk production during the lactation period through mathematical modelling helps to understand the pattern of milk production and biological efficiency of the cow. The graphical representation of

variation in milk production of a cow against the time is defined as lactation curve. It can also be defined as the trend in the milk yield over the lactation period or the curve representing the rate of milk secretion with the advancement in lactation. A typical lactation curve has three distinct stages, viz. ascending, persistent and descending phases. Ascending phase indicates the initial steep rise in the milk yield, which is usually linear; persistent phase represents the inherent capacity of the animal to sustain its higher milk production, while descending phase is comparatively short and usually associated with the drying off stage of an animal. The dairy cows with more stable lactation curve are more persistent in milk production than the cows having rapid increase in daily milk yield from calving to peak followed by a rapid decline. Macciotta *et al.* (2005) reported that the study of lactation curves is useful for prediction of total milk yield, designing suitable breeding and management strategies as well as for genetic evaluation of dairy cows.

Various mathematical models, viz. linear model, simple linear regression model, Wood's Gamma function, mixed log function, Brody's model, Inverse polynomial model, Quadratic model, Quadratic cum log model, Exponential decline function, Linear cum log model, Reparametrized

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Wood's model etc., have been proposed by many workers to describe the milk production pattern along the lactation of a dairy cow. The review of the earlier research reports suggested that among the available models, inverse polynomial model, exponential decline function, parabolic exponential model, gamma-type function and mixed log function gave better fit to the lactation curve of dairy cattle. However, in Frieswal cattle, very scanty work on lactation curve modelling has been reported and hence the present work was proposed to model the first lactation milk yield of Frieswal cattle using different mathematical models.

MATERIALS AND METHODS

The lactation curve modeling was done using 42,368 individual first lactation test day yields of 1,072 Frieswal cows calved during 2005 to 2014 in Ambala and Meerut Military dairy farms. The first test day milk yield was recorded on 6th day after calving while the subsequent records were collected at seven days interval and so the average 43 test day yields were taken for fitting the lactation curve models.

The following five mathematical functions were used to fit the lactation curve for first lactation in Frieswal cattle:

1. Exponential decline function (EDF), (Brody *et al.* 1923) as $Y_t = ae^{-ct}$
2. Parabolic exponential function (PEF), (Sikka 1950) as $Y_t = a \exp(bt - ct^2)$
3. Inverse polynomial function (IPF), (Nelder 1966) as $Y_t = t(a + bt + ct^2)^{-1}$
4. Gamma function (GF), (Wood 1967) as $Y_t = at^b e^{-ct}$

The constants can be derived by solving the above equation after transformation on the log scale as

$$\ln(Y_t) = \ln(a) + b \ln(t) - ct$$

The milk yields up to week t is given by

$$Y_t = a \int_0^t t^b \exp(-ct) dt$$

Thus we obtained the total 305-days milk yield (43 weeks) as the integral of the average weekly milk yields.

5. Mixed log function (MLF), (Guo and Swalve 1995) as

$$Y_t = a + bt^{1/2} + c \log t + e_t$$

where, Y_t = Average daily yield in the t^{th} week of lactation; a , initial milk yield after calving; b , ascending slope

parameter up to the peak yield; c , descending slope parameter; t , length of time since calving; e_t , residual error.

The curve functions were analysed using the non-linear regression analysis by PROC NLIN of SAS (SAS Institute, 2010) Newton method to obtain the model parameters (a , b and c) and standard errors of parameters. Convergence criterion was met for all the analysis with varying number of iterations. The most suitable model was identified on the basis of the highest adjusted R^2 -value, Akaike information criterion (AIC), Bayesian information criterion (BIC), Durbin Watson (DW) statistic and root mean square error (RMSE) estimated as follows:

$$\text{RMSE} = \sqrt{\text{RSS} / (n - p - 1)}$$

$$R^2 \text{ value} = 1 - (\text{RSS} / \text{TSS})$$

$$R^2_{\text{adj}} = 1 - [(n-1)/(n-p)] * (1-R^2) \text{ or } 1 - [\text{MS}_E / \text{MS}_T]$$

$$\text{AIC} = n * \log(\text{RSS}/n) + 2p$$

$$\text{BIC} = n * \log(\text{RSS}/n) + p * (\log(n))$$

$$\text{DW statistic} = \frac{\sum_t (e_t - e_{t-1})^2}{\sum_t e_t^2}$$

where, RSS, residual sum of squares; TSS, total sum of squares; n , number of test days; p , number of parameters estimated in the model; MS_E , error mean square; MS_T , total mean square; e_t is residual at time t ; and e_{t-1} is residual at time $t-1$.

The adjusted R^2 value ranges between 0 to 1 and values near one indicate the satisfactory fitting of the curve. The smaller RMSE, AIC and BIC estimates indicate better fitting of the model. The DW statistic is used to understand the presence or absence of autocorrelation in the residuals from the regression analysis and it ranges from 0 to 4. If the value is around 2, it indicates absence of autocorrelation while value towards 0 indicate positive autocorrelation and value towards 4 indicate negative autocorrelation.

RESULTS AND DISCUSSION

In this study, a total of five mathematical functions were used to fit the first lactation weekly test day yields of Frieswal cows. The estimated lactation curve parameters and the statistical criteria used for judging their goodness of fit like adjusted R^2 value, AIC, BIC, DW statistic and RMSE are given in Table 1. These statistical criteria help

Table 1. Lactation curve parameters and goodness of fit estimated by different functions for prediction of first lactation WTDMY

Function	Parameters of functions			Goodness of fit				
	a	b	c	Adjusted R^2	RMSE	AIC	BIC	DW
EDF	12.1541 (0.3368)	-	0.0120 (0.0013)	69.09	0.9858	7.660	3.1809	0.3656
PEF	10.6030 (0.3433)	0.0088 (0.0037)	0.0005 (0.0001)	83.68	0.7164	7.0218	-24.2715	0.5821
IPF	0.0751 (0.0062)	0.0590 (0.0018)	0.0020 (0.0001)	95.57	0.3732	5.7175	-80.3576	0.3706
GF	8.9365 (0.2740)	0.2347 (0.0197)	0.0285 (0.0015)	93.85	0.4399	8.0212	-63.536	0.7106
MLF	11.6953 (0.1838)	-3.6947 (0.1442)	5.0062 (0.2495)	96.14	0.3483	7.5540	-83.6262	0.6407

EDF, Exponential decline function; PEF, Parabolic exponential function; GF, Gamma function; MLF, Mixed log function; IPF, Inverse polynomial function; RMSE, Root mean square error; AIC, Akaike's information criteria; BIC, Bayesian information criteria.

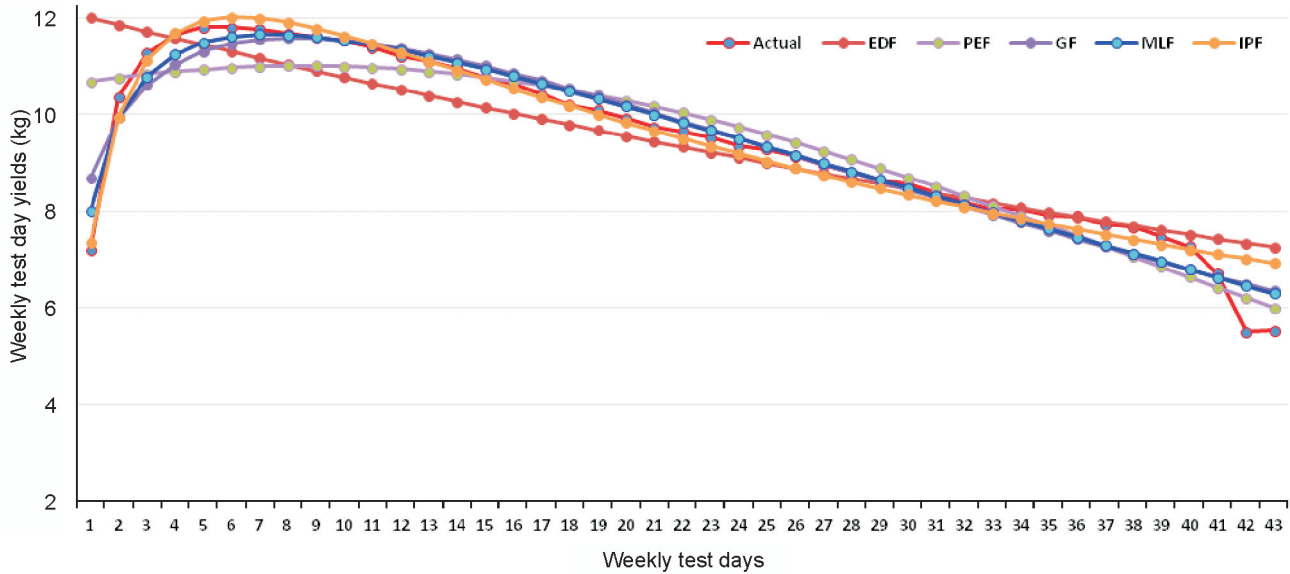


Fig. 1. Actual and predicted weekly test day milk yields for different lactation curve functions. EDF, Exponential decline function; PEF, Parabolic exponential function; GF, Gamma function; MLF, Mixed log function; IPF, Inverse polynomial function.

to compare and identify the best suited curve function which describe and characterize the first lactation milk yield in Frieswal cattle. This will help in prediction of the first lactation 305-days milk yield, understanding the persistency of lactation yield, and to modify the managerial criteria for milking dairy animals and for deciding the selection criteria. The R^2 per cent which indicates the accuracy of fitting the models revealed that the MLF (96.14) was more appropriate followed by IPF (95.57), GF (93.85), PEF (83.68) and EDF (69.09). The RMSE estimate of MLF was lowest (0.3483) as expected while the EDF had the highest RMSE value of 0.9858. The AIC criterion was lowest for IPF (5.7175) and highest for GF (8.0212). The BIC values of five functions ranged between -83.6262 for MLF to 3.1809 for EDF. All the DW estimates were positive and ranged between 0.3656 for EDF to 0.7106 for GF and were similar to the DW estimates reported by Ghavi Hossein Zadeh (2017). The moderate DW estimates obtained in the present study indicated that the residual estimates had positive autocorrelation for all the functions and based on this result it may be inferred that the positive autocorrelation among the residuals increase the chances of positive error in the predictions.

The actual averages for different weekly day milk yields of Frieswal cattle are presented in Table 2. The initial daily milk yield of 7.187 kg was recorded at sixth day (TD-1) of lactation which reached to the peak of 11.81 kg on the fifth week (TD-5) of lactation, i.e. 34 day of post calving. The rate of increase during the initial ascending phase was estimated as 64.32%. Perusal of Fig. 1 on the actual milk yield revealed that the peak yield was fairly retained up to 10th week of lactation, i.e. 69th day with the yield of 11.54 kg with a little decline of 0.27 kg. The milk yield at 43rd test day was 5.53 which was slightly higher than the 42nd test day yield of 5.50 kg. The average test day yields

observed in the present study showed gradual increase during the initial phase of lactation from calving to the peak period and decline thereafter till the end of lactation. Similar to the present findings, Jingar *et al.* (2014) also reported an increase in the fortnight yield till the peak yield at TD-3 and steadily decreased with the advancement of lactation in Karan Fries cattle. However, the shape of lactation curves may vary according to the breed, parity, health status of animal or season of calving etc. as reported by many researchers (Tekeri *et al.* 2000, Nasri *et al.* 2008, Rashia Banu 2010, Ghavi Hossein-Zadeh 2014, Ghavi Hossein-Zadeh 2017). The common trend being an increase in milk yield after parturition to the peak yield (ascending phase) which sustains for a period (persistent phase) following a gradual decrease till the end of lactation (declining phase) and this trend is very well noticed in the present study indicating that the first lactation milk yield of Frieswal crossbred cows did not deviate from the common trend of lactation curve.

The daily milk yield predicted by different functions and the corresponding prediction errors are given in Table 2. The actual daily milk yields and predicted yields by different functions are depicted Fig. 1 and the residuals obtained are depicted in Fig. 2. In the present study, the EDF was the least fitting lactation curve model as the peak yield of 12.009 kg was predicted during the TD-1 itself which declined gradually as the lactation advanced. On the contrary, all other functions predicted the peak yield in between 6th to 8th test days as PEF in TD-8, GF in TD-8, MLF in TD-7 and IPF in TD-6 against the actual peak yield of 11.81 kg observed during the TD-5. Perusal of Fig. 1 revealed that the EDF did not show the standard lactation curve pattern and the predicted yield declined gradually from TD-1 to TD-43 giving a typical straight line. Similar to the present findings, Gahlot *et al.* (1988) also reported that the EDF

Table 2. Estimated daily milk yield and prediction errors for weekly test day milk yields using different functions

Test day	Actual	EDF		PEF		GF		MLF		IPF	
		Predicted	Error	Predicted	Error	Predicted	Error	Predicted	Error	Predicted	Error
1	7.187	12.009	4.822	10.691	3.504	8.685	1.498	8.001	0.814	7.352	0.165
2	10.376	11.865	1.489	10.769	0.393	9.933	-0.443	9.940	-0.436	9.958	-0.418
3	11.271	11.724	0.453	10.836	-0.435	10.618	-0.653	10.796	-0.475	11.130	-0.141
4	11.651	11.584	-0.067	10.892	-0.759	11.041	-0.610	11.246	-0.405	11.691	0.040
5	11.810	11.445	-0.365	10.938	-0.872	11.307	-0.503	11.491	-0.319	11.943	0.133
6	11.809	11.309	-0.500	10.972	-0.837	11.470	-0.339	11.615	-0.194	12.023	0.214
7	11.770	11.174	-0.596	10.996	-0.774	11.559	-0.211	11.662	-0.108	12.000	0.230
8	11.674	11.040	-0.634	11.008	-0.666	11.592	-0.082	11.655	-0.019	11.914	0.240
9	11.611	10.908	-0.703	11.008	-0.603	11.582	-0.029	11.611	0.000	11.787	0.176
10	11.542	10.778	-0.764	10.998	-0.544	11.539	-0.003	11.539	-0.003	11.634	0.092
11	11.388	10.649	-0.739	10.976	-0.412	11.468	0.080	11.446	0.058	11.466	0.078
12	11.208	10.522	-0.686	10.943	-0.265	11.376	0.168	11.336	0.128	11.287	0.079
13	11.110	10.396	-0.714	10.899	-0.211	11.266	0.156	11.215	0.105	11.104	-0.006
14	10.955	10.272	-0.683	10.844	-0.111	11.142	0.187	11.083	0.128	10.918	-0.037
15	10.770	10.150	-0.620	10.779	0.009	11.006	0.236	10.943	0.173	10.731	-0.039
16	10.629	10.028	-0.601	10.702	0.073	10.860	0.231	10.797	0.168	10.546	-0.083
17	10.424	9.909	-0.515	10.616	0.192	10.706	0.282	10.645	0.221	10.364	-0.060
18	10.195	9.790	-0.405	10.519	0.324	10.546	0.351	10.490	0.295	10.184	-0.011
19	10.098	9.673	-0.425	10.412	0.314	10.381	0.283	10.331	0.233	10.008	-0.090
20	9.930	9.558	-0.372	10.297	0.367	10.211	0.281	10.169	0.239	9.836	-0.094
21	9.738	9.444	-0.294	10.171	0.433	10.039	0.301	10.006	0.268	9.668	-0.070
22	9.653	9.331	-0.322	10.038	0.385	9.864	0.211	9.840	0.187	9.504	-0.149
23	9.531	9.219	-0.312	9.895	0.364	9.687	0.156	9.673	0.142	9.344	-0.187
24	9.348	9.109	-0.239	9.745	0.397	9.510	0.162	9.505	0.157	9.189	-0.159
25	9.282	9.000	-0.282	9.587	0.305	9.332	0.050	9.336	0.054	9.037	-0.245
26	9.143	8.893	-0.250	9.422	0.279	9.153	0.010	9.167	0.024	8.890	-0.253
27	8.959	8.787	-0.172	9.251	0.292	8.975	0.016	8.997	0.038	8.747	-0.212
28	8.795	8.682	-0.113	9.073	0.278	8.798	0.003	8.827	0.032	8.608	-0.187
29	8.642	8.578	-0.064	8.890	0.248	8.622	-0.020	8.656	0.014	8.473	-0.169
30	8.563	8.476	-0.087	8.701	0.138	8.446	-0.117	8.486	-0.077	8.341	-0.222
31	8.387	8.374	-0.013	8.508	0.121	8.273	-0.114	8.315	-0.072	8.214	-0.173
32	8.173	8.274	0.101	8.310	0.137	8.100	-0.073	8.145	-0.028	8.089	-0.084
33	8.040	8.176	0.136	8.109	0.069	7.930	-0.110	7.975	-0.065	7.968	-0.072
34	8.046	8.078	0.032	7.905	-0.141	7.761	-0.285	7.805	-0.241	7.851	-0.195
35	7.921	7.981	0.060	7.697	-0.224	7.595	-0.326	7.636	-0.285	7.736	-0.185
36	7.898	7.886	-0.012	7.488	-0.410	7.431	-0.467	7.467	-0.431	7.625	-0.273
37	7.724	7.792	0.068	7.277	-0.447	7.268	-0.456	7.298	-0.426	7.517	-0.207
38	7.678	7.699	0.021	7.064	-0.614	7.109	-0.569	7.130	-0.548	7.411	-0.267
39	7.468	7.607	0.139	6.851	-0.617	6.951	-0.517	6.963	-0.505	7.308	-0.160
40	7.273	7.516	0.243	6.637	-0.636	6.796	-0.477	6.795	-0.478	7.208	-0.065
41	6.720	7.426	0.706	6.424	-0.296	6.644	-0.076	6.629	-0.091	7.111	0.391
42	5.501	7.338	1.837	6.210	0.709	6.494	0.993	6.463	0.962	7.016	1.515
43	5.530	7.250	1.720	5.998	0.468	6.346	0.816	6.297	0.767	6.923	1.393

Actual, Actual observed test day yields; EDF, Exponential decline function; PEF, Parabolic exponential function; GF, Gamma function; MLF, Mixed log function; IPF, Inverse polynomial function.

could not describe the initial increase in daily yield in Rathi cattle. The exclusion of inclining function (b) in the EDF model might result in poor fitting of the lactation curve. Similarly, the curve of PEF also failed to explain the initial phase of lactation significantly which resulted in second least accuracy of prediction. Singh and Bhat (1978) and Gahlot *et al.* (1988) also observed that the PEF fit poorly for the initial phase of lactation before the peak yield. Yadhav and Sharma (1985) also observed that the PEF could describe only the declining trend and hence could not

characterize the lactation curve efficiently in Harianahalf breeds. Gamma function gave better fit for the lactation curve while the MLF was the best to fit the first lactation yield in Frieswal cows followed by IPF. Bangar and Verma (2017) also reported that gamma function may be used as a model for fitting lactation curve in Gir cattle. From Fig.1 it may be inferred that all the functions included in the study showed gradual declining trend during the TD-40 to TD-43 while the actual average milk yield was declined drastically resulting in higher prediction errors (Fig. 2).

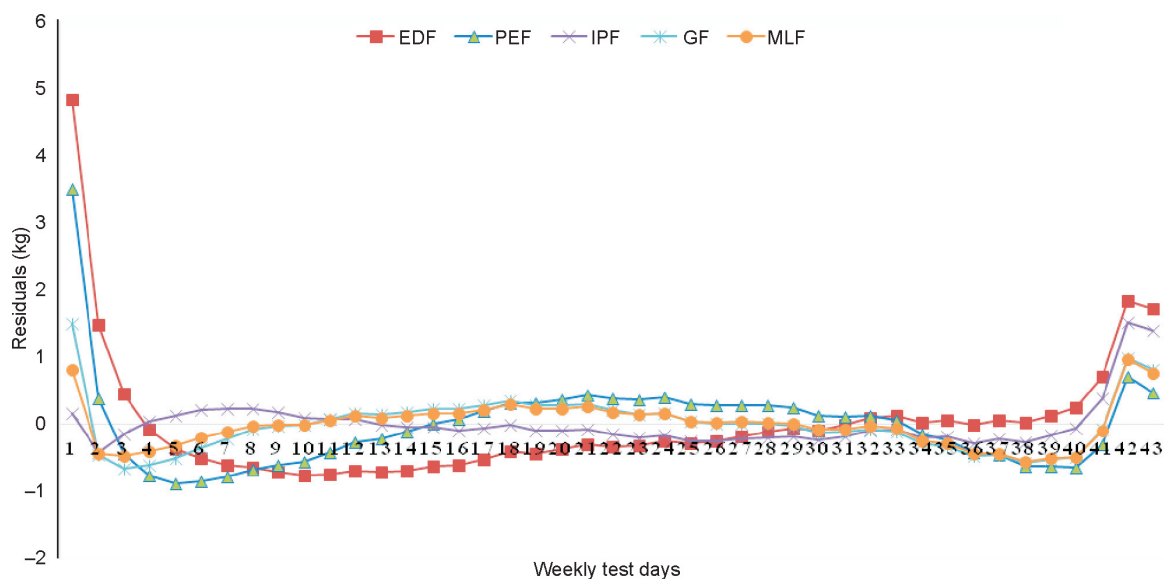


Fig. 2. Residuals (kg) of predicted WTDMY for different functions

EDF, Exponential decline function; PEF, Parabolic exponential function; GF, Gamma function; MLF, Mixed log function; IPF, Inverse polynomial function

Based on the results obtained in the present study, it may be inferred that the first lactation yield was explained accurately by the mixed log function (MLF) in Frieswal cattle. As the inverse polynomial (IPF) and gamma function (GF) also had satisfactory results, these two functions can also be used for fitting the lactation curve models in Frieswal cattle. On the other hand, exponential decline function and parabolic exponential functions least explain the first lactation curve in Frieswal cattle.

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