



Effect of mastitis on lactation curves in purebred Jersey cows

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ABSTRACT

Mastitis is a most frequently occurring disease in dairy cattle which causes severe losses in milk production. In our study, we had collected 9960 weekly test day milk yield (WTDMY) records over a period of five years (2010–2015) of 130 purebred Jersey cows reared at Central Cattle Breeding Farm, Sunabeda, Odisha under Ministry of Agriculture, Government of India. To study the lactation pattern of above milk data, we used six different lactation curve models, viz. Wilmink (WK), Wood (WD), Brody (BRD), Morant and Gnanasakthy (MG), Mitscherlich × Exponential (ME) and Ali and Schaeffer (AS). It was observed that in healthy and cows affected with mastitis, Ali and Schaeffer (AS) model showed best fit giving highest value of adjusted coefficient of determination ($R^2_{adj.} = 0.963$) and lowest value of root mean square of error (0.303), Akaike's information criterion (–97.887) and Schwartz Bayesian Information Criterion (–89.081). Testing of residuals was carried out by several tests, viz. the Shapiro-Wilk's test, the run test and the Durbin-Watson (DW). Summary measures revealed that the loss of milk production due to mastitis with respect to healthy cows was 4.43%. Lactation persistency was estimated by ratio method and Mahadevan method. Higher persistency was observed in healthy cows.

Key words: Dairy cow, Jersey, Lactation curve, Mastitis, Purebred

Lactation curves are suitable tools in analyzing different pattern of breeding and management aspects of dairy cattle (Papajcsik and Boderó 1988, Pietersma *et al.* 2001). The shape of the lactation curve provides valuable information which is essential to evaluation of biological and economical efficiency of the farm animals or dairy herd and is useful for genetic evaluation, health monitoring, nutritional management decisions and herd planning purposes (Sherchand *et al.* 1995, Kocak and Ekiz 2008). Most of the dairy farm's expenses are associated with feeding standards, nutritional supplements, physiological health checkup so suitable statistical models can able to describe the pattern of milk yield accurately. Mastitis is considered as most frequent and costly diseases which influences the quality and quantity of milk and causes the substantial economic losses (Shook 1989) and considered as most expensive disease in terms of production losses (Bardhan 2013). It is now a common practice in most of the organized dairy herds

to reduce the number of mastitis cases so as to overcome the milk losses by formulating proper mastitis management strategies. Though mastitis is more common in high yielder dairy cows, if proper preventive measures are not followed in initial stage, it may cause the serious loss from dairy farms income in later stage. Several methods of statistical models have been developed to study the shape of lactation curve. Nonlinear statistical models are more reliable in judgments and more frequently used by different authors (Silvestre *et al.* 2006, Steri *et al.* 2012, Dohare *et al.* 2015, Mohanty *et al.* 2017).

The objective of this study was to determine the shape of lactation curves and to estimate the peak milk yield, lactation persistency among healthy and animals associated with mastitis along with their comparative milk losses.

MATERIALS AND METHODS

Data: The data were 9960 weekly test day milk yield (WTDMY) records over a period of five years (2010–2015), which were collected from 130 numbers of purebred Jersey cows having 252 numbers of lactation maintained at central cattle breeding farm, Sunabeda, Odisha under Ministry of Agriculture and Farmers Welfare, Government of India. The Jersey cows over a period of 150 days of lactation length were taken into consideration. In our study, we had compared the healthy cows with mastitic cows. The data were collected from history cum pedigree sheets and disease

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record registers in above farm. Out of total numbers of lactation, only 24% were affected with mastitis and 76% were in healthy conditions during the above period.

Lactation curve models: Six different statistical models were applied to fit the milk records of individual lactations.

Wilmink model: This model was proposed by Wilmink (1987). In this model, the factor k was associated with the time of peak yield and its value was assumed to be fixed and here value was 0.05. Wilmink model is defined as:

$$Y_t = a + b \exp(-kt) + ct$$

Wood model: This model is gamma type function model and proposed by Wood *et al.* (1967). It is one of the most widely used models (Sherchand *et al.* 1995). Wood model can be defined as:

$$Y_t = at^b \exp(-ct)$$

Brody model: This model was proposed by Brody *et al.* (1924). It can be written as follows:

$$Y_t = a \exp(-bt) - a \exp(-ct)$$

Morant and Gnanasakthy model: Morant and Gnanasakthy (1989) proposed lactation curve model for proportional rate of change in milk production,

$$Y_t = a \exp(a-bt+ct^2/2+d/t)$$

Mitscherlich × Exponential model: This model was proposed by Rook *et al.* (1993). This can be defined as follows:

$$Y_t = a [1-b \exp(-ct)] \exp(-dt)$$

Ali and Schaeffer model: This model is a five parametric model, which was proposed by Ali and Schaeffer (1987) and defined as follows:

$$Y_t = a + b\delta + c\delta^2 + d\theta_1 + e\theta_1^2$$

where, Y_t , production at time t ; a , scale factor or milk yield at the beginning of lactation; b , rate of change from initial production to maximum yield; c , rate of change from maximum yield to the end of lactation; k , factor associated to the time of peak yield; d , parameter related to maximum milk yield.

$$\delta = t/305, \theta_1 = \ln(305/t) \text{ where } t = \text{week}$$

Statistical analysis: The models were fitted by nonlinear regression to the data described above using PROC NLIN statement of the statistical package SAS 9.3 version (SAS institute Inc. 2011. Cary, NC, USA) and all the parameters were estimated separately. The data were used to fit the models to determine the best fit lactation curve using Marquardt's iteration method which is known as Levenberg-Marquardt algorithm (LMA). Models were judged by using different methods of goodness of fit test. These are as follows:

Adjusted coefficient of determination (R^2_{adj})

$$R^2_{adj} = \frac{(1-R^2)(n-1)}{n-p-1}$$

Adjusted R^2 value always ranges from 0 to 1. A model having large R^2_{adj} is best fitted. However R^2_{adj} is more comparable than R^2 for model that involves different number of parameters.

Root mean square error (RMSE)

$$RMSE = \sqrt{MSE}$$

$$\text{where } MSE = \frac{1}{n-p} \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

y_i , observed value; \hat{y}_i , predicted value; p is the number of parameter in the model, n is the time in weeks.

The root of mean square error of an estimator measures the average of the squares of the errors.

Akaike's information criteria (AIC)

$$AIC = n \log_e MSE + 2P$$

The preferred model is the one which have lowest AIC value.

Schwartz bayesian information criterion (SBC)

$$SBC = n \log(MSE) + P \log(n)$$

The preferred model is the one which have lowest SBC value.

Durbin-Watson (DW) statistics was used to test the presence of autocorrelation. Shapiro-Wilk's test was used to detect the presence of normality among residuals, in which P value less than 0.05 implies the residuals were not normally distributed. Whereas randomness within residuals was checked by run test. A run is a sequence of residuals with the same sign, so a small number of runs of sign obtained when the residuals were not randomly distributed, so residuals of the same sign tend to cluster together on some parts of the curve (Motulsky and Ransnas 1987). Lactation persistency was calculated by two following methods:

Ratio method by Rao and Sundarsen (1982)

$$P_{ratio} = \frac{305 \text{ days milk yield}}{\text{Peak weekly yield}}$$

Method proposed by Mahadevan (1951)

$$P_{Mahadevan} = \frac{A-B}{B}$$

where, A, total milk yield; B, initial milk yield up to attainment of peak yield.

RESULTS AND DISCUSSION

Estimation of parameters and measures of goodness of fit: The estimated parameters along with their standard errors and goodness of fit for the different models are presented in Table 1. The average Weekly test day milk yield (WTDMY) was predicted with high degree of accuracy (R^2_{adj}) and lowest value of RMSE, AIC and SBC by all the models. Burnham and Anderson (2004) suggested that AIC has strong statistical and philosophical basis for judging the quality of fit of models. As far as healthy cows were concerned, all the models had more than 88% degree of accuracy (R^2_{adj}), and AS model gave quite high degree of accuracy with lowest value measures of goodness of fit in all aspects. In case of cows affected with mastitis, the accuracy was somewhat higher than healthy cows fitted by all the models. A similar finding by Dohare *et al.* (2015) who observed that AS model gave 99.62% and 97.85% degree of accuracy while studying in healthy and mastitic crossbred cows respectively. Steri *et al.* (2012) and Mohanty *et al.* (2017b) also studied more than 95% of accuracy by

Table 1. Estimated parameters with measures of goodness of fit of the different lactation curve models fitted for purebred Jersey cows

Animal	Model	Estimated parameters					Measures of goodness of fit			
		a	b	c	d	e	R ² _{adj}	RMSE	AIC	SBC
Healthy	WK	14.264 (0.164)	-8.592 (0.816)	-0.136 (0.006)	-	-	0.917	0.449	-66.011	-60.727
	WD	10.442 (0.307)	0.185 (0.019)	0.022 (0.001)	-	-	0.910	0.470	-62.064	-56.781
	BRD	-14.306 (0.232)	0.982 (0.087)	0.011 (0.001)	-	-	0.880	0.539	-50.220	-44.937
	MG	2.641 (0.030)	0.004 (0.003)	0.000 (0.000)	-0.370 (0.057)	-	0.934	0.401	-74.751	-67.706
	ME	14.804 (0.322)	0.526 (0.081)	0.441 (0.102)	0.013 (0.001)	-	0.894	0.508	-54.449	-47.404
	AS	-46.908 (7.463)	37.157 (7.273)	-17.868 (2.906)	24.646 (3.368)	-2.651 (0.394)	0.963	0.303	-97.887	-89.081
Mastitis	WK	14.386 (0.172)	-10.278 (0.857)	-0.162 (0.007)	-	-	0.934	0.471	-61.917	-56.633
	WD	9.908 (0.340)	0.226 (0.022)	0.027 (0.002)	-	-	0.918	0.527	-52.278	-46.995
	BRD	-14.694 (0.271)	0.799 (0.067)	0.015 (0.001)	-	-	0.904	0.569	-45.589	-40.305
	MG	2.695 (0.036)	0.008 (0.003)	0.000 (0.000)	-0.519 (0.071)	-	0.936	0.465	-61.979	-54.934
	ME	15.248 (0.369)	0.618 (0.079)	0.418 (0.084)	0.016 (0.001)	-	0.917	0.530	-50.768	-43.723
	AS	-42.313 (10.717)	21.514 (10.44)	-11.003 (4.174)	21.541 (4.837)	-2.177 (0.566)	0.943	0.435	-66.857	-58.051

The figures in parenthesis are the asymptotic standard errors. WK, Wilmlink; WD, Wood; BRD, Brody; MG, Morant and Gnanasakthy; ME, Mitscherlich × Exponential; AS, Ali and Schaeffer; a,b,c,d,e, parameters of the models; R²_{adj}, adjusted coefficient of determination; RMSE, root means square error; AIC, Akaike's information criterion; SBC, Schwartz bayesian information criterion.

AS model for Italian Holstein cows and crossbred (Jersey × Red Sindhi) cows respectively. Whereas for Irish dairy cows, accuracy observed by AS model was 68% (Quinn *et al.* 2005).

The values of degree of accuracy by six models ranged from 88% to 96.3% and 90.4% to 94.3% for healthy and cows affected with mastitis respectively. Similar trend was also found in other goodness of fit measures for both types of cows (Table 1). In both the cases (healthy and mastitis), it was noticed that AS model gave best fit in comparison to other models with more than 94% of R²_{adj}. As this model (AS) was a five parametric model, it may be the cause of higher accuracy. Similarly, though MG model had second best fit followed by AS model but if we consider three parametric equations then MG showed higher than ME model. Rodriguez-Zas *et al.* (2000) studied that based on AIC and SBC, the MG model was better followed by AS and WK in Holstein cows. In mastitic cows, AS model was best fitted followed by MG model. These results were partially similar with the findings obtained by Dohare *et al.* (2015) in crossbred dairy cows. MG model was one of the best fitted (R²_{adj}>91%) model in purebred Red Sindhi cows (Mohanty *et al.* 2017a). In agreement with our findings, Dematawewa *et al.* (2007) studied a close value between these two models fitted for Holstein cows. In healthy and mastic cows, BRD model gave lowest degree of accuracy

but its R²_{adj} was more than equal to 88%. So being lowest, this model can't be under looked. Overall observed milk production is shown in the Fig. 1. The estimated milk yield patterns by six models were plotted for healthy and cows affected with mastitis (Fig. 2).

Examination of residuals: The residuals were dependent to each other in healthy cows whereas in diseased conditions all the models didn't give same results due to irregular pattern of milk yield caused by health disorder. For healthy cows, normality test done by Shapiro-Wilk's test showed that the all six models were significantly differing (P<0.05). Hence the residuals of all models were not normally distributed (Table 2). Whereas in cows affected with mastitis, except WD and AS models (P>0.05) all other models were not normally distributed. The autocorrelation patterns tested by DW statistics for different models were comparable with those reported by other studies (Pollott and Gootwine 2000, Grossman and Koops 2003, Mohanty *et al.* 2017a). In all cases, the values of DW statistics by all models ranged from 0 to 2; which indicated positive autocorrelation among them. In healthy cows, the WK, WD and MG models gave a similar kind of run within them whereas, in cows affected with mastitis, similar results were obtained in WK, WD and instead of MG model here ME model gave the same results (Table 2). The residuals of all models fitted for healthy cows were not independent

Table 2. Testing of the residuals in purebred Jersey cows by different models

Animal	Name of test		Model					
			WK	WD	BRD	MG	ME	AS
Healthy	S-W	Statistics	0.831	0.899	0.818	0.88	0.827	0.894
		P-value	< 0.001	0.001	< 0.001	< 0.001	< 0.001	0.001
	Run	Runs	11	11	8	11	7	14
		P-value	0.001	0.001	< 0.001	0.001	< 0.001	0.014
Mastitis	D-W	Statistics	0.512	0.635	0.403	0.641	0.475	0.975
		P-value	0.894	0.956	0.881	0.926	0.878	0.956
	S-W	Statistics	0.894	0.956	0.881	0.926	0.878	0.956
		P-value	0.001	0.098	< 0.001	0.009	< 0.001	0.103
	Run	Runs	15	15	12	17	15	20
		P-value	0.031	0.031	0.002	0.123	0.031	0.539
D-W	Statistics	1.162	1.077	0.893	1.232	0.998	1.431	

WK, Wilmink; WD, Wood; BRD, Brody; MG, Morant and Gnanasakthy; ME, Mitscherlich × Exponential; AS, Ali and Schaeffer; S-W, Shapiro Wilk's test; D-W, Durbin-Watson test.

Table 3. Summary measures of estimated lactation curve models fitted for purebred Jersey cows

Animal	Summary measures	Observed value	Model					
			WK	WD	BRD	MG	ME	AS
Healthy	FWMY(kg)	65.26	66.22	71.54	61.53	67.55	67.69	64.33
	PP (wk)	4	6	8	5	8	7	5
	PY (kg)	91.22	92.54	90.23	93.80	89.88	92.54	90.79
	LWMY(kg)	45.19	58.80	57.61	61.11	56.49	60.13	52.78
	TMY(kg)	3318.47	3318.49	3318.91	3318.21	3318.91	3318.98	3318.35
	P _{Ratio}	36.38	35.86	36.78	35.38	36.93	35.87	36.55
	P _{Mahadevan}	9.16	5.52	3.89	6.82	3.88	4.54	6.93
Mastitis	FWMY(kg)	60.39	60.48	67.48	55.09	61.11	62.30	59.22
	PP (wk)	5	6	8	5	7	7	6
	PY (kg)	90.65	92.05	89.53	93.66	89.88	92.26	90.23
	LWMY(kg)	38.66	51.94	50.96	54.67	50.75	53.69	48.23
	TMY(kg)	3171.54	3171.49	3171.77	3171.00	3172.12	3172.33	3171.49
	P _{Ratio}	34.99	34.45	35.43	33.86	35.29	34.38	35.15
	P _{Mahadevan}	6.84	5.38	3.76	6.74	4.46	4.42	5.40

FWMY, First weekly milk yield; PP, peak period; PY, peak yield; LWMY, last weekly milk yield; TMY, total milk yield; Wk, weeks; WK, Wilmink; WD, Wood; BRD, Brody; MG, Morant and Gnanasakthy; ME, Mitscherlich × Exponential; AS, Ali and Schaeffer. P_{Ratio}, persistency in ratio method; P_{Mahadevan}, persistency in Mahadevan method.

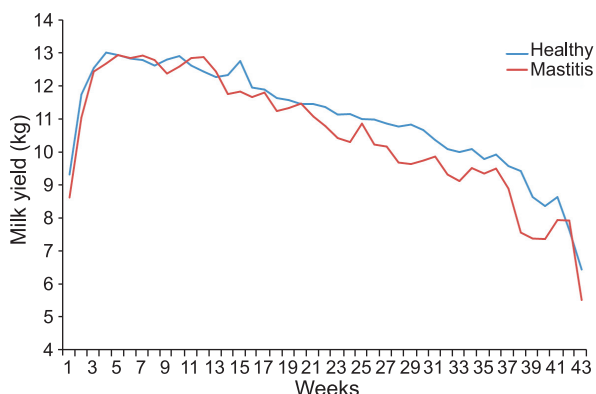


Fig. 1. Comparison of overall milk yield of purebred healthy Jersey cows with cows affected with mastitis.

(P<0.05). In mastitis case the residuals of AS and MG models (P>0.05) were independent to each other except WK, WD, BRD and ME models in which residuals were

not independent (P<0.05). The shape of lactation curves of residuals were plotted in Fig. 3.

Summary measures

For healthy Jersey cows: The average weekly milk yield increased from 65.26 kg on 1st week to a peak yield of 91.22 kg on 4th week and subsequently declined to 45.19 kg on last week with total milk yield of 3318.47 kg (Table 3). The predicted peak yields by different models ranged from 89.88 kg to 93.80 kg. Similarly, peak period also ranged from 5 to 8 week. It was observed that the best fitted AS model predicted very close peak yield (90.79 kg) to observed value on 5th week with total milk yield of 3318.91 kg. The second best fitted model MG showed peak yield on 8th week with 89.88 kg of peak milk yield and total milk yield (3318.91 kg) was very close to the observed value. In case of BRD model which was least fitted but here it showed slight more peak yield than observed yield on 5th week.

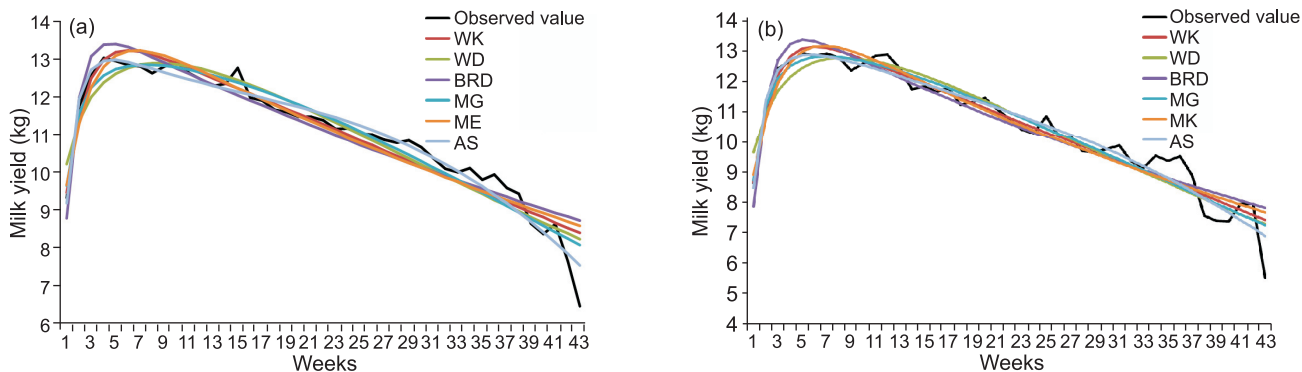


Fig. 2. Lactation curves estimated by different models (WK, Wilmink; WD, Wood; BRD, Brody; MG, Morant and Gnanasakthy; ME, Mitscherlich \times Exponential; AS, Ali and Schaeffer) for purebred Jersey cows; (a) for healthy cows, (b) for cows affected with mastitis.

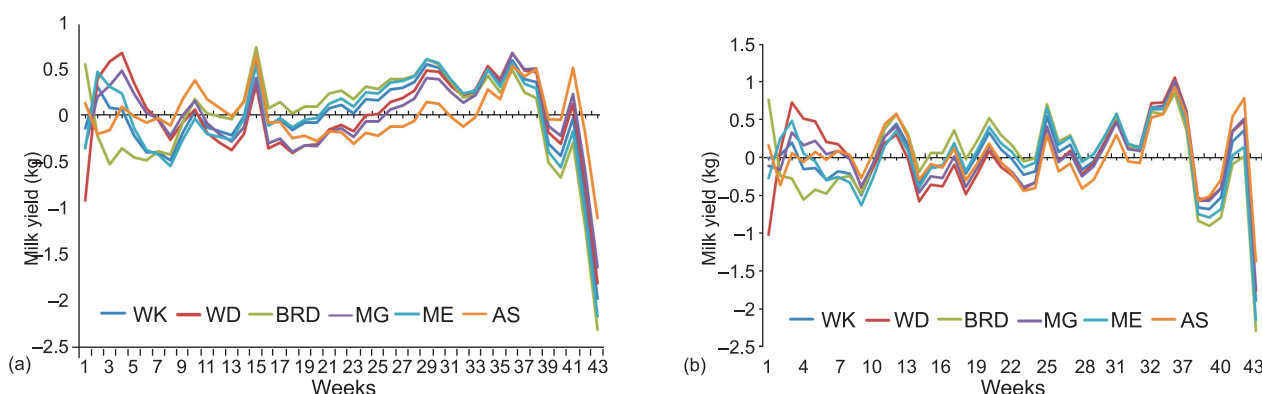


Fig. 3. Measures of residuals fitted by different models (WK, Wilmink; WD, Wood; BRD, Brody; MG, Morant and Gnanasakthy; ME, Mitscherlich \times Exponential; AS, Ali and Schaeffer) for purebred Jersey cows; (a) for healthy cows (b) for cows affected with mastitis.

However the total milk yields of all models were very close to the observed total milk yield.

For cows affected with mastitis: The average weekly milk yield increased from 60.39 kg on first week to a peak yield of 90.65 kg on fifth week and subsequently declined to 38.66 kg on last week with total milk yield of 3171.54 kg (Table 3). The predicted peak yields by different models ranged from 89.53 kg to 93.66 kg with peak period 5 to 8 week. The best fitted AS model predicted very close peak yield to observed value with total predicted milk yield of 3171.49 kg. However the total milk yields of all models were close to the observed total milk yield.

Lucey *et al.* (1986) concluded that the mean reduction in milk yield of 540 kg when mastitis occurred before the week of peak yield. Houben *et al.* (1993) estimated the production loss due to clinical mastitis was 527 kg of milk (8.1%) in the second lactation. Seegers *et al.* (2003) stated that the mastitis losses milk production was 375 kg for a clinical case. Bar *et al.* (2008) estimated the milk losses due to mastitis were 164 kg and 253 kg in primiparous and multiparous dairy cows respectively. Andersen *et al.* (2011) studied modified Wilmink model to describe shape of lactation curve in mastitis in Norwegian dairy cows. But these cows had more initial or final weekly milk yield and lowest peak yield than other cows. So due to declined rate

of peak yield total milk yield were less than healthy and mastitic cows. Dimauro *et al.* (2005) found that there were highest peak on 33 days, 26 days and 29 days for WD, WK and AS models while studying in Italian water buffalo. In comparison to our study, somewhat late peak (60 days) was obtained by Dohare *et al.* (2015) while studying in crossbred cows. However like this study, exact similar peak at 5th weeks was observed by AS model in crossbred cows observed by Mohanty *et al.* (2017b).

It was observed that there was delay in peak yield and lower last weekly milk yield in the cows affected with mastitis. However the losses of milk yield due to mastitis with respect to healthy cows were 4.43%.

Persistency: In both ratio and Mahadevan methods, higher persistency was observed in healthy cows compared to diseased cows (Table 3). The estimated higher lactation persistency by ratio method was marked in MG and WD models for healthy and mastitic cows respectively. In other hand, the estimated higher lactation persistency by Mahadevan method was marked in AS and BRD models for healthy and mastitic cows respectively (Table 3). Ratio method of calculation of persistency was more accurate however healthy cows were more persistent in milk production than diseased cow. Similar findings were also observed by Mohanty *et al.* (2017b).

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