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Robust Estimation of Single Exponential Smoothing through Kalman Filter: An Application to Agricultural and Allied Commodities

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SUMMARY

Time series modelling utilizes previous values to forecast the future values. Exponential smoothing is one of the approaches to make forecast as well as to smooth the time series data. Among the various exponential smoothing model, Single Exponential Smoothing (SES) is the most popular model in time series due its simplicity of understanding and implementation. On the other hand, state space methodology is a very useful technique to solve various problems in time series which is required to improve a system over time. This state space methodology can be used to represent various time series models including Autoregressive Integrated Moving Average (ARIMA). Kalman filter technique is an approach to estimate the time-dependent parameters. One heartening feature of Kalman filter is that it provides the minimum mean squared error (MSE) estimates for linear model. In present study, an attempt has been made to represent the SES in state space form and parameters are estimated using Kalman filter in conjunction with prediction error decomposition form of the likelihood function. An illustration has been given with different applications in agricultural domain. It has been seen that state space form of SES provides lower MSE compared to traditional SES. This integration of SES with state space formulations in agricultural domain will open a new era in agricultural modelling and forecasting.

Keywords: SES; State space methodology; Kalman filtering technique; MSE; Time series.

1. INTRODUCTION

Exponential smoothing is one of the most preferred forecasting methods for a wide range of time series data in business industry (Bunn, 1982; Newbold and Bos, 1994), agriculture field (Kumar *et al.* 2011), ecology (Su *et al.* 2018), traffic engineering (Ghosh, 2005), tourism (Lim and McAleer, 2001), environmental and biological sciences (Ishii, 1981), telecommunication (Gardner and Diaz-Saiz, 2008) etc. Exponential smoothing is generally useful for short term forecasting. This method was initially developed by Brown (1956) during the computation of the location of submarines for fire-control information through designing a tracking system, after that, many works has been done on exponential smoothing.

The exponential smoothing is categorized into different types based on trend and seasonal components in the input data. These methods are SES method, Holt-Winter's method and Holt's trend corrected exponential smoothing method (Holt, 1960). SES works in absence of trend and seasonal pattern. If there is linear trend and seasonal pattern, then the Holt's trend corrected exponential smoothing method is useful. Holt-Winter's method is useful for forecasting with the presence of both seasonal and trend pattern. But, in broad sense single SES is the most useful method among the other exponential methods.

Different reasons behind its popularity among the others are: its simplicity to understand; its easiness to implement with a simple numerical program, its

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reliability to forecast in a wide variety of applications and data storage and computing requirements for exponential smoothing are very minimal.

There are many significant facts about exponential smoothing models. First, this methodology lacks objective statistical identification, viz.., the smoothing constants of this methodology are not based on any statistical tests of hypothesis and exponential smoothing models are determined by fit of the model. So, it is an ad hoc model. Secondly, most exponential smoothing methods can be written as special cases of the class of Box-Jenkins models (Box *et al.* 1976, 2007). But, standard Box-Jenkins model is mainly used for forecasting as it is not an ad-hoc process. That's why, many well-known forecasters are not supporting to any special case of a Box-Jenkins model. On the other hand, it has been proved by Makridakis *et al.* (1979) and Makridakis *et al.* (1982) that exponential smoothing methods can provide almost an accurate forecasting as compare to non-ad hoc Box-Jenkins models. But, the main thing is that one has to choose exponential smoothing models very carefully. Another important thing is that exponential smoothing methods takes very less time than Box-Jenkins models to build the models. But, Box-Jenkins models are more accurate than exponential smoothing model in many cases. Sometimes, the adjustment between time and accuracy favours the exponential smoothing model.

State space methodology is a very useful technique to solve many problems in time series. The relation between observations and unobserved series is represented by this methodology. These unobserved series are known as a state variable which determines the improvement of the system over the time under study. This methodology has two equations viz., state equation and measurement equation. State equation models the evaluation of state variable over time and measurement equation relates the state variable to the observations (Hamilton, 1994).

Kalman (1960) has given a recursive analytical tool to compute the optimal estimator of the state vector, known as Kalman filter, which minimizes MSE for linear models (Meinhold and Singpurwalla, 1983) in the presence of Gaussian noise and also provides consistent estimator of the parameters.

The term filter is given for filtering out the noise in the process of finding out the best estimate. It is also very much popular for its simplicity in implementation, recursive algorithm and computationally efficiency. The estimates of the state vector are to be continually updated by the Kalman filter as new observations become available. In this way, an effort has been made for a state space formulation of SES with Kalman filter to recursively update the estimates as the new observations become available. Also, the study investigates the use of the state space framework and the Kalman filter along with exponential smoothing for improvement of forecast accuracy. The present study aims to answer this question by comparing a modified scenario (state space formulation of SES and Kalman filtering for updating the estimates) with the base scenario (SES alone without Kalman filtering).

SES can be written in the state space form to improve their forecasting accuracy using Kalman filtering technique. The present study is an attempt in this direction, where the basic equation of exponential smoothing is used as the state equation with Kalman filtering. State space formulation of ARIMA models, with the Kalman filter, is most popular (de Jong and Penzer, 2004). Kalman filter methodology was applied in numerous forecasting problems such as wind speed forecast (Louka *et al.* 2008), predicting soil temperature (Huang *et al.* 2008), forecasting GDP (Banbura and Runstler, 2011), electricity demand forecasting (Harvey and Koopman, 1993), water demand forecasting (Nasser *et al.* 2011) and exchanges rates (Wolff, 1987).

Till date SES has not been suggested in state space form for parameter estimation. The study will try to represent SES in state space form and estimate the parameters using Kalman filter in conjunction with Prediction error decomposition form of the likelihood function. In this regard, Kalman filter will be used for parameter estimation as it provides the optimal estimate of the states.

2. MATERIALS AND METHODS

2.1 Data Description

Time-series data on Pulse, Oil of Sardine, Cumin, Soybean, Livestock and Fiscal deficit has been collected from different sources and are treated as the study variable. The wholesale price data on pulse crop has been collected from the *agmarknet.nic.in.* for the period from January, 2005 to October, 2009. Monthly data on oil of sardine catch has been collected from ICAR-Central Marine Fisheries Research Institute (CMFRI) for the period from April, 2004 to February, 2009.

The future price daily data of Cumin of Unjah market has been collected from the *agmarknet.nic.in* for the period from 1st January, 2016 to 23rd March, 2016. Soybean data on the spot price time series data has been collected from Kota market. It has been taken from *agmarknet.nic.in* for the period from 25th march, 2015 to 19th June, 2015. Monthly export data of livestock from India has been taken from *indiastat.com* for the period from September, 2007 to June, 2012. Fiscal deficit data has been taken from *data.gov.in*. for the period from September, 2008 to June, 2013.

2.2 Single Exponential Smoothing

Suppose we have observed data up to and including time (t – 1), and we wish to forecast the next value of our time series s_t, the forecast is denoted by $\hat{s}_t$. When the observation s_t becomes available, the forecast error is $(s_t - \hat{s}_t)$. Then the method of simple exponential smoothing predicts for the next time period (t +1), using the forecast from the previous period ($\hat{s}_t$) and adjusts it using the forecast error $(s_t - \hat{s}_t)$. That is,

$$\hat{s}_{t+1} = \hat{s}_t + \alpha\left(s_t - \hat{s}_t\right) \tag{1}$$

Where α is a constant between 0 and 1.From Eq.(1), it can be seen that the new forecast is simply the previous period forecast plus an adjustment for the error that occurred in the last forecast. When, α has a value close to 1, the new forecast will include a substantial adjustment for the error in the previous forecast. Conversely, when α is close to 0, the new forecast will include very little adjustment Eq. (1) can be rewritten as

$$\hat{s}_{t+1} = \alpha s_t + (1-\alpha)\hat{s}_t \tag{2}$$

Thus, Eq. (2) can be interpreted as a weighted average of the most recent forecast and the most recent observation. The reason for using the word “exponential” is explained below. Eq. (2) can be explained by replacing s_t with its components, as follows:

$$\hat{s}_{t+1} = \alpha s_t + (1-\alpha)\left[\alpha s_{t-1} + (1-\alpha)\hat{s}_{t-1}\right]$$

$$= \alpha s_t + \alpha(1-\alpha)s_{t-1} + (1-\alpha)^2 \hat{s}_{t-1} \tag{3}$$

If this situation process is repeated by replacing $\hat{s}_{t-1}$ with its components, $\hat{s}_{t-2}$ with its components and so on, Eq. (3) results in

$$\hat{s}_{t+1} = \alpha s_t + \alpha(1-\alpha)s_{t-1} + \alpha(1-\alpha)^2 s_{t-2} + \alpha(1-\alpha)^3 s_{t-3} + \alpha(1-\alpha)^4 s_{t-4} + \ldots + \alpha(1-\alpha)^{t-1} s_1 + (1-\alpha)^t s_1 \tag{4}$$

Thus, represents a weighted moving average of all past observations with the weights decreasing exponentially; hence the name “exponential smoothing”. Sometimes, this method is also called as “single exponential smoothing” or “simple exponential smoothing” and this method will give accurate forecasts only when the input time series has no trend or seasonal patterns, i.e., a stationary series. Thus, the first step in applying exponential smoothing to a given set of data is to test whether it is stationary or not.

2.3 State Space Formulation of the SES Method

In general, a state space model for a time series $\{Z_t, t = 1, 2, \ldots\}$ consists of two equations. The first is called the state space equation and determines the state X_{t+1} at time (t+1) in terms of the previous state X_t and a noise term. The state equation is

$$X_{t+1} = F_t X_t + W_t \text{, t= 1, 2...,} \tag{5}$$

where, $\{F_t\}$ is a sequence of $v \times v$ matrices, $\{W_t\} \sim N(0, \{Q_t\})$. Sometimes, the state equation may also have an external input. The second equation, called the observation equation, expresses the w-dimensional observation Z_t as a function of v-dimensional state variable X_t plus noise. Thus

$$Z_t = G_t X_t + V_t \text{, t= 1,2…,} \tag{6}$$

Where, $\{V_t\} \sim N(0, \{R_t\})$ and $\{G_t\}$ is a sequence of $w \times v$ matrices and $\{W_t\}$ *and* $\{V_t\}$ are uncorrelated.

According to eq. (3), a time series $\{Z_t\}$ has a state space representation if there exists a state space model for $\{Z_t\}$ as specified by equations (5) and (6). Thus, the basic equation of the simple exponential smoothing method, as given in eq. (2) can be represented in the state space form similar to Eq. (5) and Eq. (6) as given below. The state and measurement equations will then be

$$X_{t+1} = (1-\alpha)X_t + \alpha U_t + W_t, \tag{7}$$

$$Z_t = X_t + V_t, \tag{8}$$

Where αU_t in Eq. (7) is the external input with U_t being equal to $\widehat{X_t}$, estimate of the observation at time ‘t’. Once this equivalent state space model is established, the next step is to use Kalman filtering to recursively update the estimates. The following section presents Kalman filter briefly.

2.4 Kalman Filter Algorithm

The Kalman filter allows a unified approach to estimation and prediction for all processes that can be given a state space representation. State space representations and associated Kalman filter have a profound impact on time series analysis and many related areas. The Kalman filter estimates a process by using feedback procedure: the filter estimates the process state at some time and then improves the same using feedback in the form of measurements. The Kalman filter equations can fall into two groups: time update equations and measurement update equations. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain a priori estimates for the next time step. The measurement update equations are responsible for the feedback, i.e., for incorporating a new measurement into the a priori estimate to obtain an improved a posterior estimate. The time update equations can also be thought of as "predictor" equations, while the measurement update equations can be thought of as "corrector" equations.

The specific equations for implementing the Kalman filter with eq. (8) are given below

$$\hat{s}^{-}_{(k+1)} = (1-\propto)\hat{s}^{+}_{k} + \propto s_k , \quad (9)$$

$$P^{-}_{(k+1)} = (1-\propto) P^{+}_{(k)} (1-\propto) + Q_k , \quad (10)$$

$$K_{k+1} = P^{-}_{(k+1)} \left[P^{-}_{(k+1)} + R_{(k+1)} \right]^{-1} , \quad (11)$$

$$\hat{s}^{+}_{(k+1)} = \hat{s}^{-}_{(k+1)} + K_{k+1} \left(Z_{k+1} - \hat{s}^{-}_{(k+1)} \right) , \quad (12)$$

$$P^{+}_{(k+1)} = \left[1 - K_{k+1} \right] P^{-}_{(k+1)} . \quad (13)$$

Equations (9) and (10) are the time series update equations and eqs. (11) to (13) are the measurement update equations. The superscripts '–' and '+' denote the priori and posterior estimates respectively. In eqs. (10) and (13), $P^{-}_{(k+1)}$ and $P^{+}_{(k+1)}$ are priori and posterior error variance respectively. In eq. (11), K_{k+1} is the Kalman gain and the Q and R in Eqs. (10) and (11), are the process and measurement noise variance respectively. This set of steps is repeated for each time interval to obtain the corresponding estimates.

2.5 Augmented Dickey-Fuller Test

In Dickey-Fuller (DF) test, it has been assumed that the disturbance term u_t is uncorrelated. But when u_t is correlated, the test can be further augmented by preceeding three equations by adding the lagged values of the dependent variable ΔY_t . That's why the test coined the term Augmented Dickey Fuller (ADF) test. The ADF test here consists of estimating the following regression equation:

$$\Delta Y_t = \alpha + \beta t + \delta Y_{t-1} + \sum_{i=1}^{m} \gamma_i Y_{t-1} + u_t \quad (14)$$

In ADF, test of the null hypothesis is set as $\delta = 0$ and the test follows the same asymptotic distribution as the DF statistic.

2.6 Prediction Error Decomposition

State space methods are used to draw valid inference in state space models. It provides the estimate of parameters with the states for forecasting future states and observations. Kalman filter (Kalman and Bucy 1961) is useful for building the one-step-ahead predictor of Z_t and its mean square error matrix. The likelihood can be evaluated via the prediction error decomposition due to the independence of the one-step-ahead prediction errors.

The one-step ahead prediction errors $\left(Z_t - G_t' \hat{\alpha}_{t|t-1} \right)$ for t = *1*, *2*, ..., T are independently and identically distributed. This implies that the joint log likelihood function of prediction errors can be written as the sum of the log likelihoods at each spelling, that is,

$$\log L = -\frac{1}{2} \sum_{t=1}^{T} \log \left(G'_t \Sigma_{t|t-1} G_t + R_t \right) - \frac{1}{2} \sum_{t=1}^{T} \frac{\left(Z_t - H_t' \hat{\alpha}_{t|t-1} \right)^2}{\left(G'_t \Sigma_{t|t-1} G_t + R_t \right)} \quad (15)$$

The parameters $\left(\boldsymbol{F}_t, \boldsymbol{H}_t, R_t, \text{ and } \boldsymbol{Q}_t \right)$ are estimated by maximizing the log-likelihood function.

2.7 Procedures Used for Comparison and Validation of Different Models

Mean Squared Error (MSE)

Measurement of the difference between the values of the estimator and the estimated value is called as MSE of an estimator. It is defined as the average value of the squares of the deviations between the estimator and the estimated value.

$$\text{MSE} = \sqrt{\sum_{t=1}^{n} ((Z_t - \hat{Z}_t)^2 / n)} \quad (16)$$

Mean Absolute Error (MAE)

If there are n observations and forecast for n time periods, then there will be n error terms, and the mean absolute error is defined as:

$$\text{MAE} = \frac{1}{n}\sum_{t=1}^{n}\left|Z_t - \hat{Z}_t\right| \tag{17}$$

Mean Absolute Percentage Error (MAPE)

The average of all the percentage errors is the MAPE. It may be defined as an index of accuracy in terms of percentage of a method that can be used for developing the fitted time-series values in statistics, which can particularly be used in trend estimation. It does not take the sign of the data. The average is computed by summing their absolute values.

$$\text{MAPE} = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{Z_t - \hat{Z}_t}{Z_t}\right| \times 100 \tag{18}$$

Where, Z_t is the original value and $\hat{Z}_t$ is the forecasted value respectively.

3. RESULTS

Estimated Values of Smoothing Constant and Initial State

The following table illustrating the value of initial state and smoothing constant of various data using SES and proposed model are illustrated below; calculated values for smoothing constant and initial state in traditional approach (SES) for Pulse, Oil, Cumin, Soybean, Livestock and Fiscal data has mentioned in table 1.

Table 1. Values of smoothing constant and initial state of various data using SES

Data	Smoothing constant $(\acute{a})$	Initial State
Pulse	0.9999	102.62
Oil	0.8666	174264.50
Cumin	0.9631	14654.10
Soybean	0.9999	3316.56
Livestock	0.6857	336.60
Fiscal Deficit	0.0001	278.95

Calculated values for smoothing constant and initial state in traditional approach (SES) for Pulse, Oil, Cumin, Soybean, Livestock and Fiscal data has mentioned in table 2.

Table 2. Values of smoothing constant and initial state of various data using Kalman filter

Data	Smoothing constant $(\acute{a})$	Initial State
Pulse	0.9999	98.29
Oil	0.8460	154220.20
Cumin	0.9999	14660.00
Soybean	0.5840	3220.00
Livestock	0.4867	378.97
Fiscal Deficit	0.0983	-142.36

Comparison of SES and Integrated Model

The MSE, MAPE and MAE have been used as an error minimization criterion for comparing the base scenario with the improved scenario in this study. The scenario which gives the smallest error for the data can be considered as better method for prediction. Calculated values for statistical measures viz., MSE, MAE and MAPE for the two approaches SES and the Proposed Model for all the data are reported in the following tables.

Table 3. Goodness of fit for Pulse data

Methods	MSE	MAE	MAPE
SES (traditional)	31.09	4.54	2.54
SES (Kalman filter)	31.09	4.54	2.54

Table 4. Goodness of fit for Oil data

Methods	MSE	MAE	MAPE
SES (traditional)	2575947844	37600.80	99.24
SES (Kalman filter)	2568055778	37637.70	99.86

Table 5. Goodness of fit for Cumin future price data

Methods	MSE	MAE	MAPE
SES (traditional)	46563.42	176.03	1.18
SES (Kalman filter)	44811.83	170.77	1.14

Table 6. Goodness of fit for Soybean data

Methods	MSE	MAE	MAPE
SES (traditional)	3692.33	33.72	0.96
SES (Kalman filter)	2819.22	35.23	1.01

Table 7. Goodness of fit for Livestock data

Methods	MSE	MAE	MAPE
SES (traditional)	32299.75	157.09	11.48
SES (Kalman filter)	31893.01	145.11	10.38

Table 8. Goodness of fit for Fiscal deficit data

Methods	MSE	MAE	MAPE
SES (traditional)	196073.70	405.49	128.59
SES (Kalman filter)	168648.30	343.70	146.41

Estimated value of standard error using proposed model for Pulse, Oil, Cumin, Soybean, Livestock and Fiscal deficit data has mentioned in table 9.

Table 9. Values of standard error using proposed model

Data	Standard error
Pulse	0.010
Oil	0.008
Cumin	0.009
Soybean	0.006
Livestock	0.005
Fiscal deficit	0.003

4. DISCUSSION

The proposed model has a lower MSE compared the traditional SES for the data namely Oil, Cumin, Soybean, Livestock and Fiscal deficit. MAE and MAPE in most of the data has a lower value in proposed model than the traditional SES method. It has seen that the Kalman filter gives the lower MSE. The proposed model has lower MSE for all the data except Pulse in which it has equal MSE to the SES method. Hence, it may be concluded that SES method using the Kalman filter is proved to be much better as compared to the traditional approach for forecasting.

The study is also further proceeded for calculating standard error using Kalman filter as because SES does not provide standard error due to its ad-hoc in nature. This is one of the main disadvantage of traditional SES. Another significant restriction of this traditional SES is that it is heuristic in nature. So in this regard, Kalman filter methodology is used to correct the above problem. This method is ideally suited for finding out standard error of the estimates (Table 9).

In this SES approach, fitting of this model is usually carried out through algorithm of Grid search. The main drawback of this algorithm is that the number of possible models to be searched is extremely large and it requires a large computation time. Moreover, this method is only an approximate method and has problems with convergence of the estimates of parameters. To this end, a very efficient and powerful Kalman filter technique has been employed to rectify the above limitations.

5. CONCLUSION

In literature, most of the existing studies used only advanced time series models like Box-Jenkins ARIMA in a state space framework with Kalman filtering for improving the estimates. The present study is an attempt to fill the gap by proposing a state space approach for simple exponential smoothing, which is one of the most widely used smoothing methods. The present study also tries to answer the question of whether state space approach really improves the forecasting accuracy by applying it to various data in agricultural field. It is noted that, for the various data, proposed model has been showed with lower MSE, MAE and MAPE than the single exponential smoothing. Hence, it can be inferred that the SES with Kalman filter is performing relatively better than the traditional SES method. The proposed approach can be extended to other applications as well, when the main interest lies on to increase the forecasting accuracy of the exponential smoothing methods.

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REFERENCES

Box, G.E.P., Jenkins, G.M. and Reinsel, G.C. (1976). Time series analysis: Forecasting and control. San Fransco, Holdenday.

Box, G.E.P., Jenkins, G.M. and Reinsel, G.C. (2007). Time-Series Analysis: Forecasting and Control. 3rd edition, Pearson Education, India.

Brown, R.G. (1956). Exponential Smoothing for Predicting Demand. Cambridge, Mass, Arthur D. Little.

Holt, C.C. (1960). Planning Production, Inventories and Work Force. Prentice Hall, Inc., Englewood Cliffs, New Jersy.

Kalman, R.E. (1960). A New approach to linear filtering and prediction problems. *Trans. ASME J. Basic Eng,* **82**(1), 35-45.

Kalman, R.E. and Bucy, R.S. (1961). New results in linear filtering and prediction theory. *Journal of basic* engineering, **83**(1), 95-108.

Makridakis, S., Hibon, M. and Moser, C. (1979). Accuracy of forecasting: An empirical investigation. *Journal of the Royal Statistical Society, Series A (General)*, **142**(2), 97-145.

Makridakis, S., Andersen, A., Carbone, R., Fildes, R., Hibon, M., Lewandowski, R. and Winkler, R. (1982). The accuracy of extrapolation (time series) methods: Results of a forecasting competition. *Journal of* forecasting, **1**(2), 111-153.

Meinhold, R.J. and Singpurwalla, N.D. (1983). Understanding the Kalman filter. *The American* Statistician, **37**(2), 123-127.

Louka, P., Galanis, G., Siebert, N., Kariniotakis, G., Katsafados, P., Pytharoulis, I. and Kallos, G. (2008). Improvements in wind speed forecasts for wind power prediction purposes using Kalman filtering. *Journal of Wind Engineering and Industrial Aerodynamics* , **96**(12), 2348-2362.

Wolff, C,C. (1987). Forward foreign exchange rates, expected spot rates, and premia: a signal-extraction approach. *The Journal of* Finance, **42**(2), 395-406.

Harvey, A. and Koopman, S.J. (1993). Forecasting hourly electricity demand using time-varying splines. *Journal of the American Statistical Association*, **88**(424), 1228-1236.

Nasseri, M., Moeini, A. and Tabesh, M. (2011). Forecasting monthly urban water demand using extended Kalman filter and genetic programming. *Expert Systems with Applications*, **38**(6), 7387-7395.

Banbura, M. and Runstler, G. (2011). A look into the factor model black box: publication lags and the role of hard and soft data in forecasting GDP. *International Journal of* Forecasting, **27**(2), 333-346.

Huang, C., Li, X. and Lu, L. (2008). Retrieving soil temperature profile by assimilating MODIS LST products with ensemble Kalman filter. *Remote Sensing of* Environment, **112**(4), 1320-1336.

Bunn, D W. (1982). Short-term forecasting: A review of procedures in the electricity supply industry. *Journal of the Operational Research* Society, **33**(6), 533-545.

Gardner, J.E.S. and Diaz-Saiz, J. (2008). Exponential smoothing in the telecommunications data. *International Journal of* Forecasting, **24**(1), 170-174.

Kumar, T.L.M., Surendra, H.S. and Munirajappa, R. (2011). Holt-winters exponential smoothing and sesonal ARIMA time-series technique for forecasting of onion price in Bangalore market. *Mysore Journal of Agricultural Sciences*, **45**(3), 602-607.

Newbold, P. and Bos, T. (1994). Introductory business and economic forecasting. *International Journal of Forecasting* **16**, 451-476.

Ishii, N., Iwata, A. and Suzumura, N. (1981). Bilateral exponential smoothing of time series. *International Journal of Systems Science*, **12**(8), 977-988.

Su, Y., Gao, W., Guan, D. and Su, W. (2018). Dynamic assessment and forecast of urban water ecological footprint based on exponential smoothing analysis. *Journal of cleaner production*, **195**, 354-364.

Lim, C. and McAleer, M. (2001). Forecasting tourist arrivals. *Annals of Tourism Research*, **28**(4), 965-977.

Ghosh, B., Basu, B. and O'Mahony, M. (2005). January. Time-series modelling for forecasting vehicular traffic flow in Dublin. *In 84th Annual Meeting of the Transportation Research Board,* Washington, DC.

Hamilton, J.D. (1994). State-space models. *Handbook of econometrics* **4**, 3039-3080.

De Jong, P. and Penzer, J. (2004). The ARMA model in state space form. *Statistics and Probability Letters*, **70**(1), 119-125.