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Applications of Weibull Lomax Distribution: Review

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SUMMARY

In this paper, we reviewed the various works by different authors on Weibull Lomax distribution. The historical research based on some classical papers as well as ongoing research and advancements on Weibull Lomax distribution and its applications based on some latest papers were thoroughly reviewed and discussed in this paper. We mainly focused on various extensions of Weibull and Lomax models that have been used for developing a bathtub-shaped hazard function model for lifetime data. We also gave importance to study about the wide real applications of Weibull Lomax distribution in various fields including survival and lifetime data.

Keywords: Bathtub hazard rate; Lifetime data; Lomax distribution; Monotone hazard rate; Weibull distribution; Weibull lomax distribution.

1. INTRODUCTION

The two most widely utilized distributions for modeling lifetime data are the Weibull and Lomax distributions. The invention of Waladdi Weibull's Weibull distribution (WD) in 1939 made lifetime distribution modeling with monotone failure rates highly flexible. However, it was inadequate to provide a sufficient parametric fit for modeling instances of unimodal failure rates and non-monotone failure rates such as the bathtub and upside-down bathtub forms. Mudholkar and Hutson (1996) stated that it cannot be used to model lifetime data such as machine life cycles and human mortality with a bathtub-shaped hazard function.

The hazard function of traditional WD can only be either increasing, decreasing or constant. Consequently, lifetime data with a bathtub-shaped hazard function cannot be modeled using it. Many real-world data points in reliability engineering exhibit bathtub-shaped failure rates. Reliability engineering's priorities include things like minimizing the rate of device failures at earlier stage. It resulted in trying to figure out when it is appropriate to stop using the outdated technology rather than putting it to use. Therefore, it was crucial to look for a life distribution that can successfully handle the bathtub-shaped failure rate. The key objective of the majority of the works we investigated was to propose different extensions of the Weibull and Lomax models that have been applied to the modeling of any one of the three bathtub curve components. Several researchers had suggested various extensions, exponentiated and modified forms of WD by adding parameters ranging from two to five. Adding parameters resulted in a more complex and adaptable distribution that can model various types of data. An obvious reason for generalizing a standard distribution is the fact that the generalization will provide more flexibility to analyze real life data.

The Lomax distribution which was also known as "Pareto type II" (the shifted Pareto) by Lomax (1954) was first developed to model data on business failures. Several authors across a range of subjects have used the Lomax distribution. The Lomax model is an aspect of the decreasing failure rate family. Lomax (1954) added a fourth category of failure theory that explained business mortality in addition to David (1952) which explained the other three failure theories. The concept of record values for a set of independent, identically

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distributed Lomax distributed random variables was introduced by Ahsanullah (1991) who also inferred some of their distributional criteria. Campbell and Ratnaparkhi (1993) used the Lomax distribution as a model for rating data and derived the corresponding Receiver Operating Curve (ROC). They demonstrated how to use this method to analyze rating data generated during the investigation of neurological issues.

A number of extensions of the Weibull distribution have been developed to simulate bathtub-shaped failure rates as the distribution is inadequate for modeling phenomena with non-monotone failure rates. These include the additive Weibull by Xie and Lai (1995) and the modified Weibull extension by Xie *et al.* (2002) and the Exponentiated Weibull (EW) by Mudholkar *et al.* (1995). Mudholkar and Srivastava (1993) generalized the Weibull distribution which was ideal for modelling bathtub failure rate and demonstrated that the family can be used to verify that the Weibull model fits well. They introduced a simple version of the Weibull family known as the exponentiated-Weibull family which offered a broader class of monotone failure rates in addition to bathtub and unimodal failure rate distributions. A generalization of the Weibull family was proposed by Mudholkar *et al.* (1995) which included all four types of hazard functions. It allowed testing the goodness of fit of the Weibull distribution as sub models which was not achievable in most models with non-monotone hazard functions. The importance and adaptability of the family were demonstrated by examining five historical data sets on bus-motor failures from David (1952), that were representative of data in repair-reuse conditions and data related to a head-and-neck cancer clinical trial. These illustrations demonstrate unimodal, censored, bathtub, increasing and perhaps non-Weibull hazard-shape models.

A simple model was studied by Xie and Lai (1995) by integrating two Weibull survival functions. In their proposed additive Weibull model, two Weibull distributions, one with a decreasing failure rate and the other with an increasing failure rate are combined. The graphical estimating method is also demonstrated. To describe bathtub-shaped failure rate life time data, Xie *et al.* (2002) suggested an extended new distribution that was connected to the exponential and Weibull distributions. Corbelini *et al.* (2007) used the Pareto II distribution to represent queueing constraints and business size in the Italian chemical sector between 1999 and 2004.

Ghitany *et al.* (2007) studied the properties of a novel parametric distribution derived from the Lomax model and the Marshal and Olkin extended family of distributions. Using mixing exponential models, they suggested that the distribution could be characterized as a compound distribution. Abd-Elfattah and Alharbey (2010) used the generalized probability weighted moments (GPWM) approach to calculate the Lomax distribution's parameters. Their objective was to study the estimation problem for the parameters of Lomax distribution using the GPWM method. The most significant hazard rate forms namely constant, growing, decreasing, bathtub and upside-down bathtub can be produced by Tahir *et al.*'s (2010) three parameter Weibull Pareto distribution.

Exponentiated Lomax distributions (ELD) which was a new class of distributions that Moniem and Hameed (2012) introduced and were used in statistical analyses. They obtained the probability density function for the Exponentiated Pareto, Paretoand Lomax distribution from ELD. Ashour and Eltehiwy (2013) suggested a generalization of the Lomax distribution called transmuted Lomax distribution that can be applied for modeling reliability data.

Lemonte and Cordeiro (2013) presented the McDonald Lomax distribution which was a five-parameter continuous distribution that extended the Lomax distribution. They showed that a linear combination of the Lomax densities may be used to express the McLomax density function. Ramos *et al.* (2013) developed Exponentiated Lomax Poisson distribution which was a flexible tool for analyzing positive data. It was created by compounding the exponentiated Lomax distribution and Poisson distribution. They suggested that the model might find more widespread use in survival analysis for the modeling of positive real data sets.

By adding two more shape factors into the Lomax model, several researchers examined the Weibull-Lomax distribution which was a compound distribution made up of the Weibull distribution and Lomax distribution. The Weibull Pareto distribution was defined by Alzaatreh *et al*. (2013) who also acquired numerous features such as moments, hazard functions and moment generating functions. They found that the

Weibull Pareto distribution was unimodal and had a form that may be negatively or positively skewed.

Salem (2014) used four estimate techniques such as maximum likelihood, quasi likelihood, Bayesian under symmetric square loss function and quasi-Bayesian estimationto examine the parameters of the Exponentiated Lomax distribution.El-Bassiouny *et al.* (2015) established a novel version of the Lomax distribution known as the Exponential Lomax distribution. They moreover focused a number of statistical aspects such as the quantile and median. By using various loss functions, Ahmad *et al.* (2015) investigated the Lomax distribution's shape parameter and its Bayes estimator.

The Weibull-Pareto distribution was examined by Nasiru and Luguterah (2015) who used the maximum likelihood method to estimate the distribution's parameters and derived several of its properties. The modified Weibull model was further developed by Aryal and Elbatal (2015)who added exponentiated versions of the standard distributions including exponential, Rayleigh, Weibull and linear failure. They also derived several mathematical features like generating function, order statistics and ordinary moments.

The Weibull Lomax (WL) distribution was introduced by Tahir *et al.* (2015) as an extension of the Lomax distribution with a hazard rate function that exhibits both decreasing and increasing shapes. Their motivation was to use the Weibull-G generator as defined by Bourguignon *et al.* (2014)to provide an additional extension of the Lomax distribution. Additionally, some structural features of the WL distribution were examined including the mean residual life, mean waiting time, probability weighted moments, ordinary and incomplete moments, generating function and quantile function. Furthermore, they resulted at explicit formulations for q entropy, renyi entropy and order statistics. The proposed model is expected to be used more widely in fields including engineering, survival and lifetime data, hydrology, economics (income inequality) and others. In order to generalize the Weibull Lomax distribution introduced by Tahir *et al.* (2015), Afify *et al.* (2015) suggested a five-parameter model called Transmuted Weibull Lomax (TWL) distribution. The aim of the author was to provide more flexible extension of the Weibull Lomax (WL) distribution using the transmutation map technique. The TWL model will be applied in a broader range of areas such as engineering, meteorology, hydrology, survival and longevity statistics, economics (income inequality) and others.

The gamma Lomax distribution is an extension of the Lomax distribution proposed by Cordeiro *et al.* (2015) who further looked at the distribution's structural features. The potential of the concept is illustrated through an application to a cancer data. A three parameter Power Lomax distribution (POLO) with more flexibility for lifetime data specifically exhibiting a falling, inverted bathtub hazard rate function was introduced by Rady *et al.* (2016). A comparison of the POLO distribution with other competing distributions using real bladder cancer data showed that it would fit better than a set of Lomax distribution extensions. The Beta Exponentiated Lomax (BEL) distribution was a five-parameter continuous model proposed by Mead (2016) who also examined it. In addition, he estimated ordinary and incomplete moments, generating functions, quantile functions, mean residual life, mean waiting time, mean deviations, Bonferroni, Lorenz and Zenga curves and Renyi entropy. The BEL model could be used more widely in numerous fields including engineering, survival analysis, hydrology, economics and others.

Based on the Weibull-G family, the Exponentiated Weibull Lomax (EWL) distribution was a new class of distribution introduced by Hassan and Abd-Allah (2018). Tahir *et al.* (2015)'s Weibull Lomax distribution was generalized by the EWL distribution and presented few new models. Moments, mean residual life, order statistics, quantile, Renyi and q entropies were among the properties that were determined. A new Weibull Lomax (T-X) distribution was introduced by Hashmi (2018) who also established a number of characteristics for it including moments, a survival function, limiting behavior of its probability density and hazard rate functions. Furthermore, he illustrated the relationship between the Weibull Lomax distribution and the type I, Weibull and exponential extreme value distributions.

The Topp-Leone Weibull Lomax distribution which have four parameters was proposed by Jamal *et al.* (2019) who also looked into the distribution's mathematical properties such as the stress strength model, ordinary and incomplete moments, conditional moments, quantile function, probability weighted moment and order statistics. Regression modeling and residual analysis were also discussed. Falgore *et al.*

(2019) presented the Weibull Inverse Lomax (WIL) distribution which is a four-parameter probability model with decreasing, increasing and bathtub hazard functions. The WIL distribution is J-shaped, inversely J-shaped, and positively skewed. Moments, variance, MGF and quantile function were a few of the statistical properties that were discussed. Its application to the bladder cancer data set demonstrated the distribution's significance.

Baset and Ghazal (2020) proposed the exponentiated additive Weibull distribution, which included a set of exponentiated distributions such as the exponentiated Weibull and exponentiated Exponential distributions in addition to some established distributions such as the additive Weibull and modified Weibull distributions. It offered a reasonable parametric fit for modeling data with bathtub failure rates and was highly helpful in modeling reliability analysis including cost analysis, decision-making reliability and firmware reliability. A five-parameter distribution known as the Weibull power Lomax distribution was created by Hussain *et al.* (2020) to extend the power Lomax distribution using the Weibull G family. Additionally,they generated a number of structural properties, such as the hazard function, quantile function, moments, probability weighted moments and distribution of order statistics.

Exponentiated Lomax Weibull (ELW) distribution was established by Osagie and Osemwenkhae (2020) utilizing a mixture of two constructing approaches for developing lifetime distributions. They created various current and new distributions as sub models and determined a number of properties, including entropy measures, order statistics, residual and inverted residual lifetimes, moments, quantile function and mean deviation. Applications of the ELW distribution to data sets on bladder cancer and flood peaks demonstrated its flexibility and usefulness showing why it was preferable to some competing distributions.

A new lifetime distribution known as the Alpha Power Transformed Exponentiated Lomax distribution(APTEL) was introduced by Hassan *et al.* (2021). Alpha power transformed Lomax, alpha power transformed Pareto, and alpha power transformed exponentiated Pareto were among the novel models used. Additionally, it included exponentiated versions of the Lomax and Pareto distributions. The APTEL distribution›s various characteristics were also examined. They also developed an analytical bias correction that lowers relative mean squared error while concurrently reducing the estimator's one or two orders of magnitude in percentage bias.

The Alpha Power Lomax (APL) distribution was introduced by Bulut *et al.* (2021) as a new expansion of the Lomax distribution using the alpha power transformation technique. The APL distribution's parameters were estimated using the maximum likelihood estimation technique. The superiority of APL distribution over the existing Lomax distribution extensions was demonstrated by using the actual data from bladder cancer patient's remission durations. Rana *et al.* (2022) suggested an innovative Pareto X family of distributions and a four-parameter sub model of the proposed family known as the Pareto Weibull distribution. Along with distributional elements like moments, moment generating function, characteristic function, quantile function, random number generation and reliability function, the distribution of order statistics was also discussed.

The asymmetric double Lomax distribution, which is the ratio of two independent and identically distributed conventional Laplace distributions, was developed by Punathumparambath and Kulathinal (2022). By combining the Lomax and Weibull distributions, Alsuhabi *et al.* (2022)provided a continuous lifetime model with four parameters known as the Extended Odd Weibull Lomax (EOWL) distribution. The parameters of the EOWL model were estimated using unconventional techniques including Bayesian analytical approaches as well as traditional methods like maximum likelihood and maximum product of spacing. They made use of three already-existing COVID-19 data sets from UK, USA and Italy to demonstrate the efficiency of the corresponding distribution.

1.1 Remarks

1. For a = 1, the EWL distribution (Hassan and Abd-Allah, 2018)reduces to WL distribution (Tahir *et al.*, 2015).
2. For $\lambda = 1$ and $\alpha = 0$, the ELW distribution (Osagie and Osemwenkhae, 2020) reduces to Lomax distribution (Lomax, 1954).
3. For $\lambda = 1$ and $\beta = 0$, ELW distribution (Osagie and Osemwenkhae, 2020) reduces to Weibull distribution.

4. For β = 1, the APTEL distribution (Hassan *et al.,* 2021) reduces to EL distribution (Moniem and Hameed, 2012).
5. For β = 1 and α = 1, the APTEL distribution (Hassan *et al.,* 2021) reduces to Lomax distribution (Lomax, 1954).
6. For Y = $\left[\left(1+\dfrac{X}{\lambda}\right)^{\theta}-1\right]$, where x follows EWL distribution (Hassan and Abd-Allah, 2018), then Y reduces to EW distribution (Mudholkar and Srivastava, 1993).

2. APPLICATION

We illustrate a real data interpretation on the data representing 128 bladder cancer patients chosen at random and their remission durations were recorded. (Lee and Wang, 2003). We compared some important distributions such as Exponentiated Lomax Weibull (ELW) distribution, Exponential Lomax (EL) distribution, Weibull distribution, Lomax distribution, Weibull Lomax (WL) distribution and Weibull Inverse Lomax (WIL) distribution.

The parameter estimates values and the performances of the distributions are shown in the Table 1. The goodness of fit statistics used to compare the performances are the Akaike Information Criterion (AIC) and the Consistent Akaike Information Criterion (CAIC).AIC and CAIC are computed as follows:

$$AIC = -2ll + 2k$$

$$CAIC = -2ll + \frac{2kn}{n-k-1},$$

where n is the sample size, k is the number of parameters and ll is the log likelihood function. To estimate parameters, we utilize R's optimal package. Better model fits are indicated by smaller values of the AIC and CAIC statistics. Among all fitted models, we observe that the EL model has the lowest AIC and CAIC values. Fig. 1 shows the data histogram and the predicted PDFs for the fitted models. The dataset analysis's findings demonstrate the EL distribution's superiority over the comparison distributions. Thusamong the compared distributions, the EL model may be selected as the most appropriate model for the dataset in consideration.

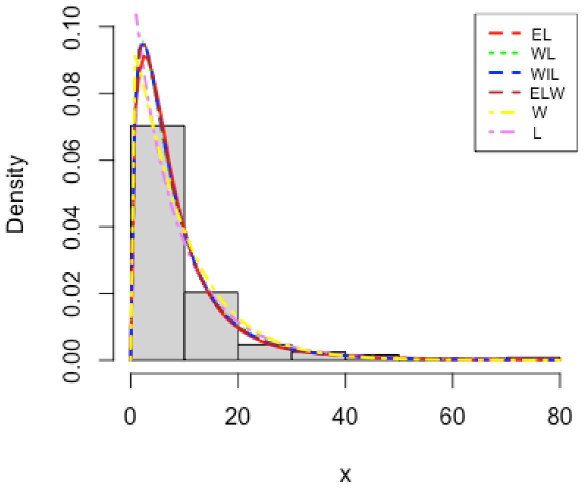

Fig. 1. Estimated pdfs for the Cancer dataset

Table 1. Parameter estimates, loglikelihood, information criteria and goodness-of-fit values of compared distributions for the data set

Model	**Estimates**					**Statistic**			
	γ (error)	**β (error)**	**α (error)**	**θ (error)**	**λ (error)**	**-2loglik**	**AIC**	**CAIC**	**KS (pvalue)**
ELW	0.0706 (0.0986)	3.2982 (2.5767)	0.5850 (3.231	0.1893 (0.4987)	4.9315 (22.4444)	813.5626	823.5626	824.0544	0.0301 (0.9998)
EL	0.0450 (0.0269)	4.3350 (1.8328)	–	–	1.6386 (0.2816)	815.0204	821.0203	821.2139	0.0408 (0.9835)
WEIBULL	–	–	0.0947 (0.0191)	1.0513 (0.0675)	–	823.7850	827.7849	827.8809	0.0720 (0.5201)
LOMAX	0.0125 (0.0067)	9.6614 (4.7930)	–	–	–	823.4134	827.4135	827.4135	0.1043 (0.1237)
WL	16.7811 (14.7253)	1.7275 (1.2112)	0.0439 (0.0926)	–	0.6160 (0.4311)	825.1196	833.1195	833.4447	0.0743 (0.4804)
WIL	21.5206 (14.3133)	4.1645 (3.6316)	7.1872 (3.6600)	–	0.3742 (0.3381)	814.3908	822.3909	822.7161	0.0356 (0.9969)

3. CONCLUSION

We introduced a literature review of Weibull Lomax distribution and its associated distributions that have been used to model lifetime data with a bathtub shaped hazard function. We have also provided a section of real data analysis. In this section, we have considered a real dataset on bladder cancer patients with their remission times and six lifetime models to assess these models' feasibility. We have given its model section criterion as AIC and CAIC, log-likelihood value, KS statistic and p-value and parameter estimations. Researchers studying lifetime distribution will find this review useful.

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REFERENCES

Abdul-Moniem, I.B. and Abdel-Hameed, H.F. (2012). On exponentiated Lomax distribution. *International Journal of Mathematical Archive,* **3(3)**, 1129-1138.

Abdel-Baset, A.A. and Ghazal, M.G.M. (2020). Exponentiated additive Weibull distribution. *Reliability Engineering and System Safety,* **193(5)**, 106634.

AbdElfattah, A. (2010). Estimation of Lomax parameters based on generalized probability weighted moment. *Journal of King Abdulaziz University–Science,* **22**, 171-184.

Afify, A., Nofal, Z., Yousof, H.M., Gebaly, Y. and Butt, N. (2015). The transmuted Weibull Lomax distribution: Properties and application. *Pakistan Journal of Statistics and Operation Research,* **11**, 135-152.

Ahmad, S.P., Ahmad, A. and Ahmed, A. (2015). Bayesian analysis of shape parameter of Lomax distribution using different loss functions. *International Journal of Statistics and Mathematics,* **2(1**), 55-65.

Ahsanullah, M. (1991). Record values of the Lomax distribution. *StatisticaNeerlandica,* **45**, 21-29.

Alsuhabi, H., Alkhairy, I., Almetwally, E.M., Almongy, H.M., Gemeay, A.M., Hafez, E.H., Aldallal, R.A. and Sabry, M. (2022). A superior extension for the Lomax distribution with application to COVID-19 infections real data. *Alexandria Engineering Journal,* **61(12)**, 11077-11090.

Alzaatreh, A., Famoye, F. and Lee, C. (2013). Weibull–Pareto distribution and its applications. *Communications in Statistics–Theory and Methods*, **42(9)**, 1673-1691.

Aryal, G. and Elbatal, I. (2015). On the exponentiated generalized modified Weibull distribution. *Communications for Statistical Applications and Methods,* **22(4)**, 333-348.

Ashour, S. and Eltehiwy, M. (2013). Transmuted Lomax distribution. *American Journal of Applied Mathematics and Statistics,* **1**, 121-127.

Bourguignon, M., Silva, R.B. and Cordeiro, G. M. (2022). The Weibull-G family of probability distributions. *Journal of Data Science,* **12(1)**, 53-68.

Bulut, Y.H., Dogru, F.Z. and Arslan, O. (2021). Alpha power Lomax distribution: Properties and application. *Journal of Reliability and Statistical Studies,* **14(1)**, 17-32.

Campbell, G. and Ratnaparkhi, M.V. (1993). An application of Lomax distributions in receiver operating characteristic (ROC) curve analysis. *Communications in Statistics–Theory and Methods,* **22(6)**, 1681-1687.

Corbellini, A., Crosato, L., Ganugi, P. and Mazzoli, M. (2010). Fitting Pareto II distributions on firm size: Statistical methodology and economic puzzles. In *Proceedings* , 321-328.

David, D.J. (1952). An analysis of some failure data. *Journal of the American Statistical Association,* **47**, 113-150.

El-Bassiouny, A., Abdo, N. and Salah, H. (2015). Exponential Lomax distribution. *International Journal of Computer Applications,* **121**, 24-29.

Falgore, J., Doguwa, S. and Isah, A. (2019). The Weibull-inverse Lomax (WIL) distribution with application on bladder cancer. *Biometrics & Biostatistics International Journal,* **8**, 195-202.

Ghitany, M.E., Al-Awadhi, F.A.and Alkhalfan, L.A. (2007). Marshall–Olkin extended Lomax distribution and its application to censored data. *Communications in Statistics–Theory and Methods,* **36(10)**, 1855-1866.

Giles, D.E., Feng, H.and Godwin, R. T. (2013). On the bias of the maximum likelihood estimator for the two-parameter Lomax distribution. *Communications in Statistics–Theory and Methods,* **42(11)**, 1934-1950.

Hashmi, S. (2018). A new Weibull-Lomax (T-X) distribution and its application. *Pakistan Journal of Statistics,* **34**, 495-512.

Hassan, A., Khaleel, M.and Elmorsy, R. (2021). An extension of exponentiated Lomax distribution with application to lifetime data. *Journal of Reliability and Statistical Studies,* **19**, 484-500.

Hassan, A.S.and Abd-Allah, M. (2022). Exponentiated Weibull-Lomax distribution: Properties and estimation. *Journal of Data Science,* **16(2)**, 277-298.

Hussain, N., Doguwa, S.and Yahaya, A. (2021). The Weibull-power Lomax distribution: Properties and application. *Journal of Statistics,* **6**, 869-881.

Jamal, F., Reyad, H., Nasir, M., Chesneau, C., Abdul Nasir, J.& Ahmed, S. (2021). Topp Leone Weibull Lomax distribution: Properties, regression model, and applications. *NED University Journal of Research,* **17(1)**, 1-15.

Lee, E.T.and Wang, J.W. (2003). *Statistical methods for survival data analysis* (3rd ed.). Wiley.

Lemonte, A.J.and Cordeiro, G.M. (2013). An extended Lomax distribution. *Statistics,* **47(4)**, 800-816.

Lomax, K.S. (1954). Business failures: Another example of the analysis of failure data. *Journal of the American Statistical Association,* **49(268)**, 847-852.

M. E. (2016). On five-parameter Lomax distribution: Properties and applications. *Pakistan Journal of Statistics and Operation Research,* **12**, 185.

Mudholkar, G.S.and Srivastava, D.K. (1993). Exponentiated Weibull family for analyzing bathtub failure-rate data. *IEEE Transactions on Reliability,* **42(2)**, 299-302.

Mudholkar, G. S., Srivastava, D. K.and Freimer, M. (1995). The exponentiated Weibull family: A reanalysis of the bus-motor-failure data. *Technometrics,* **37(4)**, 436-445.

Mudholkar, G.S.and Hutson, A.D. (1996). The exponentiated Weibull family: Some properties and a flood data application. *Communications in Statistics–Theory and Methods,* **25(12)**, 3059-3083.

Nasiru, S.and Luguterah, A. (2015). The new Weibull-Pareto distribution. *Pakistan Journal of Statistics and Operation Research,* **11(1)**, 103-116.

Osagie, S.and Osemwenkhae, J. (2023). Exponentiated Lomax-Weibull distribution with properties and applications. *Journal of Statistics,* **6(2)**, 814-833.

Popović, B., Cordeiro, G.and Ortega, E. (2013). The gamma-Lomax distribution. *Journal of Statistical Computation and Simulation,* **85**, 1441-1454.

Punathumparambath, B.&Kulathinal, S. (2015). Double Lomax distribution and its applications. *Statistica,* **75**, 107-121.

Rady, E. H., Hassanein, W.and Elhaddad, T. (2016). The power Lomax distribution with an application to bladder cancer data. *SpringerPlus,* **5(1)**, 857.

Ramos, M., Marinho, P.R., Silva, R.and Cordeiro, G. (2013). The exponentiated Lomax Poisson distribution with an application to lifetime data. *Advances and Applications in Statistics,* **34**, 1-15.

Rana, M.S., Hanif, S.S., Qaiser, M.S.and Rahman, M.M. (2022). Pareto Weibull distribution with properties and applications: A member of Pareto-X family. *Pakistan Journal of Statistics and Operations Research,* **18(1)**, 121-132.

Salem, H. (2014). The exponentiated Lomax distribution: Different estimation methods. *International Journal of Applied Mathematics and Statistics,* **2**, 364-368.

Tahir, M., Cordeiro, G., Mansoor, M.and Zubair. (2014). The Weibull-Lomax distribution: Properties and applications. *Hacettepe University Bulletin of Natural Sciences and Engineering Series B: Mathematics and Statistics,* **43(2)**, 385-402.

Tahir, M.H., Gauss, A.A.M.M., Cordeiro, G.M.and Zubair, M. (2016). A new Weibull–Pareto distribution: Properties and applications. *Communications in Statistics–Simulation and Computation,* **45(10)**, 3548-3567.

Xie, M. and Lai, C.D. (1996). Reliability analysis using an additive Weibull model with bathtub-shaped failure rate function. *Reliability Engineering & System Safety,* **52(1)**, 87-93.

Xie, M., Tang, Y. and Goh, T.N. (2002). A modified Weibull extension with bathtub-shaped failure rate function. *Reliability Engineering & System Safety,* **76(3)**, 279-285.