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Some Competing Accelerated Life Testing Models with a Bayesian Perspective*

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SUMMARY

Several competing accelerated life testing models have been studied for modeling accelerated failure times by many authors (see Meeker and Escobar, 1998). This article briefly reviews several aspects and properties of some competing accelerated life testing models namely the log-normal, the inverse Gaussian and the gamma models which are widely recognized in reliability theory. In these situations, modeling of the mean time to failure is modeled as a function of the stress variable, while assuming other parameter(s) to be constant. In this paper we considered the modeling of coefficient of variation also as a function of the stress variable, that has not been considered in the literature, in the best knowledge of the authors. We propose a Bayesian approach for finding a point estimate as well as credible interval estimate for the coefficient of variation under the assumption that the data are drawn from one of the above mentioned distributions.

Key words and phrases: Inverse Gaussian Model; Log-normal Model; Gamma Model; Coefficient of Variation; Bayesian Approach.

Dedication

The authors would like to dedicate this article to the memory of Dr. Manindra Nath Das (Dr. M.N. Das). Dr. Das was born on February 1, 1923 at Khalishpur, district Khulna, Bangladesh. He received his education with a Master's degree in Statistics in 1945 and a Ph. D. degree in 1965, both from Calcutta University, Kolkata. After obtaining his M.Sc. degree, he joined the Indian Agricultural Statistics Research Institute (IASRI) in 1946. At IASRI, he began his research and teaching innovations and served in the positions of Professor, Senior Professor and its Director. The innovations carrying Dr. Das's mind-prints were particularly in design and analysis of experiments and included reinforced incomplete block designs (Das 1958), a new class of designs called circular designs, a unified method of constructing confounded designs for asymmetrical factorial experiments, methods of construction of rotatable designs and the use of balanced incomplete block designs (BIBD) in their construction, BIBDs for biological assays, designs for experiments with mixtures, authored/coauthored several books including a textbook on Design and Analysis of Experiments. The contributions of Dr. Das in research and teaching were very influential to would be researchers and statisticians for further development. During the Green Revolution years of 1960s and 1970s, there was a lot of scope and emphasis on contributing to agricultural research, particularly in Design of experiments, and many young researchers were inspired to seek the guidance of Dr. M.N. Das. The dream of the third co-author [MS] also came true in 1974 when he was accepted to pursue his Ph. D. degree at IASRI, New Delhi during 1974-1976 for a specialization in Design of Experiments under the guidance of Dr. Das. But soon after the start of the session, Dr. Das joined the Central Water Commission as its Director at New Delhi. MS was then duly assigned the opportunity of receiving the guidance of another eminent research professor, namely, Dr. Aloke Dey who was Professor Das's student. Part of the information in this section has been incorporated from the document with link https://ssca.org.in/media/Brief_biodata_of_Dr_MN_Das.pdf (accessed 20 November 2025.)

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1. INTRODUCTION

'Accelerated life test' (ALT) applies to the type of study where failure times can be accelerated by applying higher 'stress' to the component, that is, where higher stress may bring quicker failure of the component. For example, some components may fail quicker at a higher temperatures, however, it may take a longer time before failure occurs at lower temperatures. The variable or a combination of variables, such as temperature and pressure, that may accelerate failure is generally called 'stress' factor. The need for an accelerated life test in practice arises where the objective is to study the relation between failure and stress conditions. At low 'stress' conditions, the time required may be sufficiently large for its reliability estimation which can be tested under higher stress factors terminating the experiment in a relatively shorter time. By this process the failures which in normal conditions would occur only after a long testing can be observed quicker and the size of data can be increased without a large cost and long time. In contrast, accelerated life testing provides economic collection of data in order to be able to estimate the reliability that can be projected at the lower stress level, through appropriate models. In ALTs, test units are subjected to the elevated stress levels, and then the test results are extrapolated from the test conditions to the normal operating condition using a reasonable statistical model. The common failure mechanism is considered as a necessary condition for the extrapolated procedure in ALT.

For the extrapolation of higher stress mechanism to the lower stress situation, the unit lifetime is assumed to follow a parametric distribution, assuming the mean time to failure (MTTF) to be a function of the stress variable, while assuming other parameter(s) to be constant. For example, assuming the lognormal distribution as the failure time model, engineers can conclude that the failure mechanisms are the same if the shape parameters at different stress levels are equal. In this paper, we propose (see Section 2) modeling of the Coefficient of Variation (CV) that is often used as a measure of heterogeneity or lack of uniformity in a population in practice where variable of interest is positive, as a function of the stress variable as well. Let the mean and standard deviation of the random variable X be denoted by ν and σ respectively. The coefficient of variation (CV) is defined as the ratio of σ/ν. In practice, CV is used in situations yielding positive measurements (Johnson and Welch, 1940). Gaussian distribution based approaches are presumably applied in such studies, even though Gaussian distribution is defined on the whole real line. An obvious estimate of CV based on a random sample of size n is given by the ratio of the sample standard deviation S and the sample mean $\bar{X}$, namely the sample CV $\hat{\theta} = S/\bar{X}$.

Several authors have discussed the estimation of common CV for normal populations. There are several real life situations in industry and agriculture, where the variable of interest is in positive range [Chhikara and Folks (1977, 1989), Takagi *et al.* (1997), Chaubey *et al.* (2017), Singh (1993), Chapter 15 of Johnson *et al.* (1994), among others] and follows inverse Gaussian (IG) distribution. In fact, the key parameters to verify whether the failure mechanisms are same under the parametric model are generally related to the coefficient of variation. For example, Meeker and Escobar (1998) analyzed the temperature-accelerated lifetime data of a particular kind device by using lognormal model, and assessed the reasonableness of the constant scale parameter. Yin *et al.* (2017) studied specimens of solid epoxy electrical insulation in an accelerated voltage life test using power-Weibull distribution based on the assumption of the fixed shape parameter. The models presented here are commonly used in modelling ALTs as they are appropriate for modelling positive observations and positively skewed distributions. The inverse Gaussian and gamma models offer more flexibility in contrast to log-normal models as they provide a variety of shapes through inherent shape and scale parameters, where as for the log-normal models (used by Nelson (1971) and others), only the location and scale changes, and thus, the shape remains fixed as it results from a symmetric distribution. Further, inverse Gaussian (Chhikkara and Folks 1977) and Gamma (Sen and Chaubey 2011) models are flexible models for modelling the tails of positively skewed data.

In lognormal and Weibull cases, the coefficients of variation only depends on the scale parameter and the shape parameter respectively. Thus, it is reasonable to compare the coefficients of variation at different stress levels in order to show the consistency of the failure mechanisms. Earlier Bayesian approaches proposed in literature modelled mean and variance. Since CV is the parameter of relatively higher importance, the

Bayesian approach in this study models in terms of CV in addition to mean.

This paper considers the Bayesian modeling of the failure times, in the setting of accelerated life tests, by modelling the mean (ν) and coefficient of variation (θ) as simple functions of the stress variable. This is motivated in Section 2 through an example based on a data set that has been used by various authors in the literature on accelerated life testing. Section 3 summarizes three common models using the lognormal (LN), Gamma and the inverse Gaussian distributions for the life times, in terms of mean and coefficient of variation in this set-up. This section also explores a simple way of specifying priors on the parameters of the model, which have been explored in analyzing the data using Gibbs sampling. The final section (Section 4) presents the conclusions of the study.

2. MODELS FOR FAILURE TIME DATA

In stochastic modeling of failure time, the fatigue life time distribution leads to a prominent role in the engineering literature. Bhattacharyya and Fries (1982) assumed the fatigue to be governed by a Wiener process, and the time to failure distributed as the first passage time distribution, providing an inverse Gaussian model for the failure time. For a stochastic relation of the failure time (T) to the intensity of stress X, (often assumed to be inversely proportional to the stress level), we have assumed here that the severity of the stress levels does not change the form of the life time distribution but the stress levels have an influence on the value of the parameters. In this case, the parameter ν, representing the mean time to failure (MTTF), is taken to play most important role and has a direct relation to x because it measures the mean fatigue growth per unit time. Since, in accelerated life testing, it may be assumed that higher stress produces smaller mean failure time, they considered the following simple model

$$\nu(x) = \gamma_0 + \gamma_1 x$$

In a practical situation, we only consider $\gamma_0 + \gamma_1 x > 0$ on a finite interval of x which corresponds to the range of stress x. But we assume that the origin is taken at the lower point of this interval, i.e. γ_0. In this paper we explore only the simple linear form, though this may be generalized to polynomial forms, to describe nonlinear relations.

Traditionally, in the accelerated life testing, mean time to failure (MTTF) is modelled using the linear regression model for $Y = \log(T)$, that is supported by a stochastic formulation of the failure process. Nelson (1975) presented statistical methods for planing and analyzing accelerated life tests with the Arrhenius model. This model should be of interest for estimating the life which is assumed to be a function of temperature, the life distribution is log-normal. In this case, the logarithmic standard deviation of the distribution is assumed to be constant, and the mean function $\mu(x)$ of the logarithmic life $Y = \log(T)$ is a linear function *i.e.*

$$\mu(x) = \alpha + \beta x.$$

In this model, the coefficient α is the average value of the logarithm of the life time when stress level $x = 0$ and β is the slope of the line, i.e., change in the mean value of Y for a unit change in stress level x.

Table 1. Hours to failure (T) for class-H insulation material

190^0C	220^0C	240^0C	260^0C
7228	1764	1175	600
7228	2436	1175	744
7228	2436	1521	744
8448	2436	1569	744
9167	2436	1617	912
9167	2436	1665	1128
9167	3108	1665	1320
9167	3108	1713	1464
10511	3108	1761	1608
10511	3108	1953	1896

Table 2. Mean and CV of the hours to failure (T) at different stress level

	Temperature Level			
	190^oC	220^oC	240^oC	260^oC
Mean	8782.20	2637.6000	1581.4000	1116.0000
S. D.	1244.0117	453.5654	244.2745	439.2357
CV	14.165	17.196	15.447	39.3587

To estimate this regression function, a lognormal model may simplify the analysis, however this may not be appropriate in some cases, as we demonstrate in the following example. Here we consider the data from Gnedenko *et al.* (1999) on life times of new class-H motor insulation (see the data in Table 1) that had been earlier reported in Nelson (1971, Fig. 1). In this example, 10 motorettes were periodically examined for insulation failure at four elevated temperature settings at 190^0C, 220^0C, 240^0C and 260^0C. Then the given failure time is found midway between the inspection time when the failure was found and the time of the previous inspection.

The linear regression model required to estimate the mean time to failure at different stress levels assumes a constant error variance which necessarily implies a constant coefficient of variation, for the lognormal model (see Sec. 3.1). However, in the above data this may be violated (see Table 2 and Figure 1), and therefore we may be tempted to use alternative regression relations and/or other lifetime distributions. A regression relation of $Y* = \log(T)$ on $X = 10^5/(8.6171\ Temp)$ where the temperature *Temp* is measured on the Kelvin scale, i.e.,

Temp = *Temp in Celsius* + 273.15.

The regression equation is

$y* = -7.28 + 0.649x$

This output shows a good fit of a simple linear model on the log-scale (R^2= 91%). A quadratic model, i.e., regressing $Y* = \log(T)$ on X and X^2, provides only a slight improvement with R^2 to 92%. On the other hand, regressing directly the failure time T on X (simple linear model) results in R^2 = 83% while T on X and X^2 (a quadratic model) produces R^2 = 95%.

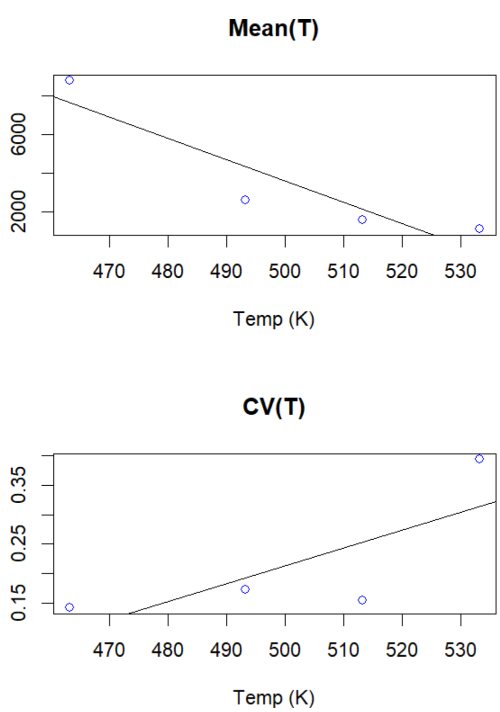

Fig. 1. Mean and CV of the time to failure (T) in hours at different temperature levels (observed means as points and fitted lines).

With the above example in the background, we propose to investigate modeling the mean (ν) and coefficient of variation (CV) θ of life times *T*, directly. We will consider the models based on simple regression relationships for ν and θ, in terms of *x*, in the present article. However, we may consider more general polynomial or nonlinear relationships in other studies.

$\nu(x) = \gamma_0 + \gamma_1 x;\ \theta(x) = \delta_0 + \delta_1 z;\ z = 1/x.$

Next we describe the most common accelerated failure probability models of the distribution of T that we will consider for further discussion in the following subsections.

3. SOME ACCELERATED LIFE TESTING MODELS WITH ν, θ PARAMETRIZATION

3.1 Log-normal model

The lognormal density of T considers the mean μ and variance parameter σ^2 for the distribution of $\log(T)$ and is given by

$$f(t\,|\,\mu,\sigma) = \frac{1}{t\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln(t)-\mu}{\sigma}\right)^2},\ (-\infty < \mu < \infty, \sigma, t > 0)$$

with

$$\mu = \log\frac{v}{\sqrt{1+\theta^2}} \text{ and } \sigma = \log(1+\theta^2).$$

The re-parametrized form in terms of (ν, θ) follows due to the following results on the log-normal random variable.

$$\nu = E(T) = \exp\left(\mu + \frac{1}{2}\sigma^2\right) \text{ and}$$

$$\theta = \sqrt{\frac{V(T)}{v^2}} = \sqrt{e^{\sigma^2} - 1}$$

where $Var(T) = (e^{\sigma^2} - 1)e^{2\mu+\sigma^2} = v^2(e^{\sigma^2} - 1)$.

Therefore, the one-to-one relation between (μ, σ) and (ν, θ) is given by the expressions in the above.

In the context of Bayesian inference, we consider direct modeling of MTTF as

$T_{ij} \sim$ lognormal $(\nu_j = \gamma_0 + \gamma_1 x_j;\ \theta_j = \delta_0 + \delta_1 z_j)$

with the likelihood function given by

$$L(\gamma_0, \gamma_1, \delta_0, \delta_1|\mathbf{T}) = \prod_{j=1}^{k}\prod_{i=1}^{n_j} f\,(t_{ij}|\nu_j = \gamma_0 + \gamma_1 x_j;\ \theta_j = \delta_0 + \delta_1 z_j)$$

where $\mathbf{T} = \{\mathbf{T_1}, \mathbf{T_2}, ..., \mathbf{T_k}\}$; $\mathbf{T_j} = \{\mathbf{t_{1j}}, \mathbf{t_{2j}}, ..., \mathbf{t_{n_j}j}\}$ and

$$f(y\,|\,\nu,\theta) = \frac{1}{t\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln(t)-\mu}{\sigma}\right)^2}$$

with $\mu = \log\frac{\nu}{\sqrt{1+\theta^2}}$; $\sigma^2 = \log(1 + \theta^2)$.

Specification of priors for $\gamma_0, \gamma_1, \delta_0, \delta_1$**:**

For simplicity we propose vague priors for each component,

$p(\gamma_0, \gamma_1, \delta_0, \delta_1) = p(\gamma_0)p(\gamma_1)p(\delta_0)p(\delta_1)$

where

$$p(\gamma_0) = \frac{1}{b_0 - a_0} I_{[a_0, b_0]}(\gamma_0)$$

$$p(\gamma_1) = \frac{1}{b_1 - a_1} I_{[a_1, b_1]}(\gamma_1)$$

$$p(\delta_0) = \frac{1}{d_0 - c_0} I_{[c_0, d_0]}(\delta_0)$$

$$p(\delta_1) = \frac{1}{d_1 - c_1} I_{[c_1, d_1]}(\delta_1)$$

where a_0, b_0, a_1, b_1, c_0, d_0, c_1, d_1 are specified by expert opinion. More flexible modeling with normal priors (with mean and precision information) may be specified, however we have not considered it here. With these specifications we propose to use Gibbs sampler to find the posterior of the parameters and the corresponding credible intervals using HPD region.

3.2 Gamma model

For the gamma model with shape parameter α and scale parameter β, the density is given by

$f(y|\alpha, \beta) = 1/\beta^{\alpha}\,\Gamma(\alpha)t^{\alpha-1}\,e^{-y/\beta}$, $(\alpha, \beta, y > 0)$

We may note that the (ν, θ) parametrization is obtained by the relations

$\nu = \alpha\beta$; $\theta^2 = 1/\alpha$

and the corresponding likelihood function and prior similar to those given in the lognormal case.

where

$$L(\gamma_0, \gamma_1, \delta_0, \delta_1|\mathbf{T}) = \prod_{j=1}^{k}\prod_{i=1}^{n_j} f\,(T_{ij}|\nu_j = \gamma_0 + \gamma_1 X_j;\ \theta_j = \delta_0 + \delta_1 Z_j)$$

$$f(y|\nu, \theta) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} y^{\alpha-1} e^{-y/\beta},$$

for $\alpha = 1/\theta^2$; $\beta = \nu\theta^2$.

3.3 Inverse Gaussian model

For the inverse Gaussian model the density with (ν, θ) parametrization is given by

$$f(t|\nu, \theta) = \left(\frac{\nu}{2\pi\theta^2 t^3}\right)^{1/2} \exp\left\{-\frac{1}{2\theta^2}\frac{(t-\nu)^2}{\nu t}\right\}. \qquad (\nu, \theta, t > 0)$$

and the likelihood function is given by

$$L(\gamma_0, \gamma_1, \delta_0, \delta_1|\mathbf{T}) = \prod_{j=1}^{k}\prod_{i=1}^{n_j} f\,(T_{ij}|\nu_j = \gamma_0 + \gamma_1 X_j;\ \theta_j = \delta_0 + \delta_1 Z_j)$$

and the Bayesian computation may follow naturally.

To examine the quality of fit to the observed data offered by the models considered here, we computed: (a) three commonly used goodness-of-fit statistics, K-S, A-D, C-von-M (the p-values not presented) and (b) two goodness-of-fit information criteria, AIC and BIC (to account and penalize for the number of model parameters in addition to the estimated likelihood) (Table 3). The estimates of mean, scale and shape parameters of these three distributions are given in Table 4. These statistics are readily available in R software (Ihaka and Gentleman, 1996). Here, K- S: Kolmogorov-Smirnov statistic, C-von-M: Cramer-von Mises statistic, A-D: Anderson-Darling statistic, AIC: Akaike's Information Criterion, BIC: Bayesian Information Criterion.

Table 3. Goodness-of-fit Statistics and Goodness-of-fit Information Criteria

Temperature		LN	Gamma	IG
190^oC	Kolmogorov-Smirnov Statistic	0.2508	0.2428	0.2489
	Cramer-von Mises Statistic	0.1120	0.1065	0.1095
	Anderson-Darling Statistic	0.7358	0.6974	0.7061
	Akaike's Information Criterion	174.0	173.9	174.0
	Bayesian Information Criterion	174.6	174.5	174.6
220^oC	Kolmogorov-Smirnov Statistic	0.2629	0.2650	0.2550
	Cramer-von Mises Statistic	0.1763	0.1761	0.1695
	Anderson-Darling Statistic	1.0957	1.0622	1.0112
	Akaike's Information Criterion	154.6	154.2	154.6
	Bayesian Information Criterion	155.2	154.8	155.2
240^oC	Kolmogorov-Smirnov Statistic	0.2230	0.2149	0.2326
	Cramer-von Mises Statistic	0.1162	0.1052	0.1186
	Anderson-Darling Statistic	0.7995	0.7029	0.7336
	Akaike's Information Criterion	142.5	142.0	142.4
	Bayesian Information Criterion	143.1	142.6	143.0
260^oC	Kolmogorov-Smirnov Statistic	0.2268	0.2129	0.2163
	Cramer-von Mises Statistic	0.0716	0.0619	0.0637
	Anderson-Darling Statistic	0.4296	0.3758	0.3813
	Akaike's Information Criterion	151.6	151.8	151.4
	Bayesian Information Criterion	152.2	152.4	152.0

LN: Lognormal, IG: Inverse Gaussian

Table 4. Fitted distribution parameters' estimates

Distribution	Parameter	190^oC	220^oC	240^oC	260^oC
LN	meanlog sdlog	9.072 0.133	7.864 0.162	7.355 0.145	6.952 0.361
Gamma	shape rate	55.380 0.006	37.57 0.01425	46.57 0.029	7.17 0.0064
IG	mean shape	8782 467627	2638 86888	1581 63791	1116 7631

LN: Lognormal, IG: Inverse Gaussian

Table 5. The intervals for uniform distribution as vague priors for all the distributions

(a_0, b_0)	(a_1, b_1)	(c_0, d_0)	(c_1, d_1)
(-51420.67, -51258.21)	(2362.965, 2369.946)	(-1.304923, -1.29477)	(35.20763, 35.44266)

Intervals (Estimate - K1*SE, Estimate + K2*SE) used for all the three distributions lognormal, gamma and inverse Gaussian, where SE = estimated standard error of the intercept and slope parameters in $\nu(x) = \gamma_0 + \gamma_1 x$; $\theta(x) = \delta_0 + \delta_1 z$; $z = 1/x$. $K1 = 0.001$, $K2 = 0.01$

γ_0 : Estimate $= -51405.9$, SE $= 14769.2$. γ_1 : Estimate $= 2363.6$, SE $= 634.6$. δ_0 : Estimate $= -1.304$, SE $= 0.923$. δ_1 : Estimate $= 35.229$, SE $= 21.366$.

4. POSTERIOR DISTRIBUTIONS OF THE CV AT VARIOUS STRESS LEVELS

As proposed in Section 3.1 (Lognormal distribution) the vague priors for each component from (γ_0, γ_1, δ_0, δ_1), a set of values of a_0, b_0, a_1, b_1, c_0, d_0, c_1 and d_1 obtained by fitting a linear regression of associated means and CVs on stress levels (x and z) are given in Table 5.

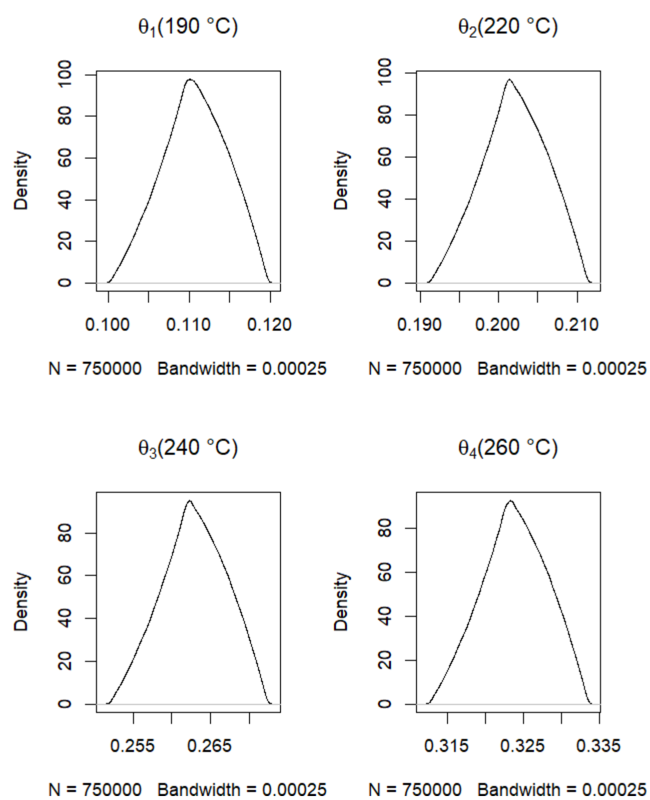

Fig. 2. Posterior density for CV for different stress levels for lognormal model

Table 6. Posterior means and 95% CI (credible interval) for the CV at different stress levels

Distribution	Stress	CV	Mean	SD	Median	Coverage	LCL	UCL	Width
LN	190^oC	θ_1	0.1107	0.004	0.1107	0.9503	0.1032	0.1182	0.015
	220^oC	θ_2	0.202	0.0041	0.202	0.9504	0.1943	0.2098	0.0155
	240^oC	θ_3	0.2629	0.0042	0.2629	0.9503	0.255	0.2708	0.0158
	260^oC	θ_4	0.3238	0.0042	0.3238	0.9501	0.3158	0.3319	0.0161
Gamma	190^oC	θ_1	0.1179	0.0013	0.1182	0.9504	0.1154	0.1197	0.0043
	220^oC	θ_2	0.2095	0.0013	0.2098	0.9501	0.2069	0.2113	0.0044
	240^oC	θ_3	0.2705	0.0013	0.2708	0.9503	0.2679	0.2724	0.0045
	260^oC	θ_4	0.3316	0.0014	0.3319	0.9502	0.3289	0.3335	0.0046
IG	190^oC	θ_1	0.1177	0.0014	0.118	0.9502	0.1149	0.1197	0.0048
	220^oC	θ_2	0.2092	0.0014	0.2095	0.9501	0.2064	0.2113	0.0049
	240^oC	θ_3	0.2703	0.0015	0.2706	0.9501	0.2674	0.2724	0.005
	260^oC	θ_4	0.3313	0.0015	0.3316	0.9503	0.3284	0.3335	0.0051

LN: Lognormal, IG: Inverse Gaussian

LCL = Lower credible limit and UCL = Upper credible limit.

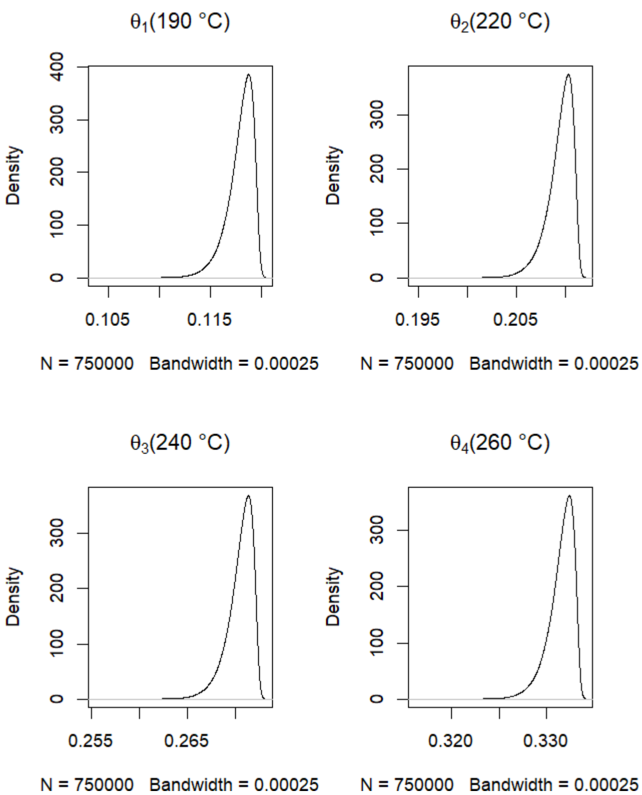

Fig. 3. Posterior density for CV for different stress levels for Gamma model

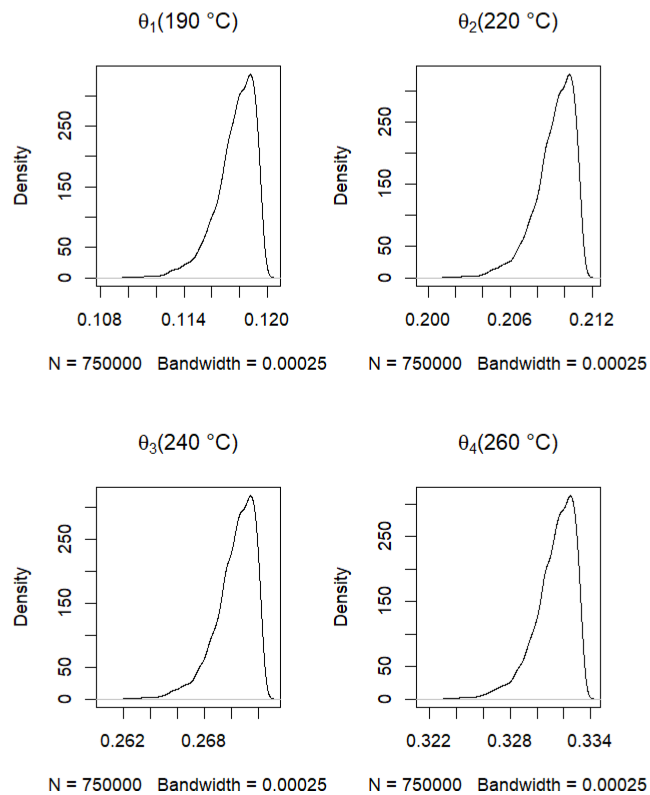

Fig. 4. Posterior density for CV for different stress levels for inverse Gaussian model

The posterior distributions of the CV at different stress levels were obtained using the bugs() function in R package R2OpenBUGS (Ihaka and Gentleman, 1996). In the bugs() function, we set three chains, each with 500,000 iterations (first 250,000 iterations discarded) resulting into a total of 750,000 iterations. The R codes and models required in the computations are given in the Appendix. The posterior mean, median and highest posterior density (HPD) interval for θ_i (i = 1...4) are presented in Table 6, and density plots in Figures 2 - 4. The bandwidth in the density plots is computed for the smooth kernel estimation with a built-in function in R software which generally uses unbiased cross-validation method based on minimizing the integrated squared error.

5. RESULTS AND DISCUSSION

Although the number of observations was small (n = 10) at each stress level, the studied distributions showed reasonably close fit at each stress level (Table 3). The estimates of the parameters of the fitted distributions are given in Table 4. The stability or relative variation measured as coefficient of variation (CV) was the main point of this study with a view that it may be constant over the stress levels. For the Bayesian estimation, we used the vague priors for the regression coefficients in relating mean (ν) and CV(θ) with the stress level. The uniform distributions were taken as vague priors over the intervals (estimate - K1*SE, estimate + K2*SE)(Table 5). The posterior mean ν showed decreasing trend with the stress level (tables not included). Posterior mean of the coefficient of variation (θ) tended to increase with the stress level, for each distribution. The widths of credible intervals for CV (θ) were found shortest for the Gamma distribution followed by the Inverse Gaussian distribution and the log-normal distribution.

In order to recommend estimates the CV (θ) values under the Bayesian framework for this data set, we may choose the best out of the three distributions – lognormal, gamma and inverse Gaussian, based on the Goodness-of-fit statistics and criterion. One way would be to use the rank out of 1 - 3 (1, with the minimum value, implies the best). The averages over all these rank values obtained from Table 3 show that the Gamma distribution stands the best followed by the inverse Gaussian followed by the lognormal distribution. The posterior means for the CV values of the hours to failure based on gamma distribution likelihood are $\theta_1 = 11.8 \pm 0.13\%$, $\theta_2 = 21.0 \pm 0.13\%$, $\theta_3 = 27.1 \pm 0.13\%$ and $\theta_4 = 33.1 \pm 0.14\%$ at temperatures 190°C, 220°C, 240°C and 260°C, respectively.

The priors assumed were vague priors and the inference is dominated by likelihood. In reality, they should be replaced by subjective priors.

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APPENDIX: R CODES

1. Common R codes for data and bugs() function

```
# 1. Read the hours to failure data y as array (y[i,j]; i=1...n; j=1...nL).

# n= number of insulation materials (10), nL=number of temperature levels (4).

n<- 10; nL<- 4; N<- n*nL

y<- array(c(y1,y2,y3,y4), dim=c(n,nL))

tempC<- c(190,220,240, 260)

temp<- tempC + 273.15 # To kelvin scale

x<- 10^5/(8.6171*temp)

z<- 1/x

# Read the uniform interval limits for a0, b0, a1, b1, c0, d0, c1, d1

data<- list("y","x", "z", "nL", "n", "a0","b0", "a1", "b1", "c0","d0", "c1", "d1")


#2. Assign initial values for random generation of gamma s gam0, gam1 and delta s del0, del1

inits1<-

list(gam0=runif(1, a0, b0), gam1=runif(1, a1, b1),

del0=runif(1, c0,d0), del1=runif(1, c1, d1))

inits2<-

list(gam0=runif(1, a0, b0), gam1=runif(1, a1, b1),

del0=runif(1, c0,d0), del1=runif(1, c1, d1))

inits3<-

list(gam0=runif(1, a0, b0), gam1=runif(1, a1, b1),

del0=runif(1, c0,d0), del1=runif(1, c1, d1))

inits <-

list(inits1, inits2, inits3)

inits

parameters <- c("theta", "nu", "gam0", "gam1", "del0", "del1")

parameters

#3. Models for the priors given in the next section and stored in file " ALT_nu_theta_bug.txt".

# Run the function bugs().

Out.sim <- bugs(data, inits, parameters,
           "ALT_nu_theta_bug.txt",
           n.chains=3,    n.iter=100000,
           debug=TRUE)

print(Out.sim, digits=12)

attach.bugs(Out.sim)
```

2. Models for various priors

```
# Model 1(Data assumed drawn from a lognormal distribution;

# uniform priors for gam0, gam1,del0, del1).

model {

# Generate random gam0, gam1,del0, del1 from unif[a, b]

   gam0 ~ dunif(a0, b0)

   gam1 ~ dunif(a1, b1)

   del0 ~ dunif(c0, d0)

   del1 ~ dunif(c1, d1)

   for(j in 1:nL){      for(i in 1:n){

#Get dloglik

   y[i,j] ~ dlnorm(mu[j],tau[j])

   }  #over i observations within j th level

nu[j]<-     gam0 + gam1 * x[j]

theta[j]<-  del0 + del1 * z[j]

tau[j]<- 1/log(1+theta[j])

mu[j]<-     log(nu[j]) - 0.5/tau[j]

}  #over j levels of Xj or zj

   } # end of BUGS codes


# Model 2 (Data assumed drawn from a gamma distribution;

# uniform priors for gam0, gam1,del0, del1).

model {
```

```
# Generate random gam0, gam1,del0, del1 from unif[a, b]
gam0 ~ dunif(a0, b0)
gam1 ~ dunif(a1, b1)
del0 ~ dunif(c0, d0)
del1 ~ dunif(c1, d1)
for(j in 1:nL){   for(i in 1:n){
y[i,j] ~ dgamma(r[j],mu[j])
}  #over i observations within j th level
nu[j]<-     gam0 + gam1 * x[j]      # x[j] <- 10^5/(8.6171*temp[j])
theta[j]<-  del0 + del1 * z[j]      # z[j] <- 1/x[j]
r[j]<- 1/pow(theta[j],2)
mu[j]<-     1/(nu[j]*pow(theta[j],2))
}  #over j levels of Xj or zj
} # end of BUGS codes

# Model 3 (Data assumed drawn from an inverse Gaussian distribution;
# uniform priors for gam0, gam1,del0, del1).
model {
# Generate random gam0, gam1,del0, del1 from unif[a, b]
gam0 ~ dunif(a0, b0)
gam1 ~ dunif(a1, b1)
del0 ~ dunif(c0, d0)
del1 ~ dunif(c1, d1)
for(j in 1:nL){   for(i in 1:n){
#Get function dloglik()
dummy[i,j]<- 0
dummy[i,j] ~ dloglik(LL[i,j])
LL[i,j]<- 0.5*log(nu[j])-log(theta[j])+
          (1 - 0.5* (y[i,j]/nu[j] + nu[j]/
          y[i,j]))/(pow(theta[j],2))
}  #over i observations within j th level
nu[j]<- gam0 + gam1 * x[j] # x[j] <- 10^5/(8.6171*temp[j])
theta[j]<- del0 + del1 * z[j] # z[j] <- 1/x[j]
# theta[j]<- ( del0 + del1 * z[j] ) * step(del0 + del1 * z[j] )
}  #over j levels of Xj or zj
} # end of BUGS codes
```