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Weibull -Fréchet Distribution for Modeling Annual Maximum Rainfall of India

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SUMMARY

Accurate modelling of extreme rainfall is fundamental to hydrological risk assessment and climate-impact studies. Although several flexible distributional families have been developed to capture complex tail behaviour, their empirical performance for rainfall extremes requires rigorous evaluation. This study provides a comprehensive empirical assessment of the Weibull–Fréchet distribution for modelling annual maximum rainfall in India. Long-term gridded monthly rainfall records spanning 1900–2016 were used to construct annual maximum rainfall series for the All-India and Peninsular regions. Model parameters were estimated using maximum likelihood method, and the performance of the Weibull–Fréchet distribution was systematically evaluated against the classical Fréchet and Weibull distribution using standard goodness-of-fit criteria and graphical diagnostics. The results demonstrate that the Weibull–Fréchet distribution offers a superior empirical fit, particularly in the upper tail, and yields balanced and physically plausible return-level estimates. These findings highlight the practical relevance of the Weibull–Fréchet distribution for modelling rainfall extremes across distinct climatic regions.

Keywords: Weibull–Fréchet distribution; Annual maximum rainfall; Extreme value analysis; Goodness-of-fit; Return-level estimation; Rainfall extremes.

1. INTRODUCTION

Extreme rainfall events often determine the most significant hydrological impacts, and therefore modelling maximum rainfall is essential for understanding the behavior of extreme observations rather than typical precipitation patterns. Maximum rainfall values generally reflect upper-tail characteristics, making them suitable for analysis using flexible statistical models that can describe heavy tails, varying shapes, and diverse hazard-rate structures. Classical probability distributions such as the Gamma, Lognormal, Weibull, and Log-Pearson Type III have long been used in practice; however, their limited adaptability often restricts their ability to represent the complex distributional features displayed by extreme rainfall data. These limitations have encouraged the development of generalized probability families that introduce additional shape parameters to improve model flexibility and tail representation.

Significant progress has been made in constructing such generalized families through parameter induction and transformation techniques. Foundational contributions include the Beta-G family (Eugene *et al.,* 2002), Kumaraswamy-G (Jones, 2004), and multiple Gamma-G variants (Zografos & Balakrishnan, 2009; Torabi & Montazeri, 2012; Ristić & Balakrishnan, 2012). Additional extensions include the Log-Gamma-G Type-2 model (Amini *et al.,* 2012), the Mc-G class (Alexander *et al.,* 2012), the exponentiated generalized family (Cordeiro *et al.,* 2013), Weibull-X and Gamma-X families (Alzaatreh *et al.,* 2013, 2014), Logistic-G (Torabi & Montazeri, 2014), odd-generalized exponential-G (Tahir *et al.,* 2015a, b), Type-I half-logistic (Cordeiro *et al.,* 2016), the new Weibull-G family (Tahir *et al.,* 2016), and the

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generalized transmuted-G distribution (Nofal *et al.,* 2017). These studies demonstrate that incorporating additional parameters significantly enhances the capacity of probability models to capture diverse distributional patterns, especially those involving strong asymmetry and heavy tails.

Recent research on extreme rainfall modelling underscores the need for probability distributions that can accommodate complex tail behavior. For example, Hussain and Abbas demonstrated that the Fréchet and Log logistic models are effective for rainfall frequency analysis in Azad Jammu and Kashmir, particularly highlighting the Fréchet distribution's strength in representing heavy tailed precipitation extremes. Likewise, the work of Ng *et al.* (2024) and Otieno *et al.* (2025) reinforces the demand for flexible statistical models capable of capturing climatic extremes under increasing variability. Although numerous generalized families have been proposed, the Weibull–Fréchet distribution—which merges the heavy-tail behavior of the Fréchet distribution with the flexibility imparted by a Weibull transformation—has not yet been examined in the context of maximum rainfall. Its structural characteristics suggest considerable potential for capturing the distributional nature of extreme observations.

Accordingly, the present study investigates the performance of the Weibull–Fréchet distribution for modelling annual maximum rainfall data. The analysis involves estimating the parameters of the model using likelihood-based numerical technique and evaluating its fit relative to commonly applied alternatives. The findings highlight the capability of the Weibull–Fréchet distribution to provide an effective representation of extreme rainfall behavior.

This paper is organized as follows. Section 2 describes the methodological approach, including the development of the Weibull–Fréchet distribution, its survival and hazard rate functions, the maximum likelihood–based parameter estimation procedure, and the benchmark distributions employed for comparison. Section 3 presents the rainfall datasets and reports the outcomes of model fitting, goodness-of-fit evaluation, diagnostic assessment, and return-level analysis for the All-India and Peninsular regions. The paper concludes in Section 4 with a summary of the key findings and a discussion of their implications for the analysis of extreme rainfall.

2. METHODOLOGY

2.1 The Weibull–Fréchet Distribution

Let X follow the Weibull–Fréchet (WF) distribution with parameter vector $\Theta = (\alpha, \gamma, \theta)$, where $\alpha > 0$ is the Weibull-type shape parameter and $\gamma > 0$, $\theta > 0$ are the scale and shape parameters of the Fréchet baseline model. (Ahmad *et al.,* 2018)

The Fréchet cumulative distribution function (cdf) is

$$F_0(x;\gamma,\theta) = \exp\left[-\left(\frac{\gamma}{x}\right)^{\theta}\right], \qquad x > 0.$$

Using the Weibull transformation, the cdf of the WF distribution becomes

$$F(x;\Theta) = 1 - \exp\left\{-\left[\frac{-\log\left(1-F_0(x)\right)}{1-F_0(x)}\right]^{\alpha}\right\}.$$

The corresponding probability density function (pdf) is

$$f(x;\Theta) = F_{0'}(x)\alpha\left[\frac{-\log\left(1-F_0(x)\right)}{1-F_0(x)}\right]^{\alpha-1}$$

$$\frac{1+H_0(x)}{[1-F_0(x)]^{\alpha+1}}\exp\left\{-\left[\frac{H_0(x)}{1-F_0(x)}\right]^{\alpha}\right\},$$

where

$$H_0(x) = -\log\left(1-F_0(x)\right).$$

2.2 Survival and Hazard Functions

The survival function of the WF distribution is

$$S(x;\Theta) = 1 - F(x;\Theta) = \exp\left\{-\left[\frac{H_0(x)}{1-F_0(x)}\right]^{\alpha}\right\}.$$

The hazard rate function is

$$h(x;\Theta) = \frac{f(x;\Theta)}{S(x;\Theta)} = \alpha\, F_{0'}(x)\frac{\left[1+H_0(x)\right]H_0(x)^{\alpha-1}}{[1-F_0(x)]^{\alpha+1}}$$

(Ahmad *et al.,* 2018)

2.3 Parameter Estimation

The parameter vector $\Theta = (\alpha, \gamma, \theta)$ is estimated using the maximum likelihood estimation (MLE) method.

For a sample $x_1, \ldots, x_n$, the log-likelihood function is

$$\ell(\Theta) = \sum_{i=1}^{n} \log f(x_i; \Theta).$$

Since the likelihood equations do not yield closed-form solutions, the estimates are obtained numerically using the Newton–Raphson algorithm.

2.4 Competing Models

To benchmark the performance of the WF distribution, two classical extreme-value models are considered:

(i) Weibull Distribution

The Weibull distribution (Weibull, 1951) is defined by the cdf

$$F_W(x; \alpha, \beta) = 1 - \exp(-\beta x^{\alpha}),$$

$$f_W(x) = \alpha \beta x^{\alpha - 1} \exp(-\beta x^{\alpha}).$$

(ii) Fréchet Distribution

The Fréchet distribution (Fréchet, 1927) is given by

$$F_F(x; \gamma, \theta) = \exp\left[-\left(\frac{\gamma}{x}\right)^{\theta}\right],$$

$$f_F(x) = \theta \gamma^{\theta} x^{-(\theta+1)} \exp\left[-\left(\frac{\gamma}{x}\right)^{\theta}\right].$$

All parameters for the competing models are estimated using MLE for consistency.

2.5 Goodness-of-Fit Evaluation

Model adequacy is assessed using the following criteria:

- Kolmogorov–Smirnov statistic (KS)
- Anderson–Darling statistic (AD)
- Cramér–von Mises statistic (CVM)
- Akaike Information Criterion (AIC)
- Bayesian Information Criterion (BIC)

Lower values indicate a better fit. Graphical tools such as Q–Q plots are also used to support comparative evaluation.

3. RESULTS AND DISCUSSION

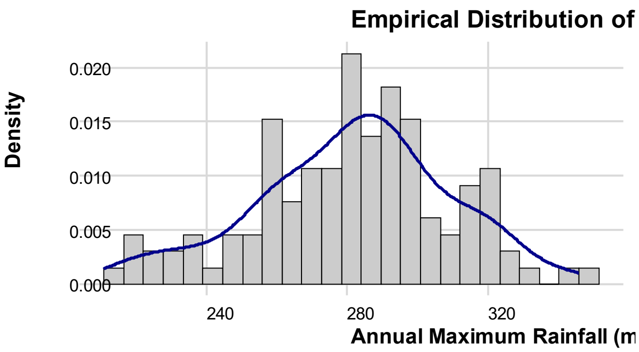
To assess the empirical performance of the Weibull–Fréchet distribution, the model was applied to two long-term annual maximum rainfall datasets corresponding to the All-India and Peninsular India regions, representing distinct climatic regimes of India. The analysis is based on gridded monthly rainfall records spanning 1900–2016, obtained from the Indian Institute of Tropical Meteorology (IITM), Pune. Annual maximum rainfall series were constructed for each region by extracting the maximum monthly rainfall value for each year, yielding 117 extreme rainfall observations per dataset. Although the source data consist of monthly rainfall measurements, the analysis focuses on annual maxima, consistent with the block-maxima approach commonly employed in extreme value analysis.

3.1 Data Description

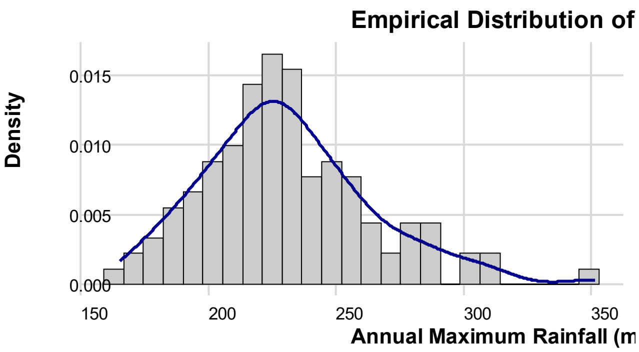
Table 1 summarizes the descriptive statistics of the two datasets. The All-India series exhibits moderate variability and near-symmetric behavior, whereas the Peninsular India series displays greater dispersion, positive skewness, and heavier tail characteristics. These distributional contrasts suggest that Peninsular India experiences more intense and irregular extreme rainfall episodes compared to the national aggregate. Fig. 1 depicts the empirical distribution of annual maxima for both regions.

Table 1. Descriptive statistics of data

	Min.	Max.	Mean	Median	SD	Skewness	Kurtosis
All India	211.00	346.00	281.39	283.60	27.81	-0.28	-0.04
Peninsular Region	165.40	351.60	230.83	226.00	33.19	0.70	0.91



(a)



(b)

Fig. 1. Histograms of annual maximum rainfall for (a) All-India and (b) Peninsular regions

3.2 Distribution Fitting and Parameter Estimation

The fitted models were evaluated through maximum likelihood estimation, and the resulting parameter estimates for both the All-India and Peninsular rainfall series are presented in Table 2. Clear differences emerge across regions and model structures. For the Fréchet distribution, the scale parameter g is appreciably higher for the All-India dataset, while the shape parameter t remains identical for both regions, indicating comparable tail behavior. In contrast, the Weibull distribution reveals more pronounced spatial variation: the All-India series exhibits substantially larger shape and scale parameters, reflecting its more compact distribution and lower inter-annual variability. The Weibull–Fréchet distribution, designed to enhance flexibility through the addition of the deformation parameter a, captures these contrasts even more effectively. Higher values of a and g for the All-India series, together with the elevated tail parameter t for the Peninsular dataset, indicate a distinctly heavier-tailed rainfall structure in the latter. Collectively, these parameter patterns highlight the contrasting extremal characteristics of the two regions and provide the basis for the comparative goodness-of-fit analysis presented in the subsequent section.

Table 2. Maximum likelihood parameter estimates (with standard errors) for the Fréchet, Weibull, and Weibull-Fréchet distributions fitted to the All-India and Peninsular India annual maximum rainfall series

Model	Parameter	All India Estimate	Peninsular Estimate
Fréchet	g	272.30 (21.84)	218.21 (17.41)
	t	5.00 (0.11)	5.00 (0.11)
Weibull	Shape	11.42 (0.80)	6.76 (0.43)
	Scale	293.72 (2.51)	245.50 (3.57)
Weibull-Fréchet	a	1.41 (0.29)	0.76 (0.17)
	g	280.72 (32.22)	234.76 (13.11)
	t	4.43 (0.11)	5.00 (0.09)

Note: *Values in parentheses denote the standard errors of the maximum likelihood estimates.*

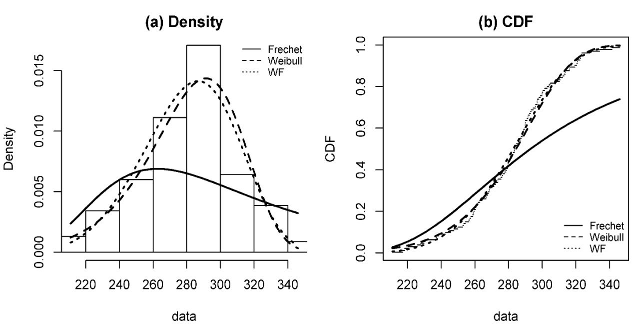
3.3 Model Performance and Diagnostic Evaluation

The empirical suitability of the Weibull–Fréchet distribution was investigated using annual maximum rainfall data from the All-India and Peninsular regions, with performance evaluated relative to the classical Fréchet and Weibull distributions. Model adequacy was assessed using standard numerical goodness-of-fit criteria, including the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Kolmogorov–Smirnov (KS), Cramér–von Mises (CM), and Anderson–Darling (AD) statistics, as summarized in Table 3. For the All-India series, the Weibull–Fréchet distribution achieves the minimum AIC and consistently lower KS, CM, and AD values, indicating improved agreement with the empirical distribution, particularly in the tail region. Although the Weibull distribution reports a slightly lower BIC, the overall evidence from distribution-based test statistics favors the Weibull–Fréchet distribution. In the case of the Peninsular dataset, the Weibull–Fréchet distribution clearly outperforms both competing distributions across all goodness-of-fit measures, reflecting its ability to capture greater variability and more pronounced extreme behavior.

These numerical results are reinforced by the graphical diagnostics presented in Fig. 2 and 3 for the All-India dataset and Fig. 4 and 5 for the Peninsular dataset. The density and cumulative distribution function comparisons [Fig. 2(a–b) and Fig. 4(a–b)] demonstrate that the Weibull–Fréchet distribution closely follows the empirical distributions over the full range of observations, while noticeable departures are observed for the Fréchet distribution, particularly at higher rainfall levels. Further insight into tail behavior is provided by the QQ and PP plots [Fig. 3 and Fig. 5], where the Weibull–Fréchet distribution exhibits near-linear alignment with the reference line across a wide range of quantiles. In contrast, the Fréchet and Weibull distributions display systematic deviations at higher quantiles, more evident for the Peninsular series. Overall, the combined numerical and graphical evidence confirms that the Weibull–Fréchet distribution provides a more flexible and reliable representation of annual maximum rainfall for both regions, supporting its use in subsequent return-level estimation and risk analysis.

Table 3. Goodness-of-fit statistics for the Fréchet, Weibull, and Weibull–Fréchet distributions fitted to the All-India and Peninsular annual maximum rainfall series

Region	Model	AIC	BIC	KS	CM	AD
All India	Fréchet	1212.49	1218.01	0.31	14.56	2.81
	Weibull	1114.53	1120.05	0.06	0.51	0.09
	Weibull-Fréchet	1113.04	1121.32	0.05	0.33	0.05
Peninsular Region	Fréchet	1186.65	1192.18	0.19	6.90	1.22
	Weibull	1175.05	1180.58	0.11	2.49	0.39
	Weibull-Fréchet	1154.07	1162.35	0.09	0.91	0.17



Note: *Lower values of AIC, BIC, KS, CM, and AD indicate better model fit.*

Fig. 2. Diagnostic plots for the All-India annual maximum rainfall series comparing the Fréchet, Weibull, and Weibull–Fréchet distributions:(a) empirical density with fitted distribution densities,(b) empirical and fitted cumulative distribution functions

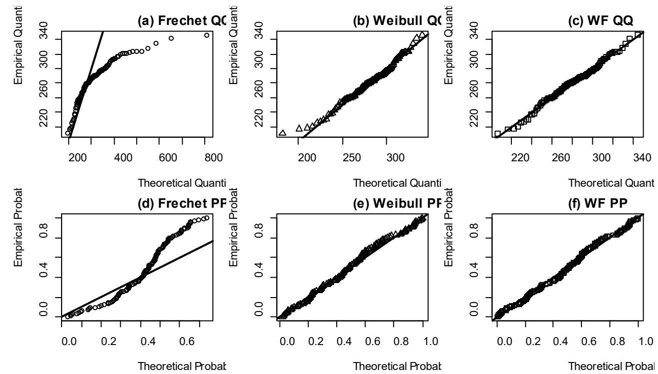


Fig. ***3.*** *Quantile–quantile (QQ) and probability–probability (PP) diagnostic plots for the fitted Fréchet, Weibull, and Weibull–Fréchet distributions based on annual maximum rainfall over the All-India region.*

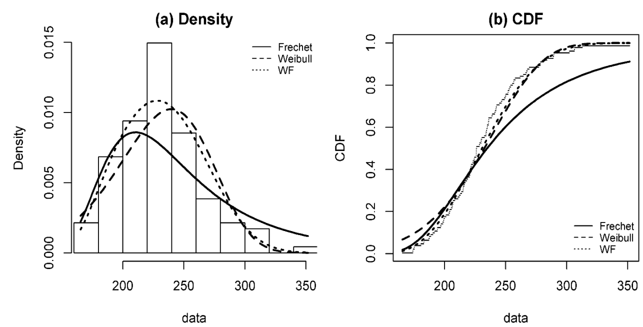


Fig. 4. Diagnostic plots for the Peninsular annual maximum rainfall series comparing the Fréchet, Weibull, and Weibull–Fréchet distributions: (a) empirical density with fitted distribution densities, (b) empirical and fitted cumulative distribution functions

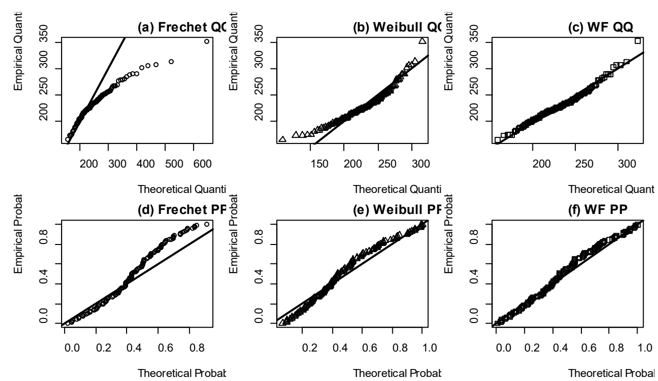


Fig. 5. Quantile–quantile (QQ) and probability–probability (PP) diagnostic plots for the fitted Fréchet, Weibull, and Weibull–Fréchet distributions based on annual maximum rainfall over the Peninsular region

3.4 Return Level Analysis

Return level analysis is commonly employed in extreme value modelling to characterize the magnitude of infrequent events associated with specified recurrence intervals. A return level denotes the rainfall amount that is expected to be exceeded, on average, once over a given period and is obtained from the fitted distribution of annual maximum rainfall. In this work, return levels are estimated using the Fréchet, Weibull, and Weibull–Fréchet models to examine differences in upper-tail behaviour relevant to design applications.

Return-level analysis was undertaken to quantify design-oriented extreme rainfall magnitudes and to examine the sensitivity of upper-tail estimates to the assumed distribution. Using the fitted Fréchet, Weibull, and Weibull–Fréchet distributions, 10, 25, 50, and 100-year return levels were derived separately for the All-India and Peninsular annual maximum rainfall series (Table 4). In both regions, return levels increase with return period, and the All-India series consistently exhibits larger values than the Peninsular series, consistent with its broader spatial domain and wetter hydroclimatic regime.

For All-India, the Fréchet distribution yields return levels of approximately 427, 517, 594, and 683 mm for the 10, 25, 50, and 100-year horizons, clearly exceeding the corresponding 316–339 mm range from the Weibull–Fréchet model and the slightly lower 316–336 mm range from the Weibull distribution in each case. A similar ordering is observed for the Peninsular series, where Fréchet estimates increase from about 342 mm (10-year) to 548 mm (100-year), whereas the Weibull–Fréchet model produces intermediate values near 278–314 mm and the Weibull distribution yields comparable or marginally smaller levels of roughly 278–308 mm. These contrasts underscore the markedly heavier tail implied by the Fréchet distribution and indicate that the Weibull–Fréchet model systematically elevates high-quantile estimates relative to the Weibull model, particularly at longer return periods.

Collectively, the results show that the Fréchet distribution provides the most conservative extrapolation and may lead to very large design estimates for rare events, while the Weibull distribution tends to understate upper-tail risk by producing comparatively modest extremes. The Weibull–Fréchet model occupies a statistically and practically attractive middle ground, capturing enhanced upper-tail risk more effectively than the Weibull model yet avoiding the pronounced over-extrapolation associated with the Fréchet distribution, and thus represents a defensible choice for 50- and 100-year design rainfall estimation in critical hydrological and infrastructure applications.

Table 4. Estimated return levels (mm) corresponding to selected return periods for the All-India and Peninsular annual maximum rainfall series based on the fitted Fréchet, Weibull, and Weibull–Fréchet distributions

Region	Model	10-yr	25-yr	50-yr	100-yr
All India	Fréchet	427.08	516.26	594.25	683.30
	Weibull	315.96	325.37	330.97	335.73
	Weibull -Fréchet	316.12	326.76	333.24	338.83
Peninsular Region	Fréchet	342.24	413.70	476.20	547.56
	Weibull	277.74	291.86	300.40	307.74
	Weibull -Fréchet	277.84	294.32	304.64	313.71

4. CONCLUSION

This study evaluated the empirical performance of the Weibull–Fréchet (WF) distribution for modelling annual maximum rainfall in India and compared its adequacy with the classical Fréchet and Weibull distribution. Using long-term rainfall records from the All-India and Peninsular regions, the analysis demonstrates that the WF distribution consistently provides a superior representation of extreme rainfall behaviour across contrasting climatic regimes. The improvement in model performance is primarily attributed to the additional flexibility introduced by the Weibull-type deformation parameter, which enables the distribution to effectively accommodate varying degrees of skewness and tail heaviness commonly observed in rainfall extremes.

Both numerical goodness-of-fit measures and graphical diagnostic tools confirm the robustness of the WF distribution, particularly in capturing upper-tail characteristics that are critical for extreme-value applications. While the Fréchet distribution tends to overstate tail heaviness and the Weibull distribution exhibits limited adaptability to extreme observations, the WF distribution achieves a balanced and reliable fit across the full range of rainfall magnitudes. Overall, the findings highlight the practical relevance of the Weibull–Fréchet distribution as an effective alternative for modelling rainfall extremes in hydrological and climate-related studies. Future research may extend this framework to non-stationary settings or explore its applicability to other environmental extreme-value datasets. Reliable estimation of extreme rainfall plays a key role in flood risk mitigation and the planning of climate-resilient infrastructure. The adoption of flexible extreme-value modelling approaches can enhance the robustness of design estimates and improve the assessment of long-term risks under growing climatic variability.

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