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Calibration-Enhanced Estimation of Population Mean with Multi-Auxiliary Information in Stratified Sampling

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SUMMARY

Calibration estimation plays an important role in improving the accuracy of population parameter estimates using available auxiliary information. In this paper, we propose two calibration estimators of the population mean using multi-auxiliary variables under stratified random sampling. These suggested calibration estimators incorporate the coefficients of variation of multi-auxiliary variables. The performance of the recommended estimators was compared with the estimators given by Rao *et al.* (2012) and Garg and Pachori (2020) through a simulation study in R software. The results showed that the recommended estimators are more effective and precise than the existing estimators under a stratified random sampling scheme.

Keywords: Auxiliary variable; Calibration estimation; Coefficient of variation; Mean; Mean Squared Error; Stratified sampling.

1. INTRODUCTION

In survey sampling, sampling allows researchers to make conclusions about a population by studying a representative subset of that population. Several sampling methods are used in practice. One important method is stratified sampling, where a heterogeneous population is divided into relatively homogeneous subgroups called strata. Samples are then drawn from each stratum, thereby generally increasing the precision of estimates for population parameters. In recent years, calibration estimation has been widely used in survey sampling to improve estimates of population characteristics such as the mean and proportion. In this approach, sampling weights are adjusted using auxiliary information so that the resulting estimates become more accurate. The calibration framework was formally developed in the early 1990s, with major contributions by Deville and Särndal (1992). Since then, many researchers have extended and applied this work under different sampling designs, including Singh *et al.* (1998), Singh (2003), Tracy *et al.* (2003), Kim *et al.* (2007), Singh and Arnab (2011), Rao *et al.* (2012), Rao *et al.* (2015), Clement and Enang (2015), Koyuncu and Kadilar (2016), and Nidhi *et al.* (2017), among others, in developing various calibration estimators of population parameters.

Most of the existing approaches make use of population means or totals of auxiliary variables. However, measures of dispersion at the stratum level, such as the standard deviation and coefficient of variation, have received comparatively less attention. This limited use motivated us to explore how the coefficients of variation of multi-auxiliary variables can be incorporated to improve the estimation of the population mean. It measures relative variability that accounts for both the mean and the variability of the auxiliary variable. Some researchers, like Rao *et al.* (2012), Ozgul (2019), and Garg and Pachori (2020), have used the coefficient of variation of an auxiliary variable to improve the accuracy or precision of estimators under a stratified random sampling scheme. This study is conducted to explore the use of the

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coefficients of variation in the multi-auxiliary variables setting to estimate the population mean of the study variable using the calibration estimation method within the stratified sampling framework. So, the coefficients of variation are used here as key parameters to develop more precise estimators of the population mean of the study variable.

This paper is organized as follows: Section 1 includes the introduction, background, and rationale for the study. Section 2 reviews existing calibration estimation methods in a similar setup. In Sections 3 and 4, we present the methodology and proposed calibration estimators. In Section 5, we present the results of a simulation study comparing the efficiency of recommended estimators with that of existing methods. Finally, Section 6 discusses the findings and results, and Section 7 presents the conclusion.

2. DEVELOPMENT OF CALIBRATION ESTIMATION UNDER STRATIFIED SAMPLING

The calibration estimation approach for generalized regression (GREG) for estimating the population total $Y=\sum_{i=1}^{N} y_i$ was first derived by Deville and Sarndal (1992) under a probability proportional to size sampling scheme. The population total of $X=\sum_{i\in U} x_i$ is considered to be known. Considering a finite population U of size N, where y_i be the ith unit of the study variable and $x_i, i=1,2,...,\mathrm{N}$, corresponding unit of the auxiliary variable. A sample $s=\{1,2,...,n\}\subset U$ of size n is selected from the population U using a probability sampling design P. The inclusion probabilities $\pi_i=\Pr(\mathrm{i}\in s)$ and $\pi_{ij}=\Pr(\mathrm{i}\ \&\ \mathrm{j}\in s)$ are strictly positive.

Deville and Sarndal (1992) represented the calibration estimator of population total as:

$$\hat{Y}_{DS}=\sum_{i\in s} w_i y_i$$

Corresponding to Horvitz and Thompson (1952) estimator:

$$\hat{y}_{HT}=\sum_{i\in s}\frac{y_i}{\pi_i}=\sum_{i\in s} d_i y_i$$

where $d_i=\dfrac{1}{\pi_i}$

Here $w_i;\, i\in s$ are the calibrated weights determined to minimize the chi-square type distance measure $\sum_{i\in s}\frac{(w_i-d_i)^2}{d_i q_i}$ that attained the calibration constraint given as:

$$\sum_{i\in s} w_i x_i = X$$

Further, the calibrated GREG estimator of population total Y is obtained as follows:

$$\hat{Y}_{DS}=\sum_{i\in s} d_i y_i+\hat{\beta}_{ds}\left(X-\sum_{i\in s} d_i x_i\right)$$

where $\hat{\beta}_{ds}=\dfrac{\left(\sum_{i\in s} d_i q_i x_i y_i\right)}{\left(\sum_{i\in s} d_i q_i x_i^2\right)}$

Assuming a finite population $U=\left(U_{1,}U_2,...,U_N\right)$ comprising N units. Let $\bar{Y}=\sum_{i=1}^{N}\frac{y_i}{N}$ denote the population mean of the study variable Y. A sample of size n has been drawn by simple random sampling without replacement (SRSWOR). Consider the study variable Y to be linearly associated with p auxiliary variables $X_j\,(j=1,2,...,p)$. In stratified random sampling, the population is divided into L non-overlapping subgroups, called strata. Let n_h be the size of selected units from the h^{th} stratum, which is comprised of N_h units where $n=\sum_{h=1}^{L} n_h$ and $N=\sum_{h=1}^{L} N_h$ denote the total sample size and overall population size, respectively. Let y_{ih} and x_{ijh} be the i^{th} unit of study variable and the j^{th} auxiliary variable in the h^{th} stratum. The other notations used in this paper are given as follows:

$\bar{y}_h=\sum_{i=1}^{n_h}\frac{y_{ih}}{n_h}$: h^{th} stratum sample mean of study variable Y

$\bar{Y}_h = \sum_{i=1}^{N_h} \frac{y_{ih}}{N_h}$: h^{th} stratum population mean of study variable Y

$\bar{x}_{jh} = \sum_{i=1}^{n_h} \frac{x_{ijh}}{n_h}$: sample mean of the j^{th} auxiliary variable X_j in the h^{th} stratum

$\bar{X}_{jh} = \sum_{i=1}^{N_h} \frac{x_{ijh}}{N_h}$: population mean of the j^{th} auxiliary variable X_j in the h^{th} stratum

$W_h = \frac{N_h}{N}$: h^{th} stratum weight

$f_h = \frac{n_h}{N_h}$: correction factor for the h^{th} stratum

$\rho_{yx_1h}, \rho_{yx_2h}, \rho_{x_1x_2h}$: population correlation coefficients between the variables represented by the respective subscripts in the h^{th} stratum

$c_{x_{jh}} = \frac{s_{x_{jh}}}{\bar{x}_{jh}}$: sample coefficient of variation of the j^{th} auxiliary variable in the h^{th} stratum

$C_{x_{jh}} = \frac{S_{x_{jh}}}{\bar{X}_{jh}}$: population coefficient of variation of the j^{th} auxiliary variable in the h^{th} stratum

$s_{yh}^2 = \frac{\sum_{i=1}^{n_h}\left(y_{ih} - \bar{y}_h\right)^2}{n_h - 1}$, $s_{x_{jh}}^2 = \frac{\sum_{i=1}^{n_h}\left(x_{ijh} - \bar{x}_{jh}\right)^2}{n_h - 1}$ be the sample variances and $S_{yh}^2 = \frac{\sum_{i=1}^{N_h}\left(Y_{ih} - \bar{Y}_h\right)^2}{N_h - 1}$, $S_{x_{jh}}^2 = \frac{\sum_{i=1}^{N_h}\left(X_{ijh} - \bar{X}_{jh}\right)^2}{N_h - 1}$ be the population variances of Y, $X_j; j = 1,2,...,p$, respectively.

The population mean of the j^{th} auxiliary variable, $\bar{X}_j = \sum_{i=1}^{N_h} W_h \bar{X}_{jh}$, is assumed to be already known.

Under a stratified sampling scheme, a multivariate calibration estimator developed by Rao *et al.* (2012) for the population mean $\bar{Y}$ is given as:

$$\bar{y}_{Rao} = \sum_{h=1}^{L} \Omega_h \bar{y}_h \tag{1}$$

To minimize the chi-square distance measure $\sum_{h=1}^{L} \frac{\left(\Omega_h - W_h\right)^2}{W_h Q_h}$, the calibrated weights Ω_h are chosen, while satisfying the specified p calibration constraints:

$$\sum_{h=1}^{L} \Omega_h \bar{x}_{jh} = \bar{X}_j; \quad j = 1,\ 2,\ ...,\ p$$

Using Lagrange's multiplier method leads to the computation of new calibrated weights, represented as:

$$\Omega_h = W_h + W_h Q_h \left(\sum_{j=1}^{p} \lambda_j \bar{x}_{jh} \right) \tag{2}$$

where $\lambda_j;\ j = 1, 2, ..., p$ are the Lagrange multipliers.

After substituting p=2 in equation (1), the optimum calibrated weights are obtained as:

$$\Omega_h = W_h + W_h Q_h \left(\lambda_1 \bar{x}_{1h} + \lambda_2 \bar{x}_{2h} \right)$$

And the final form of the estimator is reduced to:

$$\bar{y}_{Rao} = \sum_{h=1}^{L} W_h \bar{y}_h + \hat{\beta}_{R1} \sum_{h=1}^{L} W_h \left(\bar{X}_{1h} - \bar{x}_{1h} \right) + \hat{\beta}_{R2} \sum_{h=1}^{L} W_h \left(\bar{X}_{2h} - \bar{x}_{2h} \right) \tag{3}$$

where,

$$\hat{\beta}_{R1} = \frac{\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{2h}^2 \right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h} \bar{y}_h \right) - \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h} \bar{x}_{2h} \right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{2h} \bar{y}_h \right)}{\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h}^2 \right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{2h}^2 \right) - \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h} \bar{x}_{2h} \right)^2}$$

$$\hat{\beta}_{R2} = \frac{\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h}^2\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{2h} \bar{y}_h\right) - \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h} \bar{x}_{2h}\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h} \bar{y}_h\right)}{\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h}^2\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{2h}^2\right) - \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h} \bar{x}_{2h}\right)^2}$$

Garg and Pachori (2020) provided a calibrated estimator for the mean under a stratified sampling, which is:

$$\bar{y}_{\text{GP}} = \sum_{h=1}^{L} \omega_h \bar{y}_h \tag{4}$$

They derived the new calibrated weights by minimizing the chi-square distance measure

$$\sum_{h=1}^{L} \frac{\left(\omega_h - W_h\right)^2}{W_h Q_h}$$

Adhering to calibration conditions:

$$\sum_{h=1}^{L} \omega_h = \sum_{h=1}^{L} W_h$$

$$\sum_{h=1}^{L} \omega_h c_{xh} = \sum_{h=1}^{L} W_h C_{xh}$$

Subject to calibration constraints, equation (3) is minimized by applying the Lagrange multiplier method to obtain the optimum weights as:

$$\omega_h = W_h + W_h Q_h \left(\lambda_1 + \lambda_2 c_{xh}\right)$$

where λ_1 and λ_2 represents the Lagrange multipliers.

The calibration estimator proposed by Garg and Pachori (2020) can be written as:

$$\bar{y}_{GP} = \sum_{h=1}^{L} W_h \bar{y}_h + \hat{\beta}_{GP} \sum_{h=1}^{L} W_h \left(C_{xh} - c_{xh}\right) \tag{5}$$

where,

$$\hat{\beta}_{GP} = \frac{\left(\sum_{h=1}^{L} W_h Q_h\right)\left(\sum_{h=1}^{L} W_h Q_h c_{xh} \bar{y}_h\right) - \left(\sum_{h=1}^{L} W_h Q_h c_{xh}\right)\left(\sum_{h=1}^{L} W_h Q_h \bar{y}_h\right)}{\left(\sum_{h=1}^{L} W_h Q_h c_{xh}^2\right)\left(\sum_{h=1}^{L} W_h Q_h\right) - \left(\sum_{h=1}^{L} W_h Q_h c_{xh}\right)^2}$$

3. PROPOSED CALIBRATION ESTIMATORS

We introduce a modified calibration estimator for the population mean of the study variable Y under stratified sampling. Inspired by previous research, we now extend standard calibration constraints at the stratum level to improve the estimator's precision using available information on the coefficients of variation of p auxiliary variables.

The calibrated estimator for the population mean $\bar{Y}$ under stratified sampling is recommended as:

$$\bar{y}_{cv.p} = \sum_{h=1}^{L} \psi_h \bar{y}_h \tag{6}$$

where ψ_h ($\psi_h \geq 0;\ h = 1, 2, ..., L$) are the calibration weights, chosen such that the chi-square type distance function

$$\sum_{h=1}^{L} \frac{\left(\psi_h - W_h\right)^2}{W_h Q_h}$$

is minimum, subject to the p calibration constraints, given by

$$\sum_{h=1}^{L} \psi_h c_{x_1 h} = \sum_{h=1}^{L} W_h C_{x_1 h}$$

$$\sum_{h=1}^{L} \psi_h c_{x_2 h} = \sum_{h=1}^{L} W_h C_{x_2 h}$$

$$\vdots$$

$$\sum_{h=1}^{L} \psi_h c_{x_p h} = \sum_{h=1}^{L} W_h C_{x_p h} \tag{7}$$

where Q_h are the constants, which can determine various forms of the recommended estimator.

Applying the Lagrange multiplier method to minimize the following function:

$$L=\sum_{h=1}^{L}\frac{\left(\psi_h-W_h\right)^2}{W_hQ_h}-2\sum_{j=1}^{p}\Delta_j\left(\sum_{h=1}^{L}\psi_h c_{x_jh}-\sum_{h=1}^{L}W_hC_{x_jh}\right)$$

After differentiating L with respect to ψ_h, we determine the expression of the optimum calibrated weight as:

$$\psi_h=W_h+W_hQ_h\left(\sum_{j=1}^{p}\Delta_j c_{x_jh}\right);\; j=1,2,...,p \tag{8}$$

where $\Delta_j;\; j=1, 2, ..., p$ are the Lagrange multipliers.

Now, put the value of ψ_h in the considered constraints in equation (7), the system of p linear equations can be written in the form of a matrix as:

$$\mathbf{A\Delta}=\mathbf{B} \tag{9}$$

The solution to the system of p linear equations in equation (9), when the matrix $\mathbf{A}$ is invertible, i.e., $|\mathbf{A}|\neq 0$, is given by

$$\underset{\sim}{\mathbf{\Delta}}=\mathbf{A}^{-1}\underset{\sim}{\mathbf{B}} \tag{10}$$

where,

$$\mathbf{A}=\begin{bmatrix}\sum_{h=1}^{L}W_hQ_hc_{x_1h} & \sum_{h=1}^{L}W_hQ_hc_{x_2h} & \cdots & \sum_{h=1}^{L}W_hQ_hc_{x_ph}\\ \sum_{h=1}^{L}W_hQ_hc_{x_1h}^2 & \sum_{h=1}^{L}W_hQ_hc_{x_1h}c_{x_2h} & \cdots & \sum_{h=1}^{L}W_hQ_hc_{x_1h}c_{x_ph}\\ . & . & \cdots & .\\ . & . & \cdots & .\\ . & . & \cdots & .\\ \sum_{h=1}^{L}W_hQ_hc_{x_1h}c_{x_ph} & \sum_{h=1}^{L}W_hQ_hc_{x_2h}c_{x_ph} & \cdots & \sum_{h=1}^{L}W_hQ_hc_{x_ph}^2\end{bmatrix},$$

$$\underset{\sim}{\mathbf{B}}=\begin{bmatrix}\sum_{h=1}^{L}\mathbf{W_h}\left(\mathbf{C}_{x_1h}-\mathbf{c}_{x_1h}\right)\\ \sum_{h=1}^{L}\mathbf{W_h}\left(\mathbf{C}_{x_2h}-\mathbf{c}_{x_2h}\right)\\ .\\ .\\ .\\ \sum_{h=1}^{L}\mathbf{W_h}\left(\mathbf{C}_{x_ph}-\mathbf{c}_{x_ph}\right)\end{bmatrix}\; and\; \underset{\sim}{\mathbf{\Delta}}=\begin{bmatrix}\mathbf{\Delta_1}\\ \mathbf{\Delta_2}\\ .\\ .\\ .\\ \mathbf{\Delta_p}\end{bmatrix}$$

By putting the value of Lagrange's multipliers in equation (8), we obtain the optimum calibrated weights ψ_h. These weights, in turn give the expression of the final form of the calibration estimator of population mean presented in equation (6) using the coefficients of variation of the p auxiliary variables.

Special Case: When p=2

In this situation, we derive the optimum value of ψ_h as:

$$\psi_h=W_h+\Delta_1W_hQ_hc_{x_1h}+\Delta_2W_hQ_hc_{x_2h} \tag{11}$$

Here, substituting the value of ψ_h from equation (11) into the considered constraints up to 2 auxiliary variables, we can compute the values of Δ_1 and Δ_2, given as:

$$\Delta_1=\frac{\left(\sum_{h=1}^{L}W_h\left(C_{x_1h}-c_{x_1h}\right)\right)\left(\sum_{h=1}^{L}W_hQ_hc_{x_2h}^2\right)-\left(\sum_{h=1}^{L}W_h\left(C_{x_2h}-c_{x_2h}\right)\right)\left(\sum_{h=1}^{L}W_hQ_hc_{x_1h}c_{x_2h}\right)}{\left(\sum_{h=1}^{L}W_hQ_hc_{x_1h}^2\right)\left(\sum_{h=1}^{L}W_hQ_hc_{x_2h}^2\right)-\left(\sum_{h=1}^{L}W_hQ_hc_{x_1h}c_{x_2h}\right)^2}$$

and

$$\Delta_2=-\frac{\left(\sum_{h=1}^{L}W_h\left(C_{x_1h}-c_{x_1h}\right)\right)\left(\sum_{h=1}^{L}W_hQ_hc_{x_1h}c_{x_2h}\right)-\left(\sum_{h=1}^{L}W_h\left(C_{x_2h}-c_{x_2h}\right)\right)\left(\sum_{h=1}^{L}W_hQ_hc_{x_1h}^2\right)}{\left(\sum_{h=1}^{L}W_hQ_hc_{x_1h}^2\right)\left(\sum_{h=1}^{L}W_hQ_hc_{x_2h}^2\right)-\left(\sum_{h=1}^{L}W_hQ_hc_{x_1h}c_{x_2h}\right)^2}$$

which leads to the final form of optimum calibrated weights given by

$$\psi_h = W_h + \frac{\left(W_h Q_h c_{x_1h}\right)\left[\left(\sum_{h=1}^{L} W_h\left(C_{x_1h}-c_{x_1h}\right)\right)\left(\sum_{h=1}^{L} W_h Q_h c_{x_2h}^2\right) - \left(\sum_{h=1}^{L} W_h\left(C_{x_2h}-c_{x_2h}\right)\right)\left(\sum_{h=1}^{L} W_h Q_h c_{x_1h} c_{x_2h}\right)\right]}{\left(\sum_{h=1}^{L} W_h Q_h c_{x_1h}^2\right)\left(\sum_{h=1}^{L} W_h Q_h c_{x_2h}^2\right) - \left(\sum_{h=1}^{L} W_h Q_h c_{x_1h} c_{x_2h}\right)^2}$$

$$+\frac{\left(W_h Q_h c_{x_2h}\right)\left[\left(\sum_{h=1}^{L} W_h\left(C_{x_2h}-c_{x_2h}\right)\right)\left(\sum_{h=1}^{L} W_h Q_h c_{x_1h}^2\right) - \left(\sum_{h=1}^{L} W_h\left(C_{x_1h}-c_{x_1h}\right)\right)\left(\sum_{h=1}^{L} W_h Q_h c_{x_1h} c_{x_{2h}}\right)\right]}{\left(\sum_{h=1}^{L} W_h Q_h c_{x_1h}^2\right)\left(\sum_{h=1}^{L} W_h Q_h c_{x_2h}^2\right) - \left(\sum_{h=1}^{L} W_h Q_h c_{x_1h} c_{x_2h}\right)^2} \qquad (12)$$

Substitution of equation (12) in equation (6) leads to a recommended calibration estimator of population mean utilizing coefficients of variations of two auxiliary variables, expressed as follows:

$$\bar{y}_{cv.2} = \sum_{h=1}^{L} W_h \bar{y}_h + \hat{\beta}_1 \sum_{h=1}^{L} W_h\left(C_{x_{1h}} - c_{x_1h}\right) + \hat{\beta}_2 \sum_{h=1}^{L} W_h\left(C_{x_2h} - c_{x_2h}\right) \qquad (13)$$

where

$$\hat{\beta}_1 = \frac{\left(\sum_{h=1}^{L} W_h Q_h c_{x_2h}^2\right)\left(\sum_{h=1}^{L} W_h Q_h c_{x_1h}\bar{y}_h\right) - \left(\sum_{h=1}^{L} W_h Q_h c_{x_1h} c_{x_2h}\right)\left(\sum_{h=1}^{L} W_h Q_h c_{x_2h}\bar{y}_h\right)}{\left(\sum_{h=1}^{L} W_h Q_h c_{x_1h}^2\right)\left(\sum_{h=1}^{L} W_h Q_h c_{x_2h}^2\right) - \left(\sum_{h=1}^{L} W_h Q_h c_{x_1h} c_{x_2h}\right)^2}$$

$$\hat{\beta}_2 = \frac{\left(\sum_{h=1}^{L} W_h Q_h c_{x_1h}^2\right)\left(\sum_{h=1}^{L} W_h Q_h c_{x_2h}\bar{y}_h\right) - \left(\sum_{h=1}^{L} W_h Q_h c_{x_1h} c_{x_2h}\right)\left(\sum_{h=1}^{L} W_h Q_h c_{x_1h}\bar{y}_h\right)}{\left(\sum_{h=1}^{L} W_h Q_h c_{x_1h}^2\right)\left(\sum_{h=1}^{L} W_h Q_h c_{x_2h}^2\right) - \left(\sum_{h=1}^{L} W_h Q_h c_{x_1h} c_{x_2h}\right)^2}$$

When $Q_h = 1$, it reduces to the following:

$$\bar{y}_{cv.2} = \sum_{h=1}^{L} W_h \bar{y}_h + \hat{\beta}_{21} \sum_{h=1}^{L} W_h\left(C_{x_1h} - c_{x_1h}\right) + \hat{\beta}_{22} \sum_{h=1}^{L} W_h\left(C_{x_2h} - c_{x_2h}\right) \qquad (14)$$

where

$$\hat{\beta}_{21} = \frac{\left(\sum_{h=1}^{L} W_h c_{x_2h}^2\right)\left(\sum_{h=1}^{L} W_h c_{x_1h}\bar{y}_h\right) - \left(\sum_{h=1}^{L} W_h c_{x_1h} c_{x_2h}\right)\left(\sum_{h=1}^{L} W_h c_{x_2h}\bar{y}_h\right)}{\left(\sum_{h=1}^{L} W_h c_{x_1h}^2\right)\left(\sum_{h=1}^{L} W_h c_{x_2h}^2\right) - \left(\sum_{h=1}^{L} W_h c_{x_1h} c_{x_2h}\right)^2}$$

$$\hat{\beta}_{22} = \frac{\left(\sum_{h=1}^{L} W_h c_{x_1h}^2\right)\left(\sum_{h=1}^{L} W_h c_{x_2h}\bar{y}_h\right) - \left(\sum_{h=1}^{L} W_h c_{x_1h} c_{x_2h}\right)\left(\sum_{h=1}^{L} W_h c_{x_1h}\bar{y}_h\right)}{\left(\sum_{h=1}^{L} W_h c_{x_1h}^2\right)\left(\sum_{h=1}^{L} W_h c_{x_2h}^2\right) - \left(\sum_{h=1}^{L} W_h c_{x_1h} c_{x_2h}\right)^2}$$

To derive the MSE of the estimator, let us assume

$\hat{\theta}_y = \sum_{h=1}^{L} W_h \bar{y}_h$, $\theta_{C1} = \sum_{h=1}^{L} W_h C_{x_1h}$, $\hat{\theta}_{c1} = \sum_{h=1}^{L} W_h c_{x_1h}$, $\theta_{C2} = \sum_{h=1}^{L} W_h C_{x_2h}$, $\hat{\theta}_{c2} = \sum_{h=1}^{L} W_h c_{x_2h}$, $\hat{\theta}_{c1}^{(2)} = \sum_{h=1}^{L} W_h c_{x_1h}$,

$\hat{\theta}_{c2}^{(2)} = \sum_{h=1}^{L} W_h c_{x_2h}$, $\hat{\theta}_{c12} = \sum_{h=1}^{L} W_h c_{x_1h} c_{x_2h}$, $\hat{\theta}_{yc_1} = \sum_{h=1}^{L} W_h \bar{y}_h c_{x_1h}$,

$\hat{\theta}_{yc_2} = \sum_{h=1}^{L} W_h \bar{y}_h c_{x_2h}$, and $\hat{\theta}_{yc_1} = \sum_{h=1}^{L} W_h \bar{y}_h c_{x_1h}$

The new calibrated estimator can be expressed as:

$$\bar{y}_{cv.2} = \hat{\theta}_y + \left(\frac{\left(\hat{\theta}_{c2}^{(2)}\right)\left(\hat{\theta}_{yc1}\right) - \left(\hat{\theta}_{c12}\right)\left(\hat{\theta}_{yc2}\right)}{\left(\hat{\theta}_{c1}^{(2)}\right)\left(\hat{\theta}_{c2}^{(2)}\right) - \left(\hat{\theta}_{c12}\right)^2} \right)\left(\theta_{C1} + \hat{\theta}_{c1}\right) + \left(\frac{\left(\hat{\theta}_{c1}^{(2)}\right)\left(\hat{\theta}_{yc2}\right) - \left(\hat{\theta}_{c12}\right)\left(\hat{\theta}_{yc1}\right)}{\left(\hat{\theta}_{c1}^{(2)}\right)\left(\hat{\theta}_{c2}^{(2)}\right) - \left(\hat{\theta}_{c12}\right)^2} \right)\left(\theta_{C2} + \hat{\theta}_{c2}\right)$$

The expression of the estimator is reduced to:

$$\bar{y}_{cv.2} = f(\hat{\theta}_y, \hat{\theta}_{c1}, \hat{\theta}_{c2}, \theta_{C1}, \theta_{C2,}, \hat{\theta}_{c1}^{(2)}, \hat{\theta}_{c2}^{(2)}, \hat{\theta}_{c12}, \hat{\theta}_{yc1}, \hat{\theta}_{yc2}) = f(\underset{\sim}{\theta}) \quad (15)$$

Now, to derive the Mean Squared Error (MSE) of the proposed calibration estimator up to the first order of approximation (FOA) under large sample approximation, let us define,

$$\epsilon_{0h} = \frac{\bar{y}_h}{\bar{Y}_h} - 1, \quad \epsilon_{1h} = \frac{\bar{x}_{1h}}{\bar{X}_{1h}} - 1, \quad \epsilon_{2h} = \frac{s_{x_1h}^2}{S_{x_1h}^2} - 1,$$

$$\epsilon_{3h} = \frac{\bar{x}_{2h}}{\bar{X}_{2h}} - 1, \quad \epsilon_{4h} = \frac{s_{x_2h}^2}{S_{x_2h}^2} - 1$$

Such that $E(\epsilon_{ih}) = 0$ and $|\epsilon_{ih}| < 1$, for $i = 0,1,2,3,4$ and $h = 1,2...,L$

$$E\left(\epsilon_{0h}^2\right) = \left(\frac{1-f_h}{n_h}\right) C_{yh}^2, \; E\left(\epsilon_{1h}^2\right) = \left(\frac{1-f_h}{n_h}\right) C_{x_1h}^2,$$

$$E\left(\epsilon_{3h}^2\right) = \left(\frac{1-f_h}{n_h}\right) C_{x_2h}^2, \; E\left(\epsilon_{2h}^2\right) = \left(\frac{1-f_h}{n_h}\right) (\upsilon_{040h} - 1),$$

$$E\left(\epsilon_{4h}^2\right) = \left(\frac{1-f_h}{n_h}\right) (\upsilon_{004h} - 1),$$

$$E\left(\epsilon_{0h}\epsilon_{1h}\right) = \left(\frac{1-f_h}{n_h}\right) \rho_{yx_1,h} C_{yh} C_{x_1h},$$

$$E\left(\epsilon_{0h}\epsilon_{2h}\right) = \left(\frac{1-f_h}{n_h}\right) \mathrm{C}_{yh}\, \upsilon_{120h},$$

$$E\left(\epsilon_{0h}\epsilon_{3h}\right) = \left(\frac{1-f_h}{n_h}\right) \rho_{yx_2,h} C_{yh} C_{x_2h},$$

$$E\left(\epsilon_{0h}\epsilon_{4h}\right) = \left(\frac{1-f_h}{n_h}\right) \mathrm{C}_{yh}\, \upsilon_{102h},$$

$$E\left(\epsilon_{1h}\epsilon_{2h}\right) = \left(\frac{1-f_h}{n_h}\right) \mathrm{C}_{x_1h}\, \upsilon_{030h},$$

$$E\left(\epsilon_{1h}\epsilon_{3h}\right) = \left(\frac{1-f_h}{n_h}\right) \rho_{x_1x_2,h} C_{x_1h} C_{x_2h},$$

$$E\left(\epsilon_{1h}\epsilon_{4h}\right) = \left(\frac{1-f_h}{n_h}\right) \mathrm{C}_{x_1h} (\upsilon_{012h} - 1),$$

$$E\left(\epsilon_{2h}\epsilon_{3h}\right) = \left(\frac{1-f_h}{n_h}\right) \mathrm{C}_{x_2h} (\upsilon_{021h} - 1),$$

$$E\left(\epsilon_{2h}\epsilon_{4h}\right) = \left(\frac{1-f_h}{n_h}\right) (\upsilon_{022h} - 1),$$

$$E\left(\epsilon_{3h}\epsilon_{4h}\right) = \left(\frac{1-f_h}{n_h}\right) \mathrm{C}_{x_2h}\, \upsilon_{003h}$$

Therefore, by applying the Taylor-linearization method up to the first-order approximation, the MSE of the recommended estimator is obtained as:

$$MSE(\bar{y}_{cv.2}) = \sum_{h=1}^{L} W_h^2 \frac{(1-f_h)}{n_h} \Bigg\{ \bar{Y}_h^2 C_{yh}^2 + \beta_1^2 C_{x_1h} \left(C_{x_1h}^2 - C_{x_1h}\upsilon_{030h} + \frac{(\upsilon_{040h} - 1)}{4} \right) + \beta_2^2 C_{x_2h} \left(C_{x_1h}^2 - C_{x_1h}\upsilon_{003h} + \frac{(\upsilon_{004h} - 1)}{4} \right) - 2\bar{Y}_h \Big(\beta_1 C_{x_1h} C_{yh} \left(\upsilon_{120h} - \rho_{yx_1,h} C_{x_1h} \right) + \beta_2 C_{x_2h} C_{yh} \left(\upsilon_{102h} - \rho_{yx_2,h} C_{x_2h} \right) \Big) + 2\beta_1\beta_2 C_{x_1h} C_{x_2h} \left(\rho_{x_1x_2,h} C_{x_1h} C_{x_2h} - C_{x_1h} \frac{(\upsilon_{012h} - 1)}{2} - C_{x_2h} \frac{(\upsilon_{021h} - 1)}{2} + \frac{(\upsilon_{022h} - 1)}{4} \right) \Bigg\} \quad (16)$$

where,

$$\upsilon_{lmnh} = \frac{\mu_{lmnh}}{\mu_{200h}^{l/2}\, \mu_{020h}^{m/2}\, \mu_{002h}^{n/2}} \text{ and}$$

$$\mu_{lmnh} = \frac{\sum_{i=1}^{N_h} \left(\left(y_{ih} - \bar{y}_h \right)^l \left(x_{1ih} - \bar{X}_{1h} \right)^m \left(x_{2ih} - \bar{X}_{2h} \right)^n \right)}{N_h - 1}.$$

whereas l, m, and n are the positive real numbers and μ_{200h}, μ_{002h} and μ_{020h} represents the second-order moments about the means of variables Y, X_1 and X_2, respectively, and υ_{lmnh} denotes the corresponding moment ratio.

4. OTHER DEVELOPED CALIBRATION ESTIMATORS

Following Rao *et al.* (2012), we can propose an improved multivariate calibration estimator for the population mean $\bar{Y}$ as:

$$\bar{y}_{rp} = \sum_{h=1}^{L} \Omega_h \bar{y}_{rh} \tag{17}$$

where $\bar{y}_{rh} = \bar{y}_h \prod_{j=1}^{p} \left(\frac{\bar{X}_{jh}}{\bar{x}_{jh}} \right)^{\alpha_j}$

Here, $\alpha_j;\ j = 1,2,...,p$ are constants. We may choose $\alpha_j = 1$ or the ratio estimator and $\alpha_j = -1$ for the product estimator. Alternatively, we may choose a combination of values depending on the relationship between the auxiliary variables and the study variable.

To minimize the chi-square distance measure $\sum_{h=1}^{L} \frac{(\Omega_h - W_h)^2}{W_h Q_h}$, the calibrated weights Ω_h are chosen, while satisfying the specified p calibration constraints:

$$\sum_{h=1}^{L} \Omega_h \bar{x}_{jh} = \bar{X}_j; \quad j = 1,\ 2,\ ...,\ p \tag{18}$$

Using Lagrange's multiplier method leads to the computation of new calibrated weights, represented as:

$$\Omega_h = W_h + W_h Q_h \left(\sum_{j=1}^{p} \lambda_j \bar{x}_{jh} \right) \tag{19}$$

where $\lambda_j;\ j = 1,\ 2,\ ...,\ p$ are the Lagrange multipliers.

For p = 2, the final form of the estimator is reduced to:

$$\bar{y}_{rp} = \sum_{h=1}^{L} W_h \bar{y}_{rh} + \hat{\beta}_{rp1} \sum_{h=1}^{L} W_h \left(\bar{X}_{1h} - \bar{x}_{1h} \right) + \hat{\beta}_{rp2} \sum_{h=1}^{L} W_h \left(\bar{X}_{2h} - \bar{x}_{2h} \right) \tag{20}$$

where,

$$\hat{\beta}_{rp1} = \frac{\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{2h}^2 \right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h} \bar{y}_{rh} \right) - \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h} \bar{x}_{2h} \right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{2h} \bar{y}_{rh} \right)}{\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h}^2 \right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{2h}^2 \right) - \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h} \bar{x}_{2h} \right)^2}$$

$$\hat{\beta}_{rp2} = \frac{\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h}^2 \right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{2h} \bar{y}_{rh} \right) - \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h} \bar{x}_{2h} \right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h} \bar{y}_{rh} \right)}{\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h}^2 \right)\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{2h}^2 \right) - \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h} \bar{x}_{2h} \right)^2}$$

The MSE of the proposed calibration estimator for p = 2 up to FOA can be derived as:

$$\begin{aligned} MSE\left(\bar{y}_{rp2} \right) = \sum_{h=1}^{L} W_h^2 \left(\frac{(1-f_h)}{n_h} \right) \Big\{ & \bar{Y}_h^2 \left(C_{yh}^2 + \alpha_1^2 C_{x_1h}^2 + \alpha_2^2 C_{x_2h}^2 - 2\alpha_1 \rho_{yx_1,h} C_{yh} C_{x_1h} - 2\alpha_2 \rho_{yx_2,h} C_{yh} C_{x_2h} + 2\alpha_1\alpha_2 \rho_{x_1x_2,h} C_{x_1h} C_{x_2h} \right) \\ & + \beta_{rp1}^2 \bar{X}_{1h}^2 C_{x_1h}^2 + \beta_{rp2}^2 \bar{X}_{2h}^2 C_{x_2h}^2 - 2\beta_{rp1} \bar{Y}_h \bar{X}_{1h} \left(\rho_{yx_1h} C_{yh} C_{x_1h} - \alpha_1 C_{x_1h}^2 - \alpha_2 \rho_{x_1x_2h} C_{x_1h} C_{x_2h} \right) \\ & - 2\beta_{rp2} \bar{Y}_h \bar{X}_{2h} \left(\rho_{yx_2h} C_{yh} C_{x_2h} - \alpha_2 C_{x_2h}^2 - \alpha_1 \rho_{x_1x_2h} C_{x_1h} C_{x_2h} \right) \\ & + 2\beta_{rp1}\beta_{rp2} \rho_{x_1x_2h} \bar{X}_{1h} \bar{X}_{2h} C_{x_1h} C_{x_2h} \Big\} \end{aligned} \tag{21}$$

Another calibrated estimator for the population mean $\bar{Y}$ under stratified sampling can also be developed as:

$$\bar{y}_{cv.rp} = \sum_{h=1}^{L} \psi_h \bar{y}_{rh} \tag{22}$$

where $\bar{y}_{rh} = \bar{y}_h \prod_{j=1}^{p} \left(\frac{\bar{X}_{jh}}{\bar{x}_{jh}} \right)^{\alpha_j}$

Here, $\alpha_j;\ j = 1,2,...,p$ are constants. We may choose $\alpha_j = 1$ or the ratio estimator and $\alpha_j = -1$ for the product estimator. Alternatively, we may choose a combination of values depending on the relationship between the auxiliary variables and the study variable.

and ψ_h ($\psi_h \geq 0;\ h = 1, 2, ..., L$) are the calibration weights, chosen such that the chi-square type distance function $\sum_{h=1}^{L} \frac{\left(\psi_h - W_h\right)^2}{W_h Q_h}$ is minimum, subject to the p calibration constraints, given in equation (7).

If the matrix $\mathbf{A}$ is invertible, i.e., $|\mathbf{A}| \neq 0$, the Lagrange's multipliers are obtained as:

$$\underset{\sim}{\Delta} = \mathbf{A}^{-1}\underset{\sim}{\mathbf{B}}$$

In the same way as explained in Section 2, we can obtain the optimum calibrated weights ψ_h as:

$$\psi_h = W_h + \sum_{j=1}^{p} \Delta_j W_h Q_h c_{x_j h} \tag{23}$$

Then the final form of the suggested ratio-type calibration estimator of population mean using the coefficients of variation of the p auxiliary variables is given as:

$$\bar{y}_{cv.rp} = \sum_{h=1}^{L} \psi_h \bar{y}_{rh} = \sum_{h=1}^{L} W_h \bar{y}_{rh} + \sum_{h=1}^{L} \left[\sum_{j=1}^{p} \Delta_j W_h Q_h c_{x_j h} \right] \bar{y}_{rh} \tag{24}$$

Special Case: When p = 2

In the case of two auxiliary variables, the ratio-type calibration estimator of the population mean is expressed as follows:

$$\bar{y}_{cv.rp2} = \sum_{h=1}^{L} W_h \bar{y}_{rh} + \hat{\beta}_{cv.rp1} \sum_{h=1}^{L} W_h \left(C_{x_{1h}} - c_{x_1 h} \right) + \hat{\beta}_{cv.rp2} \sum_{h=1}^{L} W_h \left(C_{x_2 h} - c_{x_2 h} \right) \tag{25}$$

where

$$\hat{\beta}_{cv.rp1} = \frac{\left(\sum_{h=1}^{L} W_h Q_h c_{x_2 h}^2\right)\left(\sum_{h=1}^{L} W_h Q_h c_{x_1 h} \bar{y}_{rh}\right) - \left(\sum_{h=1}^{L} W_h Q_h c_{x_1 h} c_{x_2 h}\right)\left(\sum_{h=1}^{L} W_h Q_h c_{x_2 h} \bar{y}_{rh}\right)}{\left(\sum_{h=1}^{L} W_h Q_h c_{x_1 h}^2\right)\left(\sum_{h=1}^{L} W_h Q_h c_{x_2 h}^2\right) - \left(\sum_{h=1}^{L} W_h Q_h c_{x_1 h} c_{x_2 h}\right)^2}$$

$$\hat{\beta}_{cv.rp2} = \frac{\left(\sum_{h=1}^{L} W_h Q_h c_{x_1 h}^2\right)\left(\sum_{h=1}^{L} W_h Q_h c_{x_2 h} \bar{y}_{rh}\right) - \left(\sum_{h=1}^{L} W_h Q_h c_{x_1 h} c_{x_2 h}\right)\left(\sum_{h=1}^{L} W_h Q_h c_{x_1 h} \bar{y}_{rh}\right)}{\left(\sum_{h=1}^{L} W_h Q_h c_{x_1 h}^2\right)\left(\sum_{h=1}^{L} W_h Q_h c_{x_2 h}^2\right) - \left(\sum_{h=1}^{L} W_h Q_h c_{x_1 h} c_{x_2 h}\right)^2}$$

The MSE of the proposed ratio-type calibration estimator up to FOA can be derived as:

$$\begin{aligned}
MSE\left(\bar{y}_{cv.rp2}\right) = \sum_{h=1}^{L} W_h^2 \left(\frac{(1-f_h)}{n_h}\right) \Big\{ & \bar{Y}_h^2 \Big(C_{yh}^2 + \alpha_1^2 C_{x_1 h}^2 + \\
& \alpha_2^2 C_{x_2 h}^2 - 2\alpha_1 \rho_{yx_1 h} C_{yh} C_{x_1 h} - 2\alpha_2 \rho_{yx_2 h} C_{yh} C_{x_2 h} + \\
& 2\alpha_1 \alpha_2 \rho_{x_1 x_2 h} C_{x_1 h} C_{x_2 h} \Big) + \beta_{cv.rp1}{}^2 C_{x_1 h}^2 \Big(C_{x_1 h}^2 - C_{x_1 h} \upsilon_{030h} + \\
& \frac{(\upsilon_{040h} - 1)}{4} \Big) + \beta_{cv.rp2}{}^2 C_{x_2 h}^2 \left(C_{x_2 h}^2 - C_{x_2 h} \upsilon_{003h} + \frac{(\upsilon_{004h} - 1)}{4} \right) - \\
& 2\beta_{cv.rp1} \bar{Y}_h C_{x_1 h} \Big(\frac{1}{2} C_{yh} \upsilon_{120h} - \rho_{yx_1 h} C_{yh} C_{x_1 h} - \frac{\alpha_1}{2} C_{x_1 h} \upsilon_{030h} + \\
& \alpha_1 C_{x_1 h}^2 - \frac{\alpha_2}{2} C_{x_2 h} \left(\upsilon_{021h} - 1\right) + \alpha_2 \rho_{x_1 x_2 h} C_{x_1 h} C_{x_2 h} \Big) - \\
& 2\beta_{cv.rp2} \bar{Y}_h C_{x_2 h} \Big(\frac{1}{2} C_{yh} \upsilon_{102h} - \rho_{yx_2 h} C_{yh} C_{x_2 h} - \\
& \frac{\alpha_1}{2} C_{x_1 h} \left(\upsilon_{012h} - 1\right) + \alpha_2 C_{x_2 h}^2 - \frac{\alpha_2}{2} C_{x_2 h} \upsilon_{003h} + \\
& \alpha_1 \rho_{x_1 x_2 h} C_{x_1 h} C_{x_2 h} \Big) + 2\beta_{cv.rp1} \beta_{cv.rp2} C_{x_1 h} C_{x_2 h} \Big(\frac{(\upsilon_{022h} - 1)}{4} - \\
& C_{x_1 h} \frac{(\upsilon_{012h} - 1)}{2} - C_{x_2 h} \frac{(\upsilon_{021h} - 1)}{2} + \rho_{x_1 x_2 h} C_{x_1 h} C_{x_2 h} \Big) \Big\}
\end{aligned} \tag{26}$$

5. SIMULATION STUDY

Under a simulation study, we considered the body performance dataset using R software, consisting of N=13393 population units, which is further divided into four homogeneous strata of sizes N_1=3348, N_2=3347, N_3=3348, N_4=3350. The suggested estimator is compared with the other existing calibrated estimators: $\bar{y}_{Rao}$ in equation (2) and $\bar{y}_{GP}$ in equation (4). The study variable (Y) is assumed to be sit-up counts, i.e., the physical fitness, and the auxiliary variables X_1

and X_2 considered as age and weight, respectively. Our motive is to estimate the population mean of study variable Y and to examine the performance of the proposed calibrated estimators.

The following criteria were used to evaluate the performance of the recommended estimators:

Percentage Absolute Relative Biases (%ARBs) of the estimators are obtained as:

$$\%ARB(\bar{y}_\alpha) = \left|\frac{(\bar{y}_\alpha - \bar{Y})}{\bar{Y}}\right| \times 100; \;\; \alpha = \bar{y}_{GP}, \bar{y}_{Rao}, \bar{y}_{rp}, \bar{y}_{cv}, \bar{y}_{cv.rp}$$

Percentage Relative Root Mean Squared Errors (%RRMSEs) of the estimators are computed as:

$$\%RRMSE(\bar{y}_\alpha) = \sqrt{\frac{1}{10000}\sum_{i=1}^{10000}\left(\frac{(\bar{y}_\alpha - \bar{Y})}{\bar{Y}}\right)^2} \times 100;$$

$$\alpha = \bar{y}_{GP}, \bar{y}_{Rao}, \bar{y}_{rp}, \bar{y}_{cv}, \bar{y}_{cv.rp}$$

Percentage Relative Efficiencies (%REs) are determined to check the performance of the recommended estimator in comparison to the estimator $\bar{y}_{GP}$ as:

$$\%\mathrm{RE}(\bar{y}_\alpha) = \frac{MSE[\bar{y}_{GP}]}{MSE[\bar{y}_\alpha]} \times 100; \;\; \alpha = \bar{y}_{Rao}, \bar{y}_{rp}, \bar{y}_{cv}, \bar{y}_{cv.rp}$$

Table 1. Percentage absolute relative biases (%ARB) of the estimators

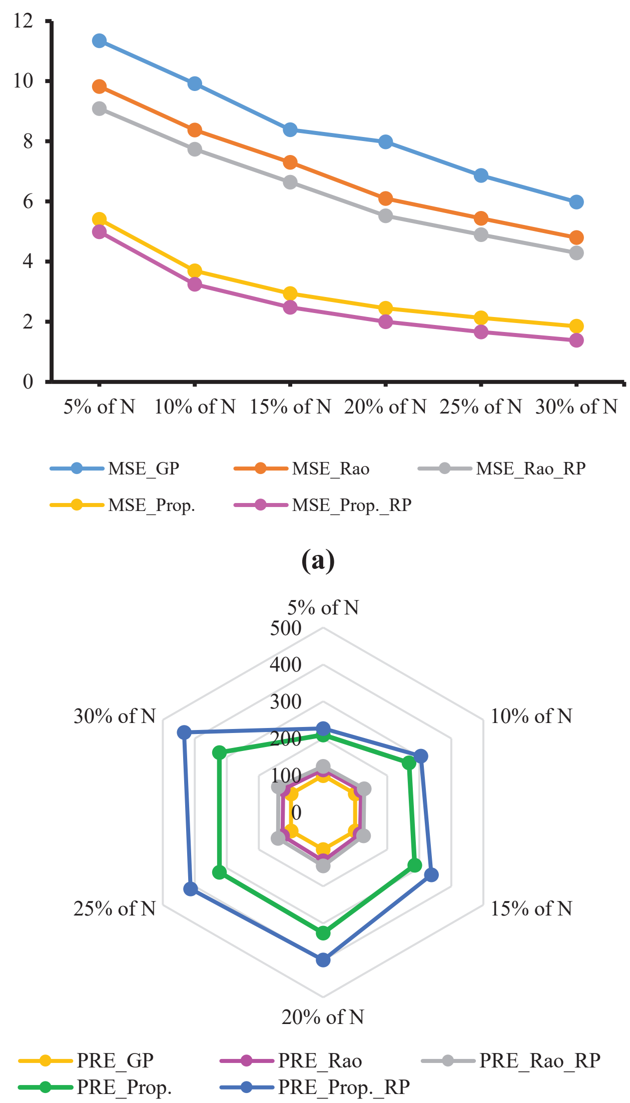
Sample Size (%)	$\bar{y}_{GP}$	$\bar{y}_{Rao}$	$\bar{y}_{rp}$	$\bar{y}_{cv}$	$\bar{y}_{cv.rp}$
5	0.1846	0.5001	0.3768	0.0452	0.0555
10	0.1476	0.1878	0.1054	0.0370	0.0463
15	0.0247	0.0578	0.0036	0.0393	0.0406
20	0.0092	0.0445	0.0755	0.0519	0.0309
25	0.0707	0.0129	0.0338	0.0059	0.0177
30	0.0008	0.0105	0.0134	0.0087	0.0132

Table 2. Percentage relative root mean squared errors (%RRMSE) of the estimators

Sample Size (%)	$\bar{y}_{GP}$	$\bar{y}_{Rao}$	$\bar{y}_{rp}$	$\bar{y}_{cv}$	$\bar{y}_{cv.rp}$
5	11.347	9.826	9.091	5.408	4.994
10	9.923	8.380	7.734	3.694	3.247
15	8.384	7.299	6.644	2.933	2.481
20	7.986	6.102	5.522	2.449	2.001
25	6.868	5.434	4.892	2.124	1.662
30	5.984	4.797	4.291	1.849	1.380

Table 3. Percentage relative efficiencies (%RE) of the estimators $\bar{y}_{Rao}$, $\bar{y}_{rp}$, $\bar{y}_{cv}$, $\bar{y}_{cv.rp}$ with respect to $\bar{y}_{GP}$

Sample Size (%)	$\bar{y}_{GP}$	$\bar{y}_{Rao}$	$\bar{y}_{rp}$	$\bar{y}_{cv}$	$\bar{y}_{cv.rp}$
5	100	115.48	124.82	209.82	227.21
10	100	118.41	128.30	268.63	305.61
15	100	114.87	126.19	285.85	337.93
20	100	130.88	144.62	326.09	399.10
25	100	126.39	140.39	323.35	413.24
30	100	124.75	139.46	323.63	433.62



(a)

(b)

Fig. 1. (a) %RRMSE and (b) %RE of the estimators

6. RESULTS AND DISCUSSION

The simulation is carried out using varying sample sizes (5%, 10%, 15%, 20%, 25% and 30%) of the given population size to assess the performance of the developed estimators $\bar{y}_{rp}$, $\bar{y}_{cv}$, and $\bar{y}_{cv.rp}$ using real data on body performance. Table 1 depicts the %ARBs of the estimators, which range from 0.0008% to 0.1846% for $\bar{y}_{\mathrm{GP}}$, 0.0105% to 0.5001% for $\bar{y}_{Rao}$, 0.0036% to 0.3768% for $\bar{y}_{rp}$, 0.0059% to 0.0519% for $\bar{y}_{cv}$, and 0.0132% to 0.0555% for $\bar{y}_{cv.rp}$. Table 2 shows %RRMSE varies from 5.984% to 11.347% for $\bar{y}_{\mathrm{GP}}$, 4.797% to 9.826% for $\bar{y}_{Rao}$, 4.291% to 9.091% for $\bar{y}_{rp}$, 1.849% to 5.408% for $\bar{y}_{cv}$, and 1.380% to 4.994% for $\bar{y}_{cv.rp}$. The %RE of the estimators shown in Table 3 varies from 124.82% to 144.62%, 209.82% to 326.09% and 227.21% to 433.62% for the recommended estimators $\bar{y}_{rp}$, $\bar{y}_{cv}$ and $\bar{y}_{cv.rp}$, respectively.

From Table 2, it is evident that the recommended estimators using the known coefficients of variation in the calibration constraints yield lower %RRMSE than the existing estimators; refer here for a comparison under similar conditions. Also, it can be observed that a decrease in %RRMSE is coupled with an increase in sample size and, in turn, results in good relative efficiency. Figure 1(a) and (b) display the %RRMSE and %RE of the estimators. The recommended estimators exhibit higher efficiency compared to $\bar{y}_{Rao}$ and $\bar{y}_{\mathrm{GP}}$ estimators under stratified random sampling across the various sample sizes considered in the provided dataset. Therefore, the recommended estimators $\bar{y}_{cv.rp}$, $\bar{y}_{cv}$ and $\bar{y}_{rp}$, will be more accurate and outperform the existing estimators, as they offer a significant efficiency gain.

7. CONCLUDING REMARKS

In this article, three calibration estimators, $\bar{y}_{rp}$, $\bar{y}_{cv}$ and $\bar{y}_{cv.rp}$ were developed for estimating the population means using the known coefficients of variation of the multi-auxiliary information for a stratified random sampling design. The %ARB, %RRMSE and %RE of the estimators were computed for comparison purposes. The theoretical development is supported by a simulation study conducted on a real dataset of Body performance. The results for %RRMSE and %RE reported in Tables 2 and 3 show that the recommended estimators exhibit lower %RRMSEs and higher %REs than the estimators developed by Rao *et al.* (2012) and Garg and Pachori (2020). The findings conclude that the recommended estimators are substantially superior to the existing estimators considered in the study.

DISCLOSURE STATEMENT

No potential conflict of interest was reported by the author(s).

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