

A New Series of Optimal General Efficiency Balance (GEB) Designs with Lesser Number of Experimental Units

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SUMMARY

An attempt has been made to construct a new series of binary GEB block designs with lesser number of experimental units. The designs are developed by using group divisible (GD) designs. The canonical efficiency factors (CEF) of each design are listed along with A, D and E optimality efficiency values. The designs require much lesser experimental units than existing BIB designs or variance balanced (VB) designs for a particular number of treatments. This article also deals with optimality criteria of balanced block designs. A list of GEB designs with their corresponding A, D and E optimal efficiency values are presented with lesser number of experimental units (n). The list shows that some GEB designs have more than 90% either A, or D or E efficiency values, which may confirm that only traditional parameters (v , b , and k) are not always acceptable for selection of a better cost-effective design.

Keywords: BIB Designs; General Efficiency Balance (GEB) Block Designs; Group Divisible (GD) design; Optimality of Block Designs; Variance Balanced Designs.

1. INTRODUCTION

Balancing is a desirable property for any block design. The notion of balance in the context of block designs especially as Balanced Incomplete Block (BIB) designs was initiated in the early work of Bose (1939). Since then different notions of balance have been proposed. The available literature reveals methods of construction of block design possessing balance of one or more kinds. The notion of X^{-1} balanced design was proposed in the celebrated paper of Calinski (1977) and further statistical discussion was made by Calinski, Cereanka and Mejza (1980). In fact this concept of balance (X^{-1} balance) contains most of earlier prevalent notions of balance. Calinski (1977) defined a set of $v-1$ treatment contrasts L_i' $X^{-1} L_i = 1$ and $L_i' X^{-1} L_j = 0$ ($i \neq j$). Here X denote a real symmetric positive definite matrix of order v , v is the number of treatments and τ is the column vector of treatment parameters. He then defined a design as X^{-1} **balanced** if the contrasts

$L_i' \tau$, ($i = 1, 2, \dots, v-1$) are all estimated with the same variance. Also, then every X^{-1} normalised linear function of treatment parameters is estimated with the same variance. Calinski (1977) proved that for connected X^{-1} balanced designs, the coefficient matrix of inter block reduced normal equations is of the form $C = \lambda [X - X11'X/1'X1]$, where $\lambda = \{n - \text{tr}(Nk^{-\delta}N')\} / \{\text{tr}(X) - 1XX1/1'X1\}$.

Here 1 denotes a vector of one's of appropriate order, N is incidence matrix, n is total number of units, k is the block size vector and X^{δ} is the vector X written in diagonal matrix form and $X^{-\delta}$ is the inverse of X^{δ} .

The definition of X^{-1} balance as given by Calinski (1977) is presented below:

Definition 1.1: A block design is said to be X^{-1} balanced if every X^{-1} normalized estimable linear function of treatment parameters can be estimated with the same variance.

Calinski (1977) established that the two most common definitions of balance, concerning the so-called variance balanced (VB) and efficiency balanced (EB) designs are particular cases of the $\mathbf{X}^{-1}$ balanced designs as $\mathbf{I}$ -balanced and $\mathbf{r}^{-\delta}$ balanced respectively.

Das and Ghosh (1985) introduced the concept of “general efficiency balance” (GEB) designs to unify the definitions of VB and EB designs and constructed several series of VB designs and EB designs as sub classes of GEB designs. These constructions are based on a technique of reinforcement, as defined by Das (1958). The definition of GEB designs, in the sense of Das and Ghosh (1985) is given as follows:

Definition 1.2: For a connected block design $D(v; b; r_1, r_2, \dots, r_v; k_1, k_2, \dots, k_b)$, the reduced normal equation (after elimination of block effects) for the treatment effects $t_1, t_2, \dots, t_v$ is given by $\mathbf{C}\boldsymbol{\tau} = \mathbf{Q}$. A connected block design is called a GEB design if for some $\theta, s_1, s_2, \dots, s_v (> 0)$, the information matrix $\mathbf{C}$ is of the form, $\mathbf{C} = \theta(\mathbf{s}^{\delta} - \mathbf{s}\mathbf{s}'/g)$, where $\mathbf{s}' = (s_1, s_2, \dots, s_v)$, $\mathbf{s}^{\delta} = \text{diag}(s_1, s_2, \dots, s_v)$ and $\mathbf{s}'\mathbf{1} = g$. A GEB design reduces to a VB design when all s_i 's are equal and to an EB design when s_i is proportional to r_i for $i = 1, 2, \dots, v$.

When we work with various complete and incomplete designs, the concept of optimality will come into the consideration. In the early stages of agricultural experimentation, only a few experimental designs were at one's disposal. But, nowadays an experimenter with modern statistical equipments can use a variety of experimental designs with same or equal parameters. Hence, whenever the conditions of an experiment allow the possibility of simultaneous existence of a number of experimental designs, the question of selection of an appropriate design i.e. a design which is more efficient as well as easy to analyse and which has some optimum properties, naturally arises.

Patil *et al.* (2016) developed a series of GEB designs with correlated observations. The A, D and E efficiency values of the developed designs are also presented.

Pal *et al.* (2024) examined the robustness properties of GEB designs. Sufficient conditions for robustness of GEB block designs for criteria I have been examined for each type of design. The declining trend of efficiency values after deletion of treatments

one by one are presented graphically for measuring the robustness of designs under study.

Experimentation is an essential part of any problem of decision making. Whenever one is facing the problem of selection of one out of a set of alternative decisions, he has to undergo some experiments or test procedures to collect observations on which the decision has to be made. In other words, it may be possible to select an optimum decision procedure; the choice of any experiment must also be optimum in some sense. This is how the problem of optimal designing of experiments arises.

In case of experimental designs fitting into usual ANOVA (Analysis of Variance) model, the existence of a sufficient experimental design would certainly settle the problem of choice of the best experimental design. However, it is easy to verify that such sufficient designs do not exist even in the simplest set-ups. In such design set-ups, therefore, the choice of an optimum experimental design is certainly governed by what the term **optimum** connotes. The study on optimal experimental design is very much relevant to the experimenters as well as theoreticians who are engaged in scientific research works.

So far, the choice of selecting a better design depends on the optimality criteria (v , b and k). According to those criteria, variance balanced (VB) designs, especially BIB designs, are always ranked first. But the number of experimental units required for above designs in most of the times very high, which leads to higher cost involvement.

Present study includes construction of new GEB block designs from group divisible (GD) designs along with canonical efficiency factors (CEF) and A, D and E optimality values. The list of newly formed designs reveal that some designs have high CEF and A, D and E optimal efficiency values. The study also presented a list of available GEB designs with higher optimal (A, D or E) efficiency values.

2. MATERIALS AND METHODS

2.1 Basic definitions on block designs

Definition 2.1.3

A-optimality: A design $d^* \in D$ is said to be A-optimal in D , iff $\text{tr}(\mathbf{V}_{d^*}) \leq \text{tr}(\mathbf{V}_d)$ for any other

Design $d \in D$. The trace of V_d is minimized in the A-optimality criterion.

D-optimality: A design $d^* \in D$ is said to be D-optimal in D , iff $\det(V_{d^*}) \leq \det(V_d)$ for any design $d \in D$.

E-optimality: A design $d^* \in D$ is said to be E-optimal in D , iff $\max$ of $\lambda_{d^*} \leq \max$ of λ_d , i.e., E optimality criteria seeks to maximize the minimum Eigen value (λ_{d^*}) of the information matrix (C), which signifies the dispersion matrix of the design d .

2.2 Procedure of obtaining optimality value

Let $\eta_1, \eta_2, \eta_3, \dots, \eta_{(v-1)}$ be non-zero Eigen values of C matrix of the resultant design D. Then,

Optimality

The design is A-Optimal, since the inequality

$$\sum_{i=1}^{(v-1)} 1/\eta_i \geq \frac{(v-1)^2}{\text{tr}(C)}$$

Holds true, which is the required condition of a variance balanced design to be A-optimal, with equal replication and unequal block sizes. Thus the variance and efficiency balanced design constructed is A-optimal.

D- Optimality

The design is D optimal since the inequality

$$\prod_{i=1}^{(v-1)} 1/\eta_i \leq \prod_{i=1}^{(v-1)} \left[\left\{ \sum_{i=1}^{(v-1)} 1/\eta_i \right\} / (v-1) \right]$$

Holds true which is required condition of a variance balanced design to be D- optimal, with equal replication and unequal block sizes.

E- Optimality

The design is D optimal since the inequality

$$\text{Min}(\eta_i) \leq \frac{\text{tr}(C)}{(v-1)}$$

Holds true which is required condition of a variance balanced design to be E- optimal, with equal replication and unequal block size.

2.3 Efficiency criterion to select good designs

The designs would have different A-, D- and E efficiencies. For a connected block design d , let $\theta_1, \theta_2, \dots, \theta_{v-1}$, be the non-zero $v-1$ Eigen values of C- matrix

of the design. Now define $\phi_A(d) = \left(\frac{1}{v-1} \sum_{i=1}^{v-1} \theta_i^{-1} \right)^{-1}$

and $\phi_D(d) = \prod_{i=1}^{v-1} \theta_i$. Then, a design is A- [D-] optimal

if it maximizes the $\phi_A(d), \phi_D(d)$ over $D(v, b, k)$.

Then, a design is A- [D- or E-] optimal if it minimizes the $\phi_A(d), \phi_D(d)$ and $\phi_E(d)$ over $D(v, b, k)$. The A-efficiency $\{e_A(d)\}$, D-efficiency $\{e_D(d)\}$ and E-efficiency $\{e_E(d)\}$ of any design d over $D(v, b, k)$ is

$$\text{defined as } e_A(d) = \frac{\phi_A(d^*)}{\phi_A(d)}, e_D(d) = \left[\frac{\phi_D(d^*)}{\phi_D(d)} \right]^{1/(v-1)}$$

and $e_E(d) = \frac{\phi_E(d^*)}{\phi_E(d)}$, where d_A^*, d_D^* and d_E^* are the

A-optimal, D-optimal and E-optimal design over $D(v, b, k)$, respectively.

2.4 Canonical Efficiency Factor (CEF)

The canonical efficiency factors of an experimental design are the Eigen values obtained from the normalized treatment information matrix, relative to a reference design such as the completely randomized design or the randomized complete block design. These factors quantify the amount of information provided by the design for the estimation of mutually orthogonal treatment contrasts.

Note: The CEF for block designs with unequal replication will be measured on the basis of average replication number of total treatments.

3. METHOD OF CONSTRUCTION OF GEB BLOCK DESIGNS WITH LESSER NUMBER OF EXPERIMENTAL UNITS

Some GEB block designs are constructed using GD designs in the following

Theorem 3.1: If there exist a GD design (D_1) with parameters $v_1 = mn, b_1, r_1, k_1, n_1 = n-1, n_2 = (m-1)n, \lambda_{11}$, and λ_{12} ($\lambda_{11} \neq \lambda_{12}, \lambda_{12} - \lambda_{11} = \theta$) and the association scheme as set of treatments (D_2) with parameters $v_2 = mn, b_2 = m, r_2 = 1, k_2 = n, \lambda_{21} = 1$, and $\lambda_{22} = 0$ imply the existence of a GEB block design (D^*) with parameters $v^* = mn+1, b_1^* = b_1, b_2^* = b_2, r_1^* = r_1+1,$

$r_2^* = r^*$, (r^* is either b_1 or b_2 or $b_1 + b_2$), λ_1^* , λ_2^* and λ^* if one of the following conditions is satisfied,

- I. $\frac{\theta}{n} = \frac{\lambda_2^*}{k_1 + 1}$
- II. $\frac{\theta}{n + 1} = \frac{\lambda_2^*}{k_1}$
- III. $\frac{\theta}{n + 1} = \frac{\lambda_2^*}{k_1 + 1}$

Here, λ_1^* denotes the occurrence of pairs of first associates of the GD design D_1 ; λ_2^* denotes the occurrence of pairs of second associates of the GD design D_1 ; and λ^* represents the occurrence of pairs of all mn treatments with the $(mn + 1)$ th treatment.

Proof (By construction)

Let us consider a GD design (D_1) with parameters $v_1 = mn$, b_1 , r_1 , k_1 , $n_1 = n - 1$, $n_2 = (m - 1)n$, λ_{11} , and $\lambda_{12} = \lambda_{11} + \theta$ (θ is any positive integer). Let us also consider a disconnected set of treatments (D_2) as the association scheme D_1 with parameters $v_2 = mn$, $b_2 = m$, $r_2 = 1$, $k_2 = n$, $\lambda_{21} = 1$, and $\lambda_{22} = 0$. D_2 be repeated θ times. Let us reinforce

Case 1: ($v_1 + 1$)th treatment to all blocks of D_1 to satisfy condition number (I).

Case 2: ($v_1 + 1$)th treatment to all m blocks of D_2 to satisfy condition number (II).

Case 3: ($v_1 + 1$)th treatment to all b_1 blocks of D_1 and m blocks of D_2 to satisfy condition number (III).

Table 3.1. List of constructed GEB block designs with optimal efficiency values and number of experimental units using theorem-3.1

Sl. No.	D_1	D_2	D^*	n^*	CEF(%)	A-Eff(%)	D-Eff(%)	E-Eff(%)	X^*
1	SR-19	Association Scheme of SR-19	$v^*=7, b_1^*=8, b_2^*=3, r_1^*=5, r_2^*=8, k_1^*=4, k_2^*=2, \lambda_1^*=1, \lambda_2^*=2, \lambda^*=4$	38	79.35	95.72	89.14	88	50,51,52, 54,55,56, 57,58,59
2	SR-26	Association Scheme of SR-26	$v^*=13, b_1^*=16, b_2^*=3, r_1^*=5, r_2^*=16, k_1^*=4, k_2^*=4, \lambda_1^*=1, \lambda_2^*=1, \lambda^*=4$	76	72	89.37	62.78	84.21	121,122, 123,124
3	SR-66	Association Scheme of SR-66	$v^*=13, b_1^*=8, b_2^*=6, r_1^*=5, r_2^*=6, k_1^*=6, k_2^*=3, \lambda_1^*=1, \lambda_2^*=2, \lambda^*=1$	66	89	96.27	83.14	92.24	121,122,123,124,126,127, 128,129,130
4	SR-52	Association Scheme of SR-52	$v^*=11, b_1^*=8, b_2^*=5, r_1^*=5, r_2^*=13, k_1^*=6, k_2^*=3, \lambda_1^*=1, \lambda_2^*=2, \lambda^*=5$	63	80.49	92.2	73.46	86.6	103,104,105
5	SR-53	Association Scheme of SR-53	$v^*=11, b_1^*=12, b_2^*=5, r_1^*=7, r_2^*=12, k_1^*=6, k_2^*=2, \lambda_1^*=1, \lambda_2^*=3, \lambda^*=6$	82	84	96.7	86.85	92.31	103,104,105,106
6	SR-99	Association Scheme of SR-99	$v^*=19, b_1^*=12, b_2^*=9, r_1^*=7, r_2^*=9, k_1^*=9, k_2^*=3, \lambda_1^*=1, \lambda_2^*=3, \lambda^*=1$	135	80	99.97	99.81	99.64	180,181,182,183
7	R-43	Transpose of Association Scheme of R-43	$v^*=7, b_1^*=12, b_2^*=3, r_1^*=7, r_2^*=3, k_1^*=3, k_2^*=3, \lambda_1^*=3, \lambda_2^*=3, \lambda^*=1$	45	73.5	81.05	45.25	33.4	51,52
8	R-144	Association Scheme of R-144	$v^*=13, b_1^*=12, b_2^*=6, r_1^*=6, r_2^*=18, k_1^*=6, k_2^*=3, \lambda_1^*=1, \lambda_2^*=2, \lambda^*=6$	90	79	91.21	67.36	86.21	121,123,124
9	R-175	Association Scheme of R-175	$v^*=13, b_1^*=12, b_2^*=6, r_1^*=8, r_2^*=12, k_1^*=8, k_2^*=3, \lambda_1^*=3, \lambda_2^*=4, \lambda^*=7$	108	88.59	98.17	90.84	95	121,123,124,127,128,129,130

Now, the resultant design D^* be constructed with D_1 and θ times D_2 . The design D^* , after satisfying any of the conditions (I), (II) or (III), will generate a binary GEB block design with parameters $v^* = v_1 + 1$, $b_1^* = b_1$, $b_2^* = m$, $r_1^* = r_1 + 1$, $r_2^* = \text{either } b_1 \text{ or } b_2 \text{ or } b_1 + b_2$, $k_1^* = \text{either } k_1 + 1 \text{ or } k_1$, $k_2^* = \text{either } n \text{ or } n + 1$, $\lambda_1^* = \lambda_{11} + \theta$ and $\lambda_2^* = \lambda_{12}$ and $\lambda^* = \text{either } r_1 \text{ or } \theta \text{ or } (r_1 + \theta)$ for all the above 3 cases, where, λ_1^* denotes the occurrence of pairs of first associates of the GD design D_1 ; λ_2^* denotes the occurrence of pairs of second associates of the GD design D_1 ; and λ^* represents the occurrence of pairs of all mn treatments with the $(mn + 1)$ th treatment.

In D^* the total number of experimental units (n^*) is either $(k_1 + 1)b_1 + \theta mn$, or $k_1 b_1 + \theta m(n + 1)$, or $(k_1 + 1)b_1 + \theta m(n + 1)$ for the cases 1, 2 and 3, respectively. It has observed that the value of n^* will be minimum when $\theta = 1$ for all the cases.

The number of experimental units (n^{**}) of a BIB design with $(v_1 + 1)$ number of treatments will be $(v_1 + 1)r$, where r is the replication number. Again, the total number of units (n^*) for the GEB design D^* will be $(v_1 + 1)\bar{r}$, where $\bar{r}$ is the average replication number. Therefore, whenever $(r - \bar{r}) > 0$, the resultant GEB design requires lesser value of n^* .

D_1 s' are taken from Clatworthy (1973), X^* is the sl. no. of the BIB designs, taken from Design Resource Server, ICAR, Indian Agricultural Statistics Research Institute, having lesser or equal efficiency but requires higher experimental units. 'Eff' means efficiency value.

Corollary 3.1: Under the realization of theorem-3.1, a GEB block design D^{**} can be constructed with parameters $v^{**} = mn + 1$, $b_1^{**} = b_1$, $b_2^{**} = m$, $r_1^{**} = r_1 + 1$, $r_2^{**} = \text{either } b_1 \text{ or } m \text{ or } b_1 + m$, $k_1^{**} = \text{either } k_1 \text{ or } k_1 + 1$, $k_2^{**} = \text{either } k_2 \text{ or } k_2 + 1$, $\lambda_1^{**} = 1$ and $\lambda_2^{**} = \lambda_{12}$, $\lambda^* = \text{either } r_1 \text{ or } 1 \text{ or } r_1 + 1$ using D_1 with parameters $v_1 = mn$, b_1 , r_1 , $k_1 = n + 1$, $n_1 = n - 1$, $n_2 = (m - 1)n$, λ_1 , and $\lambda_2 = \lambda_1 + 1$, and another design D_2 , which is the association scheme of D_1 .

Proof

Following the proof of the theorem-3.1, the proof of corollary-3.1 is straight forward.

Table 4.1. Optimal and Efficient GEB designs with lesser number of experimental units

Sl no.	Design	n	Type of Design	A-Eff (%)	D-Eff (%)	E-Eff (%)	n*
1	$v=6(5+1)$, $b=5$, $r_1=4$, $r_2=5$, $k=5$, $\lambda_1=3$, $\lambda_2=4$	25	GEB	99	98	95	30
2	$v=8(7+1)$, $b=7$, $r_1=3$, $r_2=7$, $k=4$, $\lambda_1=1$, $\lambda_2=3$	28	GEB	91	77	83	56
3	$v=8(7+1)$, $b=7$, $r_1=6$, $r_2=7$, $k=7$, $\lambda_1=5$, $\lambda_2=6$	49	GEB	99.69	98.98	97.62	56
4	$v=10(9+1)$, $b=12$, $r_1=4$, $r_2=12$, $k=4$, $\lambda_1=1$, $\lambda_2=4$	48	GEB	87.89	65.90	81.30	60
5	$v=10(9+1)$, $b=12$, $r_1=8$, $r_2=12$, $k=7$, $\lambda_1=5$, $\lambda_2=8$	84	GEB	98.68	96.60	94.60	60
6	$v=12(11+1)$, $b=11$, $r_1=5$, $r_2=11$, $k=6$, $\lambda_1=2$, $\lambda_2=5$	66	GEB	94.82	79.80	90	132
7	$v=12(11+1)$, $b=11$, $r_1=6$, $r_2=11$, $k=7$, $\lambda_1=3$, $\lambda_2=6$	77	GEB	96.82	87.70	92.80	132
8	$v=14(13+1)$, $b=13$, $r_1=4$, $r_2=13$, $k=5$, $\lambda_1=1$, $\lambda_2=4$	65	GEB	84.85	29.70	85	182
9	$v=14(13+1)$, $b=13$, $r_1=9$, $r_2=13$, $k=10$, $\lambda_1=6$, $\lambda_2=9$	130	GEB	99.31	-	96.70	182
10	$v=16(15+1)$, $b=15$, $r_1=7$, $r_2=15$, $k=8$, $\lambda_1=3$, $\lambda_2=7$	120	GEB	96.39	83	93	80
11	$v=16(15+1)$, $b=15$, $r_1=8$, $r_2=15$, $k=9$, $\lambda_1=4$, $\lambda_2=8$	135	GEB	97.65	88.50	94	80
12	$v=18(17+1)$, $b=34$, $r_1=16$, $r_2=34$, $k=9$, $\lambda_1=7$, $\lambda_2=16$	306	GEB	97.24	90.80	93.80	306
13	$v=20(19+1)$, $b=19$, $r_1=9$, $r_2=19$, $k=10$, $\lambda_1=4$, $\lambda_2=9$	190	GEB	97.23	82.50	94.40	380
14	$v=20(19+1)$, $b=19$, $r_1=10$, $r_2=19$, $k=11$, $\lambda_1=5$, $\lambda_2=10$	209	GEB	97.93	85.55	95.50	380

n^* is the number of experimental units of existing BIB designs with same number of treatments.

4. CONSTRUCTION OF OPTIMAL AND EFFICIENT INCOMPLETE BLOCK DESIGNS WITH LESSER NUMBER OF EXPERIMENTAL UNITS

Here, an effort has been made to construct Generalised Efficiency Balanced (GEB) block designs from existing Balanced Incomplete Block (BIB) designs with a least number of experimental units using the reinforcement procedure of Kageyama and Mukerjee (1986).

Theorem 4.1: The existence of a BIB design $(v, b, r, k \text{ and } \lambda)$ implies that the existence of a GEB designs with parameters $v^*=v+1$, $b^*=b$, $r^*=(rI_v, r_2)$ and $k^*=k+1$. The design requires minimum number of experimental units i.e. $n = b^*k^* = v^*(rI_v, r_2)$.

Proof (By construction)

Let D be a BIB design $(v, b, r, k \text{ and } \lambda)$ and let us construct a GEB design D^* by reinforcement of an additional treatment as $(v+1)$ th treatment to each and every block of design D . The resultant design D^* will be GEB design as stated by Kageyama and Mukerjee (1986). The parameters of the design D^* will be $v^*=v+1$, $b^*=b$, $r^*=(rI_v, r_2)$ and $k^*=k+1$. Different optimality criteria have been calculated for the design D^* . It has been observed that D^* has very high A-optimality efficiency value and the efficiency values of D-optimality and E-optimality are also high.

Following the procedures mentioned in **Theorem 4.1**, some GEB designs are constructed from BIB designs with parameters $(v, b, r, k \text{ and } \lambda)$ in table 4.1. The parameters of the developed GEB designs will be $v^*=v+1$, $b^*=b$, $r^*=(rI_v, r_2)$ and $k^*=k+1$.

5. CONCLUSION

This study presents a new class of binary GEB designs based on group divisible structures with reduced numbers of experimental units. The canonical efficiency factors and A-, D-, and E-optimality efficiency values show that some of the developed

designs retain high efficiency; often A-efficiency values are exceeding 90%, despite using fewer experimental units than corresponding BIB or variance-balanced designs. These results demonstrate that near-optimal efficiency can be achieved; even the designs are not variance balance. It makes the developed designs attractive and cost-effective alternatives for practical experimentation.

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