

Fuzzy Classification Tree Modelling with Application in Agricultural Ergonomics

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SUMMARY

Classification in agricultural systems are quite useful for planning for which decision trees like Classification And Regression Trees (CART) can be used effectively. Additionally, data in reality involves fuzziness that necessitates development of CART which can handle them. As against crisp boundaries between which elements are members and non-members of a particular set, fuzzy set theory offers degree of membership anywhere between 0 and 1 to each set element. Fuzzy based CART has been dealt with in this study considering agricultural ergonomics data with response variable taking levels as presence/ absence of discomfort for labourers during farm operation. The associated variables were both categorical: farm machinery load, operation modes, percent aerobic capacity of farm labourers and continuous: difference between working/ resting heart rates, oxygen consumption during farm operation. The data was divided into training and test sets for model building and validation respectively. Membership function for each variable has been defined with the aid of linguistic variables, using which all the variables were fuzzified. The conventional CART was obtained and complexity parameter was used to prune the tree. The set of 'if-then' rules obtained from this CART formed the knowledge base for the fuzzy inference system (FIS) to build conventional Fuzzy CART. The inputs were given to FIS and the outputs for making decisions were defuzzified into crisp values indicating the degree of discomfort. For comparison, those values greater than 0.5 were taken as presence of discomfort, absence otherwise. As an improvement to this Fuzzy CART, the bias due to simultaneous selection of the split variable and the split point in CART was overcome by using separate selection procedures, while other steps remained the same. This proposed Fuzzy CART model was found to be outperforming the existing CART approaches viz., conventional CART, a certain modified CART (method obtained by earlier workers wherein separate selection procedures for split variable and split point as mentioned above were employed in conventional CART) and Fuzzy CART methods when the results were compared using the correct classification rate. Even though model based logistic regression outperformed the proposed Fuzzy CART, the latter has many advantages over the former. Thus it has been demonstrated that Fuzzy CART can be used as a viable alternative for classification purposes in agricultural domain.

Keywords: ANOVA F statistic; Correct Classification Rate; Dalenius and Hodges stratification; Fuzzification; Logistic regression; Pearson's Chi-square statistic; Rule base; Split variable/split point selection; Triangular membership function.

1. INTRODUCTION

Classification and prediction are very useful for planning purposes in agricultural systems. Decision tree-based classification and regression procedures can be resorted to as these are not based on any stringent assumptions. One of the decision tree approaches,

more commonly referred to as Classification And Regression Trees (CART), was proposed by Breiman *et al.* (1984). Classification trees can be used when the response variable is inherently dichotomous or polytomous. Within CART, the observations are successively divided into two subsets every time based

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on an associated variable substantially related to the response variable; this method has an advantage of offering easily comprehensible decision strategies.

Data collection in the real world involves a lot of fuzziness and uncertainty. Nature of agricultural systems warrant the need for modelling systems that are robust, interpretable, noise tolerant, less time consuming, and are extensible. Therefore, there is a need for some techniques which can be applied in real-world situation and also perhaps have a better accuracy. Fuzzy logic has these characteristics and has been used in control and modelling in various domains. Conventional set theory is based on a crisp boundary between which elements are members or non-members of a particular set - here the answer of the question of whether an element is a member or not of a particular set is always yes or no. In contrast, fuzzy theory can handle the vagueness present in the data. Recently, several advanced CART techniques such as Bayesian CART, Genetic Algorithm (GA) based CART, Fuzzy CART etc. have been used for improving accuracy of the classification.

Tree-based implementations emerged in the 1960s with the introduction of AID (Automatic Interaction Detector) by Morgan and Sonquist (1963) as regression trees. Further modifications to this methodology culminated in the development of THAID (THeta AID) by Morgan and Messenger (1973) and CHAID (CHi AID) by Kass (1977). Sadhu *et al.* (2014) have employed classification tree modelling in the field of agricultural ergonomics. Classifications using modified CART models were reported to be higher than those obtained for the corresponding logistic regression and discriminant function analysis approaches. In addition, they have also proposed an alternative novel tree-based modelling classification process within the context of the CART.

Denison *et al.* (1998) have proposed a stochastic search form of CART motivated by a Bayesian model. The Bayesian analogue of ‘pruning the tree’ is achieved by putting a suitably chosen prior distribution over the number of terminal nodes in the structure, and, in the case of classification trees, a distribution over the classification of data points within each node. Chipman *et al.* (1998) and his later work have also put forward a Bayesian approach and its improved versions for

finding CART model. The basic idea is to have the prior induce a posterior distribution that will guide the stochastic search toward more promising CART models. As the search proceeds, such models can then be selected with a variety of criteria, such as posterior probability, marginal likelihood, residual sum of squares or misclassification rates. Fan and Gray (2005) have given Tree Analysis with Randomly Generated and Evolved Trees (TARGET) method for constructing regression trees via GA and compared its performance to CART, Bayesian CART, and random forests. Gray and Fan (2008) considered the construction of classification trees using TARGET. Grubinger *et al.* (2011) proposed an evolutionary learning method based on GA for building globally optimal classification and regression trees and also have uploaded in R software a package named ‘evtree’. Barros *et al.* (2012) presents a survey of evolutionary algorithms (Genetic algorithms and Genetic programming) designed for decision tree induction. Jankowski and Jackowski (2014) proposed an evolutionary algorithm for decision tree induction with population of possible tree structures decoded in the form of tree-like chromosomes and training process aims at minimizing objective functions having misclassification rate and tree size as two components. The results revealed that the CART-GA method is more accurate at classifying the condition as compared to the methods employed viz., ANN, CART, and ANN-GA. Ahmadi *et al.* (2022) proposed GA based enhanced CART approach that can provide both high interpretability and high accuracy for flood susceptibility mapping. Ramasubramanian *et al.* (2024) enhanced CT by integrating a certain GAversion to overcome inherent biases in traditional approaches demonstrating that the GA-integrated CART approach outperforming the other existing CA methods.

Zadeh (1965) did a pioneering work on fuzzy sets and said that such a set is characterized by a membership function which assigns a degree of membership ranging between zero and one to each object of the set. Klir and Yuan (1995) gave some theoretical aspects of fuzzy set theory and fuzzy logic. They have introduced elementary concepts, including basic types of fuzzy sets, which also contains a discussion of the meaning and significance of the emergence of fuzzy set theory. Connections between fuzzy sets and crisp sets have been examined. It shows how fuzzy sets can be represented

by families of crisp sets and how classical mathematical functions can be fuzzified wherein the concepts of linguistic variables and fuzzy equations also have been introduced and examined. Ross (2005) has introduced the basic concept of fuzziness and distinguishes fuzzy uncertainty from other forms of uncertainty. He has also briefed the fundamental idea of set membership and presents membership functions as the format used for expressing set membership. Basic IF–THEN rule structures have been discussed along with few fuzzy classification methods.

Fuzzy based decision tree modeling is discussed subsequently. Jang (1994) used fuzzy modelling to the structure determination problem by using CART and finally proposed a CART plus ANFIS (Adaptive-Network based Fuzzy Inference System) approach to complete the two tasks of fuzzy modelling, i.e. structure determination and parameter identification. Suarez and Luksko (1999) generated a fuzzy decision tree by replacing the Boolean tests in a standard CART decision tree by giving the degree of membership of an example in a node of the decision tree. In contrast with a crisp CART tree, where a single leaf is responsible for the prediction, the classification or regression was made jointly by all the terminal nodes of the decision tree using the full set of membership values to the terminal nodes of the tree. In nutshell, they have superimposed a fuzzy structure over the skeleton of a CART decision tree. Chiang and Hsu (2002) have proposed fuzzy classification trees along with their construction algorithm by integrating the fuzzy classifiers with decision trees for cases where the boundaries among classes are not always clearly defined. In this way, instead of determining a single class for any given instance, their method of fuzzy classification predicts the degree of possibility for every class. Haskell *et al.* (2004) have improved the performance of binary tree classifier by making each node of the tree fuzzy with the adjustments in membership functions. They have also shown that GA can be used to optimize these fuzzy trees. Andreou and Papatheocharous (2008) and Papatheocharous and Andreou (2009) have proposed the use of automatic generation and evaluation of decision trees instances enhanced with fuzzy logic for software cost estimation and built a data-driven model based on fuzzy decision trees. The stable and strong classification rules produced by them were

used to perform cost predictions. Vagliasindi *et al.* (2008) proposed an automatic identifier based on fuzzy logic together with a CART analysis to automatically select among several diagnostic signals available for the relevant inputs to be provided to the identifier. Moreover, the outputs of the CART analysis were also the basis for the membership selection and rule design of the fuzzy identifier. Yenyayla (2011) has briefed on fuzzy entropy and used the fuzzy entropy to express the mathematical values of the fuzziness in fuzzy sets.

Goel and Sehgal (2015) have proposed Fuzzy Rule-Based System (FRBS) to estimate the ripeness of tomatoes based on color depicted by red-green color difference / ratio. Fuzzy partitioning of the feature space into linguistic variables is done using Mamdani fuzzy inference system. A rule set is automatically generated from the derived feature set using CART. Pinem *et al.* (2015) used a combination of CART and Fuzzy Logic method to detect intrusion in a computer system where the CART is used to build rule or model that was in turn implemented by fuzzy inference system. Wang *et al.* (2015) provided a new architecture of a fuzzy decision tree and provided a learning algorithm. In contrast with “traditional” decision trees in which only a single feature (variable) is taken into account at each node, the node of the proposed decision trees involves a fuzzy rule which involves multiple features. Muchai *et al.* (2018) used Pearson’s chi-squared impurity measure to compare the performance of crisp and fuzzy classification trees and observed that the fuzzy classification trees allocated observations to the correct population with fewer errors than did crisp classification trees.

In the present study, a fuzzy based CART modelling procedure for classification in agricultural ergonomics has been developed. In particular, the modification done in Sadhu *et al.* (2014)’s work in conventional CART has been incorporated in the conventional Fuzzy CART in order to propose a new Fuzzy CART. The proposed method has been compared with the existing CART approaches and other methods used for classification. Section1 provides a brief introduction to the topic of CART and fuzzy theory. It is followed by motivation and objectives of the present work. While doing so, a detailed review of literature on use of CART, fuzzy theory, and their combination i.e. fuzzy CART in various fields and their salient findings in

brief are mentioned. Section 2 consists of definitions of some important terminologies related to CART and fuzzy theory viz, decision tree, split variable, split point, membership function, linguistic variable, fuzzification, fuzzy inference system, defuzzification, etc. The methods of fuzzification by use of membership functions, constructing the conventional CART, framing the set of rules from the tree obtained and using these membership functions and rule bases in the fuzzy inference system followed by defuzzification to construct the fuzzy CART have also been discussed. A novel way of constructing fuzzy based CART approach has also been proposed. Section 3 comprises of the results of the various models fitted by considering a data set followed by comparisons of the performance of these models for classification purposes. The models considered were the conventional CART, Sadhu *et al.* (2014)'s modified CART, conventional fuzzy CART, logistic regression and the proposed fuzzy based CART methodology. The paper is signed off with concluding remarks in Section 4.

2. MATERIAL AND METHODS

In this section, information about the data utilized in this study is given. Thereafter, the preliminaries of fuzzy theory and also that of the CART method will be dealt with. In addition, the methodological steps involved in conventional CART modelling in brief and Fuzzy CART modelling in detail will be done. Moreover, the improved CART method given by Sadhu *et al.* (2014) will also be discussed from the standpoint of how it differs from the conventional CART method. The development of new fuzzy based CART procedure will also be outlined. As logistic regression has also been employed for comparison purposes, the same has also been included in brief.

Division of Agricultural Engineering, ICAR-Indian Agricultural Research Institute, New Delhi provided the data collected during 2007-08 in the field of agricultural ergonomics for the present study. The variable known to be a dependent variable is the “presence” (Pr or simply, P) or “absence” (Ab or simply, A) of “discomfort” of farm laborers during farm service. Collection of qualitative predictor variables are: “load given to farm machinery (Load)”, modes of operation (Mode)” and “percent aerobic capacity of the farm labourers (A_Capacity)” and quantitative predictor variables are:

“difference between working and resting heart rates (DHR)” and “oxygen consumption at the time of farm operation (O₂ Consumption or simply, O₂_Con)”. The data collection accessible is at five rates of Load viz. “No load, i.e. 0 W, 0.90W, 1.80W, 2.70W and 3.60W” denoted as L1 through L5 respectively, where W means the S.I. unit of power, i.e. Watt. In addition, there were two Loads viz., 0.45W and 1.35W which were merged with L2 and L3 respectively. The observations on all the above mentioned predictor variables (five variables) were recorded upon nine farm labourers (subjects). The observations were independently taken at three durations under each of the “five loads of operation” considered and for each farm labourers with three distinct “modes of operations”, out of the five modes were considered for taking observations. So, a total of $9 \times 3 \times 5 \times 3 = 405$ data observations were available for the study. Within this dataset, two types of Mode viz., Predominantly operated by foot (e.g. Stepper, Pump, Wheelchair, etc.) denoted as Foot or simply, F and “other modes of operation” (e.g. Rolling, Flywheel, etc.) denoted as Others or simply, O each with 225 and 180 measurements respectively were considered. A_Capacity has been categorised as “Low” and “High” respectively according as $\leq 35\%$ and $> 35\%$ of aerobic capacity of the farm labourers.

In the conventional CART decision tree using recursive binary partitioning, the optimum split of the given dataset is determined at various stages and also the best splits of its subsets on the basis of various issues such as identifying which variable should be used to create the split (splitting variable), and determining the precise rule for the split (splitting condition for the selected splitting variable and hence splitting point), evaluating whether the tree node is a terminal one, and labeling the predicted class to each terminal node.

The CART separation principle is branching of the node using the selected splitting variable and split point at every stage which helps in the largest decline in ‘impurity’ measure in the subsequent nodes (child nodes) as compared to the previous node (parent). From among the several

impurity functions, Gini index given by has been used where p_i is the

$$Gini = 1 - \sum_{i=1}^c p_i^2$$

probability of an object (observation) classified to a particular class i ($i = 1, 2, \dots, c$) of the main study variable and c is the number of classes of the response variable.

For a response variable with two classes, Gini index reduces to

$$Gini = 2p(1 - p)$$

where, p is the probability of an object (observation) classified to one of the two classes (say, 'presence' or 'yes' class, depending on the context) of the response variable. A classification tree with the best possible classification performance is called a right-sized tree. The determination of the correct size tree involves rigour, with a completely grown tree pruned to achieve an optimal tree size by considering standard criteria such as complexity function and misclassification rate computations. For further details, refer Izenman (2008).

A membership function $\mu_A(x)$ for a fuzzy set A on the universe of discourse X is defined as $\mu_A(x): X \rightarrow [0, 1]$, where each element x of X is mapped to a value between 0 and 1. The values taken by the membership function are called the degree of membership that quantifies the grade of membership of the element in X to the fuzzy set A . The membership degree of a set element is any real number between 0 and 1. For defining these membership functions, the variables are considered as linguistic values with words or sentences in a natural or artificial language. Selection of linguistic variable depends on the triangular membership function depicted in Fig.3.1(i).

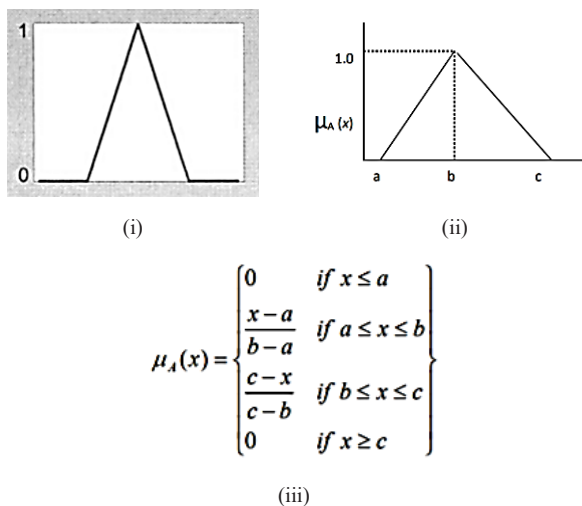


Fig. 2.1. Triangular membership function

In particular, the triangular membership function is given by $\mu_A(x)$ as in Fig.2.1(iii), where a , b and c represented in Fig. 2.1(ii) are x coordinates of the three vertices of $\mu_A(x)$ in a fuzzy set A with a as the lower boundary where membership degree is zero; b as the centre where membership degree is 1 and c as the upper boundary where membership degree is again zero.

A fuzzy inference system (FIS) is a system that uses fuzzy set theory to map inputs (features or variables) to outputs (classes or levels of the response variable). Mamdani's fuzzy inference method is the most commonly used fuzzy methodology proposed in 1975 by Ebrahim Mamdani (Ghosal *et al.*, 2017). FIS uses a collection of fuzzy membership functions and rules for reasoning data. The functional operations in FIS proceed as follows:

- i) Fuzzification of the inputs: The process of converting crisp values into fuzzy values by identifying possible uncertainties or variations in the crisp values. This conversion is represented by membership functions.
- ii) Fuzzy inference: The set of rules in a FIS known as knowledge base or rule base, obtained mostly by perception and expert's domain knowledge, is generated and is given as an input to a fuzzy classification system.
- iii) Aggregation of all outputs: The output of each rule is combined through aggregation.
- iv) Defuzzification: As a reverse process of fuzzification, fuzzy values are converted into crisp values, by centroid method in the present study.

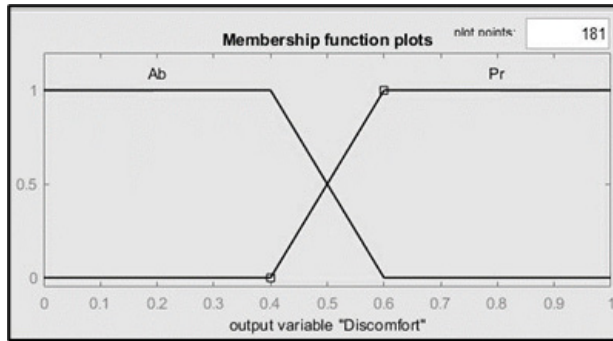
The process of designing a fuzzy rule based classification tree model involves the following steps:

- Define the input and output attributes.
- Construct membership functions (MFs).
- Generate fuzzy rule base for the system.
- Feed the MFs and rule base to fuzzy inference system (FIS).
- Obtain the output.

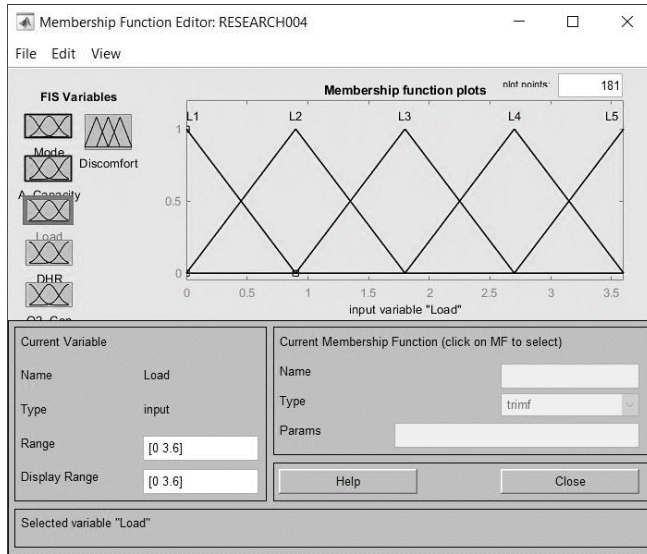
The data has been partitioned randomly into training and test datasets in the ratio of 80:20. Using R software, a random sample with size of 80% (i.e. 324 observations) of original dataset is obtained. This random sample is considered as the training dataset

and the remaining observations are assigned to the test set. The input and output variables have already been discussed earlier. Each of these variables has been fuzzified into linguistic terms and a membership function has been defined for each variable.

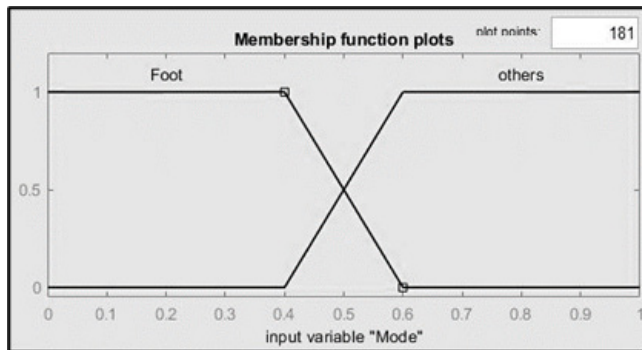
The variable Discomfort with two linguistic variables Absence (Ab) and Presence (Pr) is fuzzified using Fig.2.2(i).



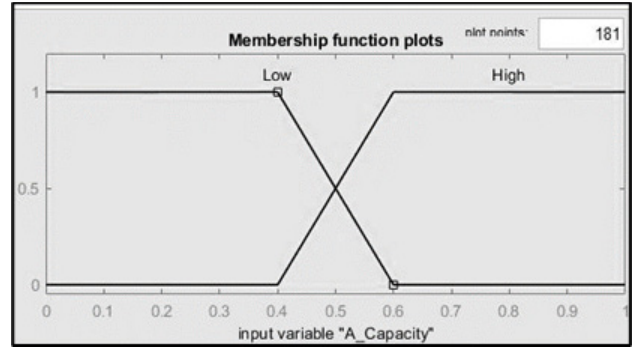
(i). Membership function for Discomfort



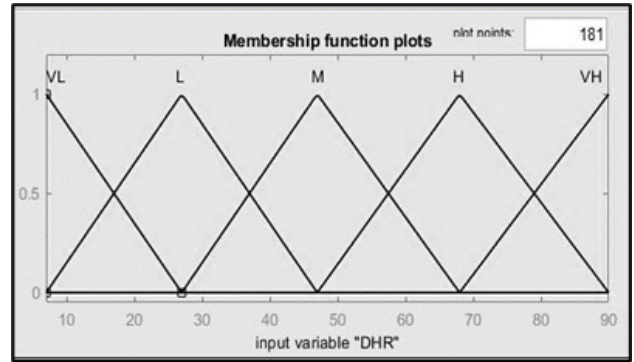
(ii). Membership function for Load



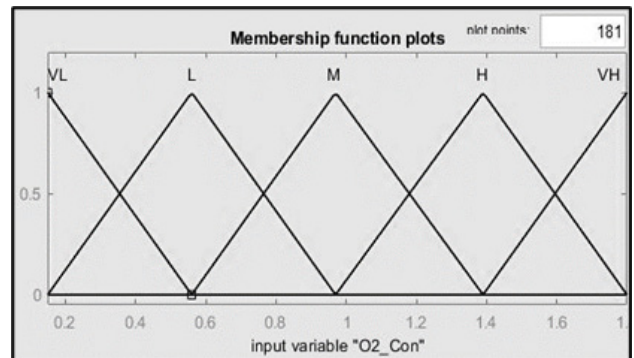
(iii). Membership function for Mode



(iv). Membership function for Aerobic capacity mentioned as A_Capacity



(v). Membership function for DHR



(vi). Membership function for Oxygen consumption mentioned as O2_Con

Fig. 2.2. Membership functions for Response and associated variables

and the equations are given as follows:

$$\mu_{Ab}(Discomfort) = \begin{cases} 1 & , x \leq 0.4 \\ \frac{0.6 - x}{0.6 - 0.4} & , 0.4 \leq x \leq 0.6 \\ 0 & , x \geq 0.6 \end{cases}$$

$$\mu_{Pr}(Discomfort) = \begin{cases} 0 & , x \leq 0.4 \\ \frac{x - 0.4}{0.6 - 0.4} & , 0.4 \leq x \leq 0.6 \\ 1 & , x \geq 0.6 \end{cases}$$

The variable Load with five linguistic variables i.e., 0.0W (L1), 0.9W (L2), 1.8W (L3), 2.7W (L4), 3.6W (L5) is fuzzified using Fig.2.2(ii). and the equations are given as follows:

$$\mu_{L1}(Load) = \begin{cases} \frac{0.9-x}{0.9-0}, & 0 \leq x \leq 0.9 \\ 0 & , x \geq 0.9 \end{cases}$$

$$\mu_{L2}(Load) = \begin{cases} \frac{x-0}{0.9-0}, & 0 \leq x \leq 0.9 \\ \frac{1.8-x}{1.8-0.9}, & 0.9 \leq x \leq 1.8 \\ 0 & , x \leq 0 \text{ or } x \geq 1.8 \end{cases}$$

$$\mu_{L3}(Load) = \begin{cases} \frac{x-0.9}{1.8-0.9}, & 0.9 \leq x \leq 1.8 \\ \frac{2.7-x}{2.7-1.8}, & 1.8 \leq x \leq 2.7 \\ 0 & , x \leq 0.9 \text{ or } x \geq 2.7 \end{cases}$$

$$\mu_{L4}(Load) = \begin{cases} \frac{x-1.8}{2.7-1.8}, & 1.8 \leq x \leq 2.7 \\ \frac{3.6-x}{3.6-2.7}, & 2.7 \leq x \leq 3.6 \\ 0 & , x \leq 1.8 \end{cases}$$

$$\mu_{L5}(Load) = \begin{cases} 0 & , 0 \leq x \leq 2.7 \\ \frac{x-2.7}{3.6-2.7}, & 2.7 \leq x \leq 3.6 \end{cases}$$

The variable Mode with two linguistic variables Foot and Others is fuzzified using Fig.2.2(iii). and equations for $\mu_{Foot}(Mode)$ and $\mu_{Others}(Mode)$ are given respectively as the RHS of that given for $\mu_{Ab}(Discomfort)$ and $\mu_{Pr}(Discomfort)$ above.

The variable A_Capacity with two linguistic variables Low and High is fuzzified using Fig.2.2(iv).

and equations for $\mu_{Low}(A_Capacity)$ and $\mu_{High}(A_Capacity)$ are given respectively as the RHS of that given for $\mu_{Ab}(Discomfort)$ and $\mu_{Pr}(Discomfort)$ above.

The variable DHR with five linguistic variables i.e., Very Low (VL), Low (L), Medium (M), High (H), Very High (VH) is fuzzified using Fig.2.2(v). and equations given below and the equations are given as follows:

$$\mu_{VL}(DHR) = \begin{cases} \frac{28-x}{28-7}, & 7 \leq x \leq 28 \\ 0 & , x \geq 28 \end{cases}$$

$$\mu_L(DHR) = \begin{cases} \frac{x-7}{28-7}, & 7 \leq x \leq 28 \\ \frac{49-x}{49-28}, & 28 \leq x \leq 49 \\ 0 & , x \leq 7 \text{ or } x \geq 49 \end{cases}$$

$$\mu_M(DHR) = \begin{cases} \frac{x-28}{49-28}, & 28 \leq x \leq 49 \\ \frac{70-x}{70-49}, & 49 \leq x \leq 70 \\ 0 & , x \leq 28 \text{ or } x \geq 70 \end{cases}$$

$$\mu_H(DHR) = \begin{cases} \frac{x-49}{70-49}, & 49 \leq x \leq 70 \\ \frac{90-x}{90-70}, & 70 \leq x \leq 90 \\ 0 & , x \leq 49 \end{cases}$$

$$\mu_{VH}(DHR) = \begin{cases} 0 & , 0 \leq x \leq 70 \\ \frac{x-70}{90-70}, & 70 \leq x \leq 90 \end{cases}$$

The membership function for O2Consumption or simply, O2_Con with five linguistic variables i.e., Very Low (VL), Low (L), Medium (M), High (H), Very High (VH) is fuzzified using Fig.2.2(vi). and the equations are given as follows:

$$\mu_{VL}(X5) = \begin{cases} \frac{0.56-x}{0.56-0.15}, & 0.15 \leq x \leq 0.56 \\ 0, & ,x \leq 0.15 \text{ or } x \geq 0.56 \end{cases}$$

$$\mu_L(O2_Con) = \begin{cases} \frac{x-0.15}{0.56-0.15}, & 0.15 \leq x \leq 0.56 \\ \frac{0.98-x}{0.98-0.56}, & 0.56 \leq x \leq 0.98 \\ 0, & ,x \leq 0.15 \text{ or } x \geq 0.98 \end{cases}$$

$$\mu_M(O2_Con) = \begin{cases} \frac{x-0.56}{0.98-0.56}, & 0.56 \leq x \leq 0.98 \\ \frac{1.39-x}{1.39-0.98}, & 0.98 \leq x \leq 1.39 \\ 0, & ,x \leq 0.56 \text{ or } x \geq 1.39 \end{cases}$$

$$\mu_H(O2_Con) = \begin{cases} \frac{x-0.98}{1.39-0.98}, & 0.98 \leq x \leq 1.39 \\ \frac{1.8-x}{1.8-1.39}, & 1.39 \leq x \leq 1.8 \\ 0, & ,x \leq 0.98 \end{cases}$$

$$\mu_{VH}(O2_Con) = \begin{cases} 0, & ,0 \leq x \leq 1.39 \\ \frac{x-1.39}{1.8-1.39}, & 1.39 \leq x \leq 1.8 \end{cases}$$

Each branch of the CART represents a rule which can be further utilized as predictor of the decision to be made. We can put the rules in the Fuzzy linguistic format as fuzzy rules. IF<fuzzy proposition>THEN<fuzzy proposition>. The set of rules are framed from the conventional CART which can be put in the form of IF... THEN... format. These rules form as a knowledge base for the fuzzy inference system which are shown below.

- IF [Load is (L1 OR L2)] THEN Discomfort is Ab
- IF (O2_Con < 0.96 AND Load is L3) THEN Discomfort is Ab
- IF [O2_Con < 0.96 AND Load is (L4 OR L5)] THEN Discomfort is Pr

- IF Load is (L3, L4 OR L5) AND O2_Con $\geq$ 0.96 THEN Discomfort is Pr

As discussed earlier, Mamdani Fuzzy Rule-Based Systems (FRBS) which is composed of four main components is constructed with a Knowledge Base (KB), an Inference System and the Fuzzification and Defuzzification Interfaces. The KB stores the available knowledge about the problem in the form of linguistic IF-THEN rules. The Inference System puts into effect the inference process on the system inputs making use of the information stored in the KB. The Fuzzification Interface establishes a mapping between crisp values in the input domain universe of the system outputs and fuzzy sets defined in the same universe of discourse. On the other hand, the Defuzzification Interface develops the opposite operation by defining a mapping between fuzzy sets defined in the output domain and crisp values defined in the same universe. Thus, the fuzzy CART model was fitted upon the training dataset and validated using test dataset. Flow chart representing the methodology is represented in the Fig.2.3.

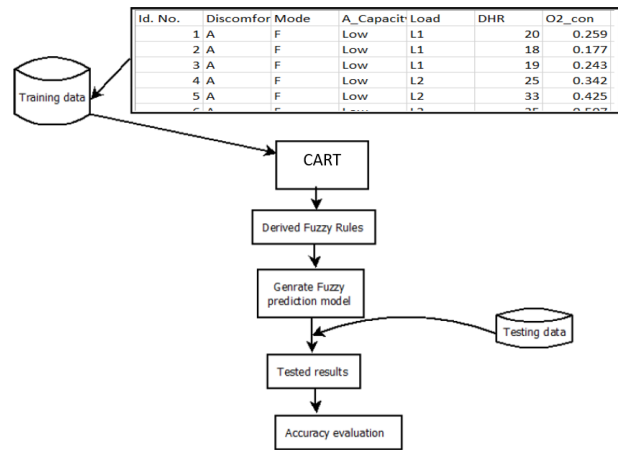


Fig. 2.3. Flow chart representing the Fuzzy CART methodology

The modified CART proposed by Sadhu *et al.* (2014) is discussed below. It has been seen that simultaneously selecting the split variable as well as the split points in the CART methodology introduces some selection bias for the variables with greater number of possible splits. To reduce this bias, a better way is to separate these two procedures by adopting different measures for the two selections. Here for variable selection, also based on the testing of significance for each variable and selecting the variable giving minimum probability value (p) for the test statistic is used. Once the variable is determined for the purpose

of splitting a node, next comes the question of value of the variable to be used as the split point. Since it is desired to divide the observations present at a node into two groups, each being more homogeneous with respect to the class of the dependent variable (i.e. each daughter node should consist of maximum number of cases belonging to the same class), the other way to look at this method is from the theory of stratified sampling perspective. It can be viewed as stratifying a heterogeneous population (the parent node) into two more homogeneous sub-populations (the daughter nodes). So the stratification methodology as proposed by Dalenius and Hodges (1959) in the field of sampling theory has been adopted to determine the split point, since this value (split value) can be used to split (divide) the observations in the parent node into two more pure (homogeneous) daughter nodes. In this way, for each node the significant variable is obtained and the split point for the node is derived using the Dalenius and Hodges stratification method. Hence, using these split variable and split point a fully grown tree classifier is obtained.

In the proposed method, all the steps as discussed for the conventional fuzzy GA for designing a fuzzy rule based classification system remains same except that selection of the split variable and split point have been done separately (as have been done in the Sadhu *et al.* (2014)'s modified CA for the conventional CA). Using the split variable and split point the CART decision tree was formed. Further the IF-THEN rules were formulated from the obtained tree. And were given for the fuzzy inference system as a knowledge base to make the predictions. The output of the inference system is defuzzified. In this way, the fuzzy CART model was obtained which was evaluated using the training dataset and validated using the test dataset. The classification accuracies of this tree is then compared with the existing CART methodology and other conventional methods of classification viz. CART, Modified CART of Sadhu *et al.* (2014), fuzzy CART and logistic regression methods for classification using the same dataset as utilized in the evaluation of the proposed methodology.

In order to remove the selection bias in case of selection of the split variable in CART methodology as mentioned earlier the following procedure is followed. For each split, the association between each explanatory variable and the dependent variable is

computed using the ANOVA F-test or Levene's test (for continuous predictors) or Pearson's contingency table χ^2 -test of independence (for nominal predictors). The explanatory variable having the highest association with the dependent variable is selected for splitting. For each node the same process is repeated with the observations present at that node until the stopping rule (i.e. until a node consists of less number of observations than a prefixed number) is triggered. With these variables under study, at the root node which consists of all the observations, each X variable is tested for their degree of association with the dependent variable Y. The categorical variables, viz., Load, Mode and A_Capacity are tested through the Pearson's chi-square statistic while for other two predictor variables, DHR and O2_Con, the ANOVA F-statistic is calculated. In case of the Pearson's contingency chi-square table, the columns of the table consist of the categories of the predictor variables and the rows constitutes of the classes of the dependent variable, i.e. of Y. Since each of the categorical predictor variable has two levels and Y also has two classes, the chi-square test will give a value of chi-square with $(2-1) \times (2-1) = 1$ degree of freedom. Once the value of the test statistics for each variable and the corresponding degrees of freedom has been found, based on these, the p-values are calculated for each variable. Selection of the split variable is then done based on the lowest p-value.

One major aim of splitting a parent node into two daughter nodes is to make each of the set of observations in the daughter nodes more homogeneous with respect to the class of the observations, i.e. the class the observations belong to. It is desired that a node should contain as much observations as possible belonging to the same class rather than a mix of different classes, in this way to make the daughter nodes more homogeneous than the parent nodes are. Here it can be looked upon as stratifying the parent node observations into two strata which are more homogeneous within themselves but more heterogeneous between them. Dalenius and Hodges (1959) proposed such a procedure of construction of stratification so that the sampling variance of the estimates for population parameters is minimized. It involves the following steps:

1. Discretize a stratification variable that is strongly correlated with the target variable into sequential intervals.

2. Compute the cumulative square root of the frequencies of the intervals.
3. Divide the range from zero to the maximum cumulative square root frequency by L , the desired number of strata, to create L intervals of equal length on the scale of the cumulative square root frequency.
4. Assign the intervals for the auxiliary variable to strata based on the closest interval found in the previous step.

Given a fixed sample size and a fixed number of strata, the Dalenius-Hodges method provides a quick means of determining strata boundaries that approximately minimizes variation. The number of strata or groups to be made for the present study is two, because for a binary split, each of the parent node (considering the entire population) will be split into only two daughter nodes (each being a stratum). As only one boundary value is required for forming two strata. At each node one boundary value has been obtained based on this method and the node is split according to the observations having values below or above the boundary point value of the split variable at that node.

As an additional method, the binary logistic regression model with dichotomous response variable and the mix of categorical and quantitative predictor variables has been considered for comparison of the CART methods considered.

3. RESULTS

In the present study, the proposed Fuzzy CART is compared with the existing CART methodology viz. CART, Modified CART of Sadhu *et al.* (2014), Fuzzy CART and logistic regression method for classification. The results of each of these methods are explained below.

3.1 Conventional CART

Load is considered as the root node based on the highest reduction in the Gini index. Further splitting was done to expand the tree and the complexity parameter (refer Table 3.1) was used to prune the tree to have an optimal sized tree. The tree obtained using this procedure is shown in the Fig.3.1.

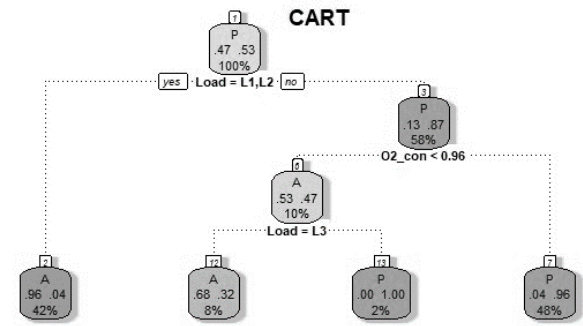


Fig. 3.1. Conventional CART

Table 3.1. Complex parameter vis-à-vis cross validation error for Conventional CART

Complexity Parameter	No. of splits	Relative error	Cross validation error
0.8039	0	1.0000	1.0000
0.0294	1	0.1960	0.1960
0.0100	3	0.1372	0.2418

This method classified 83.95% of observations correctly. The correct classification rate (CCR) is obtained by making a 2x2 confusion matrix (Table.3.2) of actual vs predicted as shown below.

Table 3.2. Confusion matrix of CART methodology

		Actual	
Predicted		Ab	Pr
	Ab	35	4
	Pr	9	33

$$\text{The CCR is} = \left(\frac{35 + 33}{81} \right) * 100 = 83.95\%$$

3.2 Modified CART of Sadhu *et al.* (2014)

The Modified CART model (Sadhu *et al.*, 2014) obtained is shown in the Fig. 3.2 where the parent nodes are represented by a rectangle and the child nodes by ellipse. Load is found to be the most significant variable hence chosen as a root node. Using Dalenius and Hodges stratification, the node was further split and for further details of tree formation, refer Sadhu *et al.* (2014).

The process continues until the stopping rule i.e. < 20 number of observation in a node or purely homogenous node was attained. In this way the tree was built. Membership function for each variable was defined with the aid of linguistic variables. Using these membership functions all the variables were converted into fuzzy value. The set of rules are framed from the above developed tree which can be put in the form of IF... THEN... format. These rules form as a knowledge base for the fuzzy inference system which are shown below.

1. IF Load is L1 & DHR is VL THEN Discomfort is Ab
2. IF Loadis L2 & DHR is VL THEN Discomfort is Ab
3. IF Loadis L1 & DHR is L THEN Discomfort is Ab
4. IF Loadis L2 & DHR is L THEN Discomfort is Ab
5. IF Loadis L1 & DHR is M &O2_Con is VL THEN Discomfort is Ab
6. IF Loadis L1 & DHR is M &O2_Conis L THEN Discomfort is Ab
7. IF Loadis L1 & DHR is H &O2_Conis VL THEN Discomfort is Ab
8. IF Loadis L1 & DHR is H &O2_Conis L THEN Discomfort is Ab
9. IF Loadis L1 & DHR is VH &O2_Conis VL THEN Discomfort is Ab
10. IF Loadis L1 & DHR is VH &O2_Conis L THEN Discomfort is Ab
11. IF Loadis L2 & DHR is M &O2_Conis VL THEN Discomfort is Ab
12. IF Loadis L2 & DHR is M &O2_Conis L THEN Discomfort is Ab
13. IF Loadis L2 & DHR is H &O2_Conis VL THEN Discomfort is Ab
14. IF Loadis L2 & DHR is H &O2_Conis L THEN Discomfort is Ab
15. IF Loadis L2 & DHR is VH &O2_Conis VL THEN Discomfort is Ab
16. IF Loadis L2 & DHR is VH &O2_Conis L THEN Discomfort is Ab
17. IF Loadis L1 & DHR is M &O2_Conis M THEN Discomfort is Ab
18. IF Loadis L1 & DHR is H &O2_Conis M THEN Discomfort is Ab
19. IF Loadis L1 & DHR is VH &O2_Conis M THEN Discomfort is Ab
20. IF Loadis L2 & DHR is M &O2_Conis M THEN Discomfort is Ab
21. IF Loadis L2 & DHR is H &O2_Conis M THEN Discomfort is Ab
22. IF Loadis L2 & DHR is VH &O2_Conis M THEN Discomfort is Ab
23. IF Loadis L1 & DHR is M &O2_Conis H THEN Discomfort is Ab
24. IF Loadis L1 & DHR is H &O2_Conis H THEN Discomfort is Ab
25. IF Loadis L1 & DHR is VH &O2_Conis H THEN Discomfort is Ab
26. IF Loadis L2 & DHR is M &O2_Conis H THEN Discomfort is Ab
27. IF Loadis L2 & DHR is H &O2_Conis H THEN Discomfort is Ab
28. IF Loadis L2 & DHR is VH &O2_Conis H THEN Discomfort is Ab
29. IF Loadis L1 & DHR is M &O2_Conis VH THEN Discomfort is Pr
30. IF Loadis L1 & DHR is H &O2_Conis VH THEN Discomfort is Pr
31. IF Loadis L1 & DHR is VH &O2_Conis VH THEN Discomfort is Pr
32. IF Loadis L2 & DHR is M &O2_Conis VH THEN Discomfort is Pr
33. IF Loadis L2 & DHR is H &O2_Conis VH THEN Discomfort is Pr
34. IF Loadis L2 & DHR is VH &O2_Conis VH THEN Discomfort is Pr
35. IF Loadis L3 & DHR is VL &O2_Conis VL THEN Discomfort is Ab
36. IF Loadis L3 & DHR is VL &O2_Conis L THEN Discomfort is Ab

37. IF Loadis L3 & DHR is VL & O2_Conis M THEN Discomfort is Ab
38. IF Loadis L3 & DHR is L & O2_Conis VL THEN Discomfort is Ab
39. IF Loadis L3 & DHR is L & O2_Conis L THEN Discomfort is Ab
40. IF Loadis L3 & DHR is L & O2_Conis M THEN Discomfort is Ab
41. IF Loadis L3 & DHR is M & O2_Conis VL THEN Discomfort is Ab
42. IF Loadis L3 & DHR is M & O2_Conis L THEN Discomfort is Ab
43. IF Loadis L3 & DHR is M & O2_Conis M THEN Discomfort is Ab
44. IF Loadis L3 & DHR is VL & O2_Conis H THEN Discomfort is Pr
45. IF Loadis L3 & DHR is VL & O2_Conis VH THEN Discomfort is Pr
46. IF Loadis L3 & DHR is L & O2_Conis H THEN Discomfort is Pr
47. IF Loadis L3 & DHR is L & O2_Conis VH THEN Discomfort is Pr
48. IF Loadis L3 & DHR is M & O2_Conis H THEN Discomfort is Pr
49. IF Loadis L3 & DHR is M & O2_Conis VH THEN Discomfort is Pr
50. IF Loadis L3 & DHR is H THEN Discomfort is Pr
51. IF Loadis L3 & DHR is VH THEN Discomfort is Pr
52. IF Loadis L4 THEN Discomfort is Pr
53. IF Loadis L5 THEN Discomfort is Pr

The inputs are given to the fuzzy inference system using the Fuzzy logic toolbox of MATLAB software. The fuzzy inference system makes the decisions based on the provided knowledge base to give the outputs. The outputs are then defuzzified using the centroid method. The defuzzified value is crisp indicating the degree of discomfortness. For comparison sake the values < 0.5 are taken as Absence of discomfort and the values > 0.5 are considered as Presence of discomfort.

It can be observed that the proposed fuzzy CART is performing better than the other CART methods employed. It classifies 86.42% of observations correctly. There is a considerable increase in the number of correct classifications and decrease in number of misclassification. The confusion matrix was tabulated on the same lines as given in Section 3.1 above and the CCR was obtained as 86.42%.

3.5 Logistic Regression

If there are s levels of a categorical variable, then $(s-1)$ number of indicator variables will be included in the model. Accordingly, for the categorical variable Load with five levels four indicator variables are included in the model i.e. L1 is taken as a base for the reference and Load_L2, Load_L3, Load_L4 and Load_L5 are used as indicator variables. Similarly, Mode has two levels hence one indicator variable Mode_O included with the level F as reference. In addition, A_Capacity Low is the indicator variable for A_Capacity with the level High as reference. Table 3.5 shows the coefficients, their standard errors, z-statistic and the associated p values obtained. DHR, Oxygen consumption, Load L3 are statistically significant at 5% level. The estimated model has been utilized for classification of the test dataset.

Table 3.5. Logistic regression model estimates

	Estimate	Std. Error	z value	Pr(> z)	
(Intercept)	-11.46	2.41	-4.75	0.00	***
LoadL2	1.12	1.18	0.94	0.34	
LoadL3	2.66	1.17	2.28	0.02	*
LoadL4	39.62	4138.61	0.01	0.99	
LoadL5	40.68	4097.84	0.01	0.99	
ModeO	-20.16	2567.59	-0.01	0.99	
A_CapacityLow	1.67	1.13	1.49	0.14	
DHR	0.07	0.03	2.40	0.02	*
O2_Con	5.85	1.69	3.46	0.00	***

The confusion matrix was tabulated on the same lines as given in Section 3.1 above and the CCR was obtained as 87.65%.

To sum up, the comparison of the methods used is given in Table 3.6.

Table 3.6. Correct classification rate for both training and test datasets for the various methods

Method	% CCR for training dataset	% CCR for test dataset
CART	93.52	83.95
Modified CART	91.67	83.95
Fuzzy CART	87.65	80.25
Proposed Fuzzy CART	88.89	86.42
Logistic Regression	93.21	87.65

4. CONCLUSIONS

For the test dataset, the proposed fuzzy CART is better than other CART approaches. Logistic Regression (LR) seems to outperform the proposed fuzzy CART model. However, it is stated here that Logistic Regression is model driven but in CART no functional form is imposed and hence Fuzzy CART can be implemented for wide range of datasets as it is not based on any stringent assumptions. CART decision tree models are better preferred than LR models as the former are easier to interpret by way of comprehensible classification rules. Moreover, for LR, a decision threshold has to be set to classify the given new case as, say, belonging to the second group and not to the first group and this brings in arbitrariness (in this study, a 0.5 cut-off for the LR output was fixed). Also CART is non-parametric while LR is parametric. LR is model-driven and requires (curvy) linear separable criterion but CART is data-driven and does not hold any such requirement and allows the data to be divided in a non-linear realistic manner. In addition, as Izenman (2008) pointed out, CART can handle outliers, missing and highly skewed data effectively in comparison to LR. Thus fuzzy classification based decision tree can be employed as a viable alternative for modelling and prediction. Further, the fuzzy theory can be applied with various decision tree algorithms like ID3, C4.5 etc. to evaluate their efficacy for classification in agriculture.

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