

Applying Grey System Theory for MSW Forecasting in Data-Limited Urban Environments: A Case Study of Shillong, Meghalaya

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SUMMARY

Rapid urbanization, population growth, and shifting consumption patterns have made municipal solid waste (MSW) management a pressing environmental issue. Accurate projections of waste generation are critical for planning collection, transportation, treatment, and disposal systems. However, conventional forecasting methods typically require large datasets, which are often unavailable in developing cities.

This study applies the Grey System Theory GM(1,1) model to forecast MSW generation in Shillong, Meghalaya, using annual data from the Shillong Municipal Board for 2021–2025. GM(1,1) was chosen for its capacity to produce reliable forecasts with limited data. Parameters were estimated using the least squares method, and model performance was assessed using MAE, MAPE, and RMSE.

The model demonstrated strong accuracy with a MAPE of 1.28%, MAE of 1123.76 MT, and RMSE of 1096.66 MT. Forecasts indicate that annual MSW generation in Shillong will rise from about 75,998 MT in 2026 to 98,227 MT in 2035, an increase from 208 TPD to 269 TPD. This upward trend underscores the need for upgraded waste management infrastructure and strategic long-term planning.

The findings confirm that GM(1,1) is an effective forecasting tool for data-scarce urban settings and offers practical insights for sustainable waste management in Shillong.

Keywords: Municipal Solid Waste; Grey System Theory; GM(1,1); Forecasting; Shillong; Waste Management.

1. INTRODUCTION

Waste generation is a direct and visible consequence of modern urbanization. As cities expand and consumption patterns shift, municipal solid waste (MSW) — from households, markets, institutions, and businesses — has emerged as a key environmental challenge of the 21st century. Addressing this challenge requires more than infrastructure and funding; it also demands forward-looking planning. Estimating future waste volumes is essential for effective preparation. In India, this challenge is acute. Per capita MSW generation is projected to increase by about 1.33% annually, adding pressure on urban local bodies (Sharholy *et al.*, 2008). Small and medium-sized cities are particularly vulnerable due to limited institutional

capacity, financial constraints, and restricted access to modern waste technologies. Shillong, the capital of Meghalaya, exemplifies these issues. The Greater Shillong Planning Area generates an estimated 170–180 metric tonnes of waste daily, while urban areas across Meghalaya produce approximately 315 tonnes per day (CAG Report, 2020). The Marten landfill is nearing capacity, and waste management responsibilities are divided across multiple agencies (Mipun *et al.*, 2015). Accurate forecasts are therefore critical for planning collection systems, treatment facilities, landfill space, and budgets. However, conventional approaches such as regression, time series analysis, and machine learning require long, continuous historical datasets. In smaller Indian cities like Shillong, such data are often

absent because waste records are recent, incomplete, or inconsistently maintained, which restricts the use of these methods. Kolekar *et al.* (2016) emphasized that model choice should match data characteristics and availability, and noted that grey forecasting models perform well when data are limited and uncertain. Grey System Theory, developed by Deng (1982), addresses systems with incomplete information. It categorizes systems as white for fully known, black for fully unknown, and grey for partially known, and provides tools for modeling under uncertainty. The GM(1,1) model is a first-order, single-variable grey model that uses the Accumulated Generating Operation (AGO) to reduce noise and reveal underlying trends. It can produce reliable forecasts with as few as four data points (Deng, 1989). A key limitation is that GM(1,1) is univariate and assumes a stable exponential trend, so it does not explicitly incorporate external drivers such as population growth or economic change. This should be considered when interpreting long-term projections. Grey models have been applied extensively in energy, environmental, and engineering studies (Ma & Liu, 2017; Hsu & Wang, 2007). In MSW forecasting, they have also proven valuable. Intharathirat *et al.* (2015) demonstrated their effectiveness with sparse data, and Islam *et al.* (2022) showed that grey models can outperform traditional methods under certain data conditions. While ARIMA and machine learning models require large datasets, GM(1,1)'s minimal data requirement makes it well suited to data-scarce settings. Khan and Osinska (2023) similarly found grey models to be competitive with time series methods for small samples. Although grey forecasting has been used widely in Asia, its application in Northeast India remains limited. This study addresses that gap by exploring grey models in this region and contributing to the literature on grey forecasting in an understudied context.

This study therefore applies the GM(1,1) model to project MSW generation in Shillong using annual data from 2021 to 2025. It assesses model accuracy through MAE, MAPE, and RMSE, aiming to support better waste management planning and promote sustainable urban development.

2. MATERIALS AND METHODS

2.1 Study Area

Shillong, the capital and largest urban centre of Meghalaya, sits in the north-eastern highlands of India at 25.57°N, 91.88°E, roughly 1,525 metres above sea level. Solid waste management in the city is handled by the Shillong Municipal Board (SMB), which manages collection, transport, and disposal across 21 municipal wards. The wider Greater Shillong Planning Area (GSPA) stretches beyond these core boundaries. While the 2011 Census recorded a municipal population of around 143,000, recent estimates that include migrants, tourists, and urban growth suggest the actual daily population is much larger. Shillong's economy centres on government administration, commerce, and tourism, and its substantial floating population of students and visitors drives seasonal spikes in waste generation. The city's steep topography and crowded informal settlements complicate waste collection, since narrow, sloping roads restrict vehicle access in many areas. At present, all municipal solid waste goes to the Marten landfill, which is operating past its intended capacity and nearing saturation. This makes accurate long-term forecasting of waste generation an urgent planning priority.

2.2 Data Source

Annual data on municipal solid waste (MSW) generation in Shillong were collected from the Shillong Municipal Board for 2021–2025, giving five yearly records. The figures, expressed in metric tons (MT) per year, indicate the total amount of MSW produced annually within the study area. These annual MSW values served as the input series for the GM(1,1) model to examine past trends and project future waste generation. Annual data are particularly appropriate for the GM(1,1) framework, which requires a minimum of four observations and has been demonstrated to produce reliable results even with small sample sizes (Deng, 1989). Although a larger dataset would improve model robustness and validation, the five available observations satisfy the theoretical requirements of the GM(1,1) approach and are sufficient to capture the underlying growth pattern in Shillong's waste generation. In this study the waste generation data are used as inputs to the GM(1,1) model, the collection and efficiency indicators are retained in the dataset

to provide a more comprehensive understanding of Shillong's waste management system. Their inclusion helps to highlight the widening gap between waste generation and collection, offering important contextual insight into the city's growing operational shortfall.

2.3 Yearly Data Table

Table 2.1 Yearly MSW Generation (Gen) and Collection (Col) in Metric Tons, Shillong, 2021–2025. Source: Shillong Municipal Board

Year	Generated	Gen(Tpd)	Collected	Col(Tpd)	Uncollected	Efficiency
2021	64240	176	56210	154	8030	87.50%
2022	66430	182	61320	168	5110	92.31%
2023	71540	196	60590	166	10950	84.69%
2024	72270	198	61427	168.2932	10843	85.00%
2025	73000	200	62050	170	10950	85.00%

The dataset was initially organised in Microsoft Excel before being imported into R for statistical analysis. All analyses were performed using R version 4.6.0 (R Core Team, 2026). The GM(1,1) model was implemented entirely using base R, without the need for additional packages, as all key steps—including the Accumulated Generating Operation (AGO), parameter estimation via ordinary least squares (OLS), and the inverse accumulated generating operation (IAGO)—were executed through built-in matrix operations. Data quality checks confirmed that the five annual observations were complete and free from missing values, duplicate entries, and inconsistencies. Since all values were consistently reported in metric tons and already arranged in chronological order, no unit conversion, reordering, imputation, or further preprocessing was required prior to model estimation.

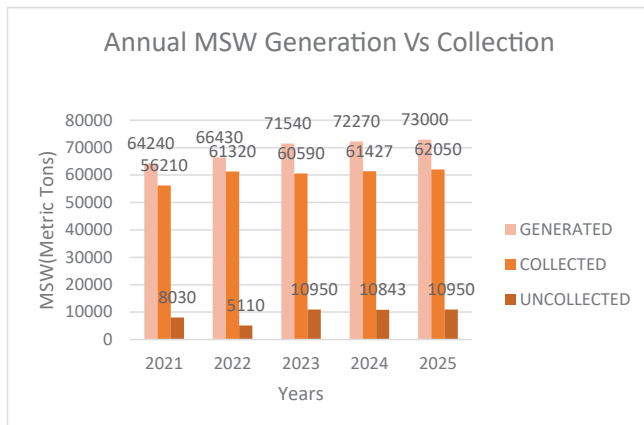


Fig. 2.1 Annual MSW Generation Vs Collection

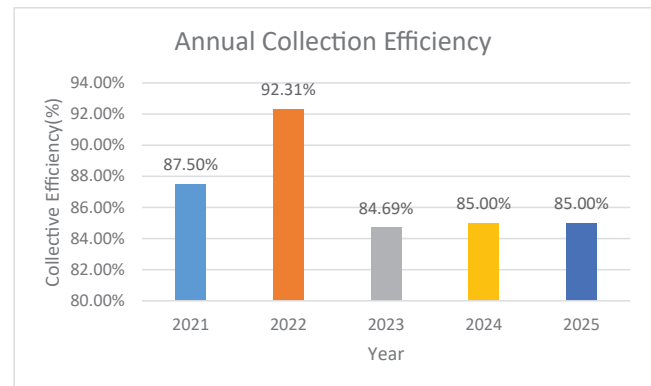


Fig. 2.2 Collection Efficiency (%). Note the 2022 efficiency peak (92.31%) and subsequent decline, and persistently large uncollected waste volumes from 2023 onwards

2.4 Forecasting model

Grey System Theory

Grey System Theory, introduced by Deng in 1982, addresses the modeling of systems with partial information and uncertainty. Its strength lies in handling small datasets that violate the assumptions of conventional statistical models.

The GM(1,1) model, a first-order single-variable grey model, is the most widely adopted technique in grey forecasting. It is particularly appropriate for municipal solid waste forecasting, where data are often sparse and irregular.

This study adopts GM(1,1) because of its capability to generate stable forecasts with limited data. Instead of requiring extensive historical records, the model employs the Accumulated Generating Operation (AGO) to reduce noise and reveal the dominant trend. Furthermore, GM(1,1) is computationally simple, data-efficient, and has seen broad application across environmental, economic, and engineering domains.

The key strengths of the GM(1,1) model include:

- Works effectively with small datasets
- Needs only a few data points to operate
- Minimizes randomness using data accumulation
- Simple and computationally efficient
- Delivers dependable short-term predictions

Construction of GM(1,1) Model

Before implementing the GM(1,1) model, it is essential to ensure that the raw data satisfy its underlying applicability conditions. The first requirement is that all

observed values must be non-negative, a condition that is clearly met in this study, as municipal solid waste generation ranges from 64,240 MT to 73,000 MT. The second requirement is that the AGO sequence should exhibit a quasi-exponential growth pattern, which can be assessed using the smooth ratio test. The computed smooth ratios, defined as

$$\frac{x^{(1)}(k)}{x^{(1)}(k-1)} \text{ for}$$

$k = 2, 3, 4, 5$, were 2.034, 1.547, 1.357, and 1.266, respectively. The steadily declining pattern of these ratios indicates a stable exponential tendency in the accumulated series, thereby confirming that the dataset satisfies the fundamental conditions required for GM(1,1) modelling.

i. Original Data Sequence

Let the original non-negative sequence be represented as:

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}$$

where:

$x^{(0)}(k)$ represents the original MSW generation value at time k

n represents the total number of observations

ii. Accumulated Generation Operation (AGO)

To reduce randomness and smooth the data, AGO is applied to the original sequence.

The accumulated sequence is defined as:

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\}$$

where:

$$x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i), \quad k = 1, 2, \dots, n$$

AGO helps transform irregular data into a smoother sequence that approximately follows an exponential trend.

iii. Background Value Generation

The background value sequence is calculated using consecutive AGO values.

$$Z^{(1)} = \{z^{(1)}(2), z^{(1)}(3), \dots, z^{(1)}(n)\}$$

where:

$$z^{(1)}(k) = \frac{1}{2} [x^{(1)}(k) + x^{(1)}(k-1)], \quad k = 2, 3, \dots, n$$

The background value represents the average behaviour of the accumulated sequence between two consecutive periods.

iv. Grey Differential Equation

The GM(1,1) model is represented by the following first-order differential equation:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b$$

where:

a = development coefficient

b = grey input coefficient

v. Parameter Estimation Using Least Squares Method

The parameters a and b are estimated using the Least Squares Method.

The matrices are constructed as:

$$Y = [x^{(0)}(2) x^{(0)}(3) \dots x^{(0)}(n)]$$

And

$$B = [-z^{(1)}(2) 1 -z^{(1)}(3) 1 \dots -z^{(1)}(n) 1]$$

The parameter vector is:

$$\hat{\theta} = \begin{bmatrix} a \\ b \end{bmatrix}$$

The least squares estimate is:

$$\hat{\theta} = (B'B)^{-1} B'Y$$

vi. Time Response Function

The solution of the grey differential equation gives the time response function:

$$\hat{x}(k+1) = \left(x^{(0)}(1) - \frac{b}{a} \right) e^{-ak} + \frac{b}{a}$$

where:

$\hat{x}(k+1)$ is the predicted AGO value

$$k = 0, 1, 2, 3, \dots, n$$

The time response function generates the predicted accumulated sequence.

vii. Inverse Accumulated Generation Operation (IAGO)

To obtain predicted values in the original form, IAGO is applied.

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1), \quad k \geq 2$$

This step converts the accumulated predicted sequence into the final forecasted MSW generation values.

Model Accuracy Evaluation

Forecast accuracy was assessed using Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and Root Mean Square Error (RMSE). These metrics were employed to capture different dimensions of predictive performance. MAE quantifies the average magnitude of errors, RMSE is more sensitive to large deviations, and MAPE provides a percentage-based measure that supports straightforward interpretation.

1. Mean Absolute Error (MAE)

$$MAE = \frac{1}{n} \sum_{k=1}^n |x^{(0)}(k) - \hat{x}^{(0)}(k)|$$

MAE indicates the average absolute deviation between actual and predicted values.

2. Mean Absolute Percentage Error (MAPE)

$$MAPE = \frac{1}{n} \sum_{k=1}^n \left| \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \right| \times 100$$

Interpretation:

MAPE Value	Forecast Accuracy
< 10%	Excellent
10%–20%	Good
20%–50%	Reasonable
> 50%	Poor

3. Root Mean Square Error (RMSE)

$$RMSE = \sqrt{\frac{1}{n} \sum_{k=1}^n (x^{(0)}(k) - \hat{x}^{(0)}(k))^2}$$

RMSE measures the magnitude of forecasting error.

The lower values of MAE, MAPE, and RMSE indicate that the GM(1,1) model provides good forecasting accuracy and that the predicted values are close to the actual observed values.

Forecasting Horizon

This study adopts a ten-year forecasting horizon from 2026 to 2035, consistent with standard municipal infrastructure planning cycles in India. The Solid Waste Management Rules (2016) and related urban development policies typically follow a 10–15 year planning framework. Projections to 2035 are therefore directly relevant for decisions on landfill expansion, treatment facility development, and upgrading waste collection capacity in Shillong.

Such long-term forecasts are essential for strategic planning and infrastructure investment in municipal solid waste management. However, uncertainty in forecasts increases with time. Consequently, the projected values should be interpreted as indicative trends of future waste generation rather than exact predictions.

3. RESULTS AND DISCUSSION

3.1 Input sequence and AGO computation in Excel

Table 3.1 presents the full GM(1,1) computational sequence including AGO values, background values (Z), OLS-fitted values, and Absolute Percentage Errors.

3.2 Model Parameters

The parameters obtained from the GM(1,1) model for waste generation were:

$$a = -0.02850815$$

$$b = 65014.1505$$

The negative development coefficient $a = -0.02850815$ confirms that Shillong's MSW generation exhibits an exponentially increasing trajectory. Parameter b represents the grey input coefficient of the model and reflects the overall level of the data series. The values obtained suggest that waste generation showed a stable upward movement during the study period.

Table 3.1 GM(1,1) computation table. AGO = cumulative sum of $x(0)$. $z(1)(k)$ = mean of consecutive $x(1)$ values

K	YEAR	$x(0)$	$x(1)$	$z(1)$	Pred $x(1)$	FITTED	MAE	MAPE
1	2021	64240	64240		64240		1377.454	2.073543
2	2022	66430	130670	97455	132047.4543	67807.45433	1771.663	2.476465
3	2023	71540	202210	166440	201815.7915	69768.33721	484.0743	0.669814
4	2024	72270	274480	238345	273601.7172	71785.92568	861.8596	1.18063
5	2025	73000	347480	310980	347463.5768	73861.85958	1123.763	1.28009

3.3 Model Fit: Actual vs Predicted Values

Figure 3.1 compares actual annual MSW generation with the GM(1,1) fitted values across the calibration period (2021-2025).

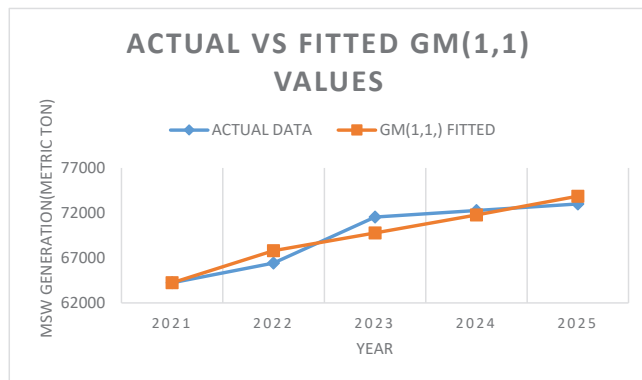


Fig. 3.1 Time Series Comparison of Actual Vs Predicted Values of MSW Generation(2021-2025)

The graphical comparison between the actual and predicted values shows that both series generally increased over time. Although the actual values fluctuated at certain periods, the predicted values followed a smoother pattern. This occurs because the GM(1,1) model focuses mainly on the long-term trend of the data and reduces random fluctuations through the Accumulated Generation Operation (AGO).

3.4 Model Accuracy

To evaluate the forecasting performance of the model, error measures such as Mean Absolute Percentage Error (MAPE), Mean Absolute Error (MAE), and Root Mean Square Error (RMSE) were calculated.

RMSE = 1096.659MT	MAE = 1123.763MT	MAPE = 1.28009%
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The MAPE of 1.28009% indicates very high forecasting accuracy for waste generation, as the error percentage is well below the 10% threshold typically regarded as excellent. The MAE of 1,123.763 MT shows that

the average deviation between actual and predicted values was approximately 1,124 MT per year, which is small relative to the overall data range of ~64,000 to ~73,000 MT. The RMSE of 1,096.659 further confirms that forecast errors remain within an acceptable range.

It is important to note, however, that these metrics reflect in-sample performance rather than independent validation, since the same five observations used to estimate the model were also used to calculate the errors. Given the limited dataset ($n = 5$), a hold-out validation was not feasible. Therefore, these statistics should be interpreted as measures of model fit rather than as proof of out-of-sample predictive accuracy.

For benchmarking, a linear regression model was also fitted to the same five observations. Both models performed well, but the regression model produced a MAPE of 1.41%, slightly higher than the 1.28% from GM(1,1). This suggests that the grey model provided a better fit. Moreover, while linear regression assumes constant growth, GM(1,1) models exponential growth, which aligns more closely with waste generation patterns in developing and urbanizing cities. This difference becomes more important when making longer-term projections. The findings are consistent with existing literature. Intharathirat *et al.* (2015) applied GM(1,1) to waste forecasting in Thailand using similarly sized datasets and reported MAPE values between 1.5% and 3.2%. The 1.28% obtained here falls at the more accurate end of that range. Likewise, Islam *et al.* (2022) found that grey forecasting methods perform competitively with traditional models, especially under data constraints, which supports the results of this study. Additionally, the estimated annual growth rate of approximately 2.8% for Shillong lies within the 2%–4% range commonly reported for medium-sized, rapidly urbanizing cities in South and Southeast Asia (Khan & Osinska, 2023). While Shillong has its own socio-economic and environmental context, its waste

generation trend broadly mirrors regional patterns. This suggests that the challenges identified here are not unique to Shillong but reflect wider urban waste management pressures in developing cities across the region.

3.5 Analysis of Collection Efficiency Trends

While the GM(1,1) model is intended to forecast future waste generation, it is equally important to examine collection capacity trends in relation to these projections. Doing so provides insight into system adequacy and the growing divergence between waste generation and management capacity.

Analysis of historical data reveals that collection efficiency peaked in 2022 at 92.31%, an increase from 87.50% in 2021 and the highest level observed during the study period. This improvement was not sustained. Efficiency declined to 84.69% in 2023 and remained stagnant at 85.00% in both 2024 and 2025. The 2022 increase may indicate improved collection operations, but the available data do not identify a definitive cause, and further investigation is warranted. The subsequent decline and plateau from 2023 onward suggest a structural limitation: while total waste generation rose sharply from 66,430 MT to 73,000 MT, collection capacity did not keep pace. Consequently, approximately 10,900 MT of waste per year remained uncollected on average after 2022. This persistent gap underscores the need for long-term infrastructure investment instead of short-term operational adjustments.

3.6 Forecasting Result: MSW Generation (2026-2035)

Table 3.2 GM(1,1) forecasted MSW generation, Shillong 2026–2035. Gen = Generation

k	YEAR	PRED _x (1)	GEN(MT)	GEN(TPD)
6	2026	423461.4	75997.8261	208.2132
7	2027	501657	78195.5615	214.2344
8	2028	582113.8	80456.8517	220.4297
9	2029	664897.4	82783.5349	226.8042
10	2030	750074.9	85177.502	233.363
11	2031	837715.6	87640.6988	240.1115
12	2032	927890.7	90175.1274	247.0551
13	2033	1020674	92782.8475	254.1996
14	2034	1116140	95465.9787	261.5506
15	2035	1214366	98226.7018	269.1143

Table 3.2 presents the GM(1,1) model forecasts for annual MSW generation in Shillong from 2026 to 2035, including daily averages (TPD) and year-on-year growth.

To account for uncertainty inherent in long-term forecasting, $\pm 10\%$ bounds were applied to the GM (1,1) projections to generate low- and high-growth scenarios. The central estimate indicates that Shillong's municipal solid waste generation will reach 98,227 MT by 2035, with a plausible range of 88,404–108,049 MT reflecting potential variations due to urban expansion, policy interventions, and changing consumption patterns. Notably, even the lower-bound estimate exceeds the city's current collection capacity, highlighting the necessity for proactive and sustained planning across all scenarios.

Table 3.3 GM(1,1) Forecast with Low and High Scenario Bounds ($\pm 10\%$), Shillong, 2026–2035

Year	Low Scenario (MT)	GM(1,1) Forecast (MT)	High Scenario (MT)
2026	68,398	75,998	83,598
2027	70,376	78,196	86,015
2028	72,411	80,457	88,503
2029	74,505	82,784	91,062
2030	76,660	85,178	93,695
2031	78,877	87,640	96,405
2032	81,158	90,175	99,193
2033	83,505	92,783	102,061
2034	85,919	95,466	105,013
2035	88,404	98,227	108,049

The GM(1,1) forecast graph presents projected municipal solid waste levels based on historical records. The results indicate a consistent upward trend, implying that waste generation in Shillong is likely to continue increasing in the coming years. Although the observed data exhibit fluctuations and irregularities, the forecast curve remains relatively smooth, demonstrating the model's capability to capture the underlying long-term pattern. The close correspondence between actual and predicted values during the historical period suggests that the model yields reasonably accurate forecasts. Overall, the findings indicate a sustained rise in municipal solid waste generation and provide useful insights for future waste management planning and policy formulation.

3.7 Forecasted Visualisation

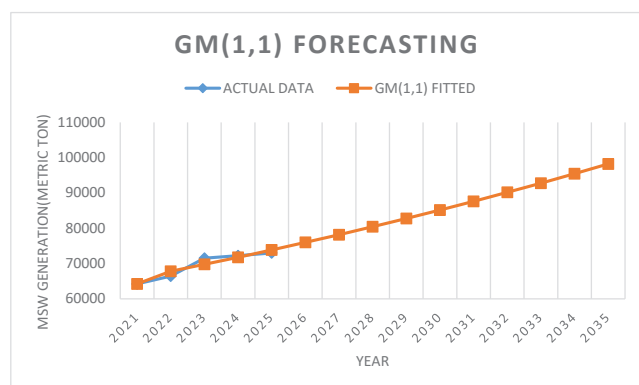


Fig. 4.2 GM(1,1) Future Forecast of Municipal Solid Waste Generation

4. CONCLUSION

This study proposes a reliable forecasting model for municipal solid waste (MSW) generation in Shillong based on the GM(1,1) model from Grey System Theory. Using the available dataset, the model achieved strong in-sample performance with a MAPE of 1.28009%, an MAE of 1,123.763 MT, and an RMSE of 1,096.659 MT, all of which indicate excellent predictive accuracy. The estimated development coefficient of $a = -0.02850815$ reflects an exponential growth trend in MSW generation, aligning with wider patterns of urbanization and rising consumption across Indian cities. Projections indicate that this growth will persist in the near future, further straining an already burdened waste management system. The Marten disposal facility is nearing capacity, waste collection remains inconsistent, and source segregation is still limited. In the absence of targeted investments in infrastructure, treatment facilities, and collection networks, these pressures are expected to worsen.

The findings position GM(1,1) as a practical instrument for urban waste planning, especially in contexts with limited data. The results can support policymakers and municipal authorities in making evidence-based decisions on collection, transportation, treatment, and landfill management. Methodologically, the study confirms that GM(1,1) performs well with sparse, irregular, or short time series, a common scenario in Shillong and other small- to medium-sized Indian cities. Its capacity to produce accurate forecasts from just five annual observations makes it particularly suitable for municipalities with constrained data and technical capacity.

To the best of our knowledge, this represents the first application of Grey System Theory to MSW forecasting in Northeast India, filling a gap in the regional literature. Future work could extend this research by employing multivariate grey models such as GM(1,N) to include drivers such as population growth, per capita income, and urbanization. Comparisons with ARIMA and machine learning models on the same dataset would also be valuable. Longer time series in the future would enable more robust validation and extended forecast horizons.

The study does have limitations. It relies on five annual data points, which may not capture short-term fluctuations in waste generation. Moreover, GM(1,1) is designed mainly for trend extrapolation and does not directly incorporate variables such as population dynamics, economic shifts, or policy interventions.

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APPENDIX

Raw Data Source

The Yearly MSW generation and collection data used in this study were obtained from the Shillong Municipal Board. Data covers January 2021 through December 2025. The complete dataset is reproduced in Table 2.1

GM(1,1) implementation in R

The GM(1,1) model was implemented in R. The annual generation totals (x0) were used as the input sequence. The complete script is:

```
# GM(1,1) MSW Forecasting: Shillong 2021-2025
x0<- c(64240, 66430, 71540, 72270, 73000)
n <- length(x0)
# Step 1: AGO - Accumulated Generating Operation
x1<- cumsum(x0)
# Step 2: Background value sequence Z
z1 <- 0.5 * (x1[-1] + x1[-n])
# Step 3: OLS parameter estimation
B <- cbind(-z1, 1)
Y <- x0[-1]
ab<- solve(t(B) %*% B) %*% t(B) %*% Y
a <- ab[1]
b <- ab[2]
cat('a:', round(a, 6), 'b:', round(b, 4), '\n')
# Step 4: Fitted values via IAGO
x1_hat <- (x0[1] - b/a) * exp(-a * (0:(n-1))) + b/a
x0_hat <- c(x1_hat[1], diff(x1_hat))
# Step 5: Accuracy metrics
mape<- mean(abs((x0 - x0_hat)/x0)) * 100
rmse<- sqrt(mean((x0 - x0_hat)^2))
mae<- mean(abs(x0 - x0_hat))
```

```
cat('MAPE:', round(mape, 4), '% | RMSE:',
round(rmse, 4), ' | MAE:', round(mae, 4),
'\n')
```

```
# Step 6: Forecast 2026–2035
```

```
k_vals<- n:(n+9)
```

```
x1_fore <- (x0[1] - b/a) * exp(-a * k_
vals) + b/a
```

```
x0_fore <- diff(c(x1_hat[n], x1_fore))
```

```
print(data.frame(Year = 2026:2035,
```

```
Forecast_MT = round(x0_fore, 2),
```

```
Forecast_TPD = round(x0_fore/365, 2)))
```

Linear Regression Benchmark

A simple linear regression was fitted to the same five observations as a benchmark comparison. The script is:

```
# Linear Regression Benchmark
```

```
actual<- c(64240, 66430, 71540, 72270,
73000)
```

```
t <- 1:5
```

```
fit<- lm(actual ~ t)
```

```
fitted_vals<- predict(fit)
```

```
mape_lm<- mean(abs((actual - fitted_vals)
/ actual)) * 100
```

```
cat("Linear Regression MAPE:", round(mape_
lm, 4), "%\n")
```

Scenario Bounds ($\pm 10\%$)

Low and high forecast scenarios were constructed by applying $\pm 10\%$ bounds to the GM(1,1) projections. The script is:

```
# Scenario Bounds
```

```
forecasts<- c(75997.83, 78195.56,
80456.85, 82783.53,
```

```
85177.50, 87640.70, 90175.13, 92782.85,
```

```
95465.98, 98226.70)
```

```
years<- 2026:2035
```

```
low<- round(forecasts * 0.90)
```

```
high<- round(forecasts * 1.10)
```

```
print(data.frame(Year = years,
```

```
Low_Scenario = low,
```

```
GM11_Forecast = round(forecasts),
```

```
High_Scenario = high))
```