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A Pioneering Approach to Population Mean Estimation Leveraging Ranked Set Successive Sampling

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SUMMARY

Ranked set sampling has been observed to be an efficient sampling method that has gained popularity in recent years. In the present work, the concept of ranked set sampling has been extended to successive sampling, which results in a novel sampling scheme called Ranked Set Successive Sampling (RSSS). The theoretical foundation of RSSS has been discussed and estimation methodologies has been modified to be applicable in RSSS environment for the estimation of population mean. RSSS has been considered for two successive occasions, and the estimators are proposed for estimating the population mean at current (second) occasion in two-occasion RSSS. The performance of the proposed estimators are examined through simulation-based evaluation including the real data applications as well as simulated populations. The simulation results shows that the concept of RSSS is practically feasible. The comparative analysis reveals that the proposed estimators under RSSS exhibit improved efficiency over the available similar successive sampling estimators, which may be applicable in various real-life situations.

Keywords: Ranked Set Successive Sampling; Auxiliary variable; Mean squared error; Relative efficiency.

1. INTRODUCTION

One of the fundamental problems in statistical inference is the estimation of population mean. The precision of this estimate has a significant impact on decision-making processes. To achieve better precision ranked set sampling (RSS) is used that involves ranking the units within each sample, rather than selecting units randomly. The ranking is termed as perfect, when it is based on the actual values of the study variable. However, it is called imperfect, when it is based on auxiliary variable or imperfect measurements. McIntyre (1952) was the first to introduce the concept of rank set sampling for estimating mean pasture and forage yield. Its theoretical justification was pioneered by Takahasi and Wakimoto (1968), and Takahasi (1970). Dell and Clutter (1972) further proved that RSS yields at least same precision as provided by the simple random sampling with replacement. Stokes (1980) explored RSS for estimating variance using judgment ordered ranked set samples. Yu and Lam (1997) discussed the regression estimator in RSS. Further the theory was explored by many authors such as Kadilar *et al.* (2009), Mandowara and Mehta (2013), Arnab and Olaomi (2015), Vishwakarma and Singh (2022), Khan *et al.* (2024), Kumar and Sinha (2026) and many others. It is well known that for studies where the variable under study varies over time, successive sampling offers a reliable method for achieving precise estimates. For example, in agricultural field, farmers use successive sampling to estimate crop yield, monitoring changes in crop growth and development over the growing season. Jessen (1942) pioneered the concept of successive sampling, which was further extended by many authors including Patterson (1950), Singh and Priyanka (2010), Priyanka *et al.* (2018), Priyanka *et al.* (2023), Singh *et al.* (2025) and many others.

It is observed that all these authors used simple random sampling and proposed estimators for estimating different parameters in successive sampling. As it is pointed out that ranked set sampling is better

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than simple random sampling. Motivated with this, the aim of the present work is to use ranked set sampling together with the successive sampling and propose ranked set successive sampling (RSSS). Many areas of real-life concerns can be addressed effectively by RSSS. For example, RSSS can be used in clinical trials to estimate the efficiency of new treatments over time. In this case ranking can be done based on patients' response to the treatment for selecting units in the sample. Similarly in epidemiological studies, RSSS can be applied to estimate disease prevalence, incidence, or transmission rates over time. In this case ranking can be done based on identifying high risk areas. Likewise, there may be many other fields where RSSS can be effectively utilized in real-life applications, providing valuable insights and improving decision-making processes. In the present work the estimators are proposed for estimating the population mean at second (current) occasion in two occasions RSSS. Section 2 describes the survey design and notations. Section 3 focuses on formulation of RSSS estimators. The properties of the proposed estimators have been studied in Section 4. However, in Section 5 numerical illustration including simulation is presented. Section 6 focuses on discussion of results and finally the study is concluded in section 7.

2. SURVEY DESIGN AND NOTATIONS

A finite population $U=\{U_1,U_2,\ldots,U_N\}$ of N identifiable units is considered for sampling over two successive occasions. The size of population is kept same while the value of units does change over occasions. Let the study character be denoted by x at the first occasion and y at the second occasion. It is further assumed that the information on an auxiliary variable z whose population mean is known and stable over occasions, is readily accessible on both the occasions.

For applying the ranked set sampling setup in two occasions successive sampling lay out, let us consider small numbers w_1, w_2, and w_3 as set sizes for selecting the sample s_n at first occasion, s_m the matched sample, and s_u the fresh sample at second occasion respectively such that it is possible to rank w_1, w_2, and w_3 units of the population with sufficient accuracy.

2.1 Selection of Samples

At first occasion, w_1^2 units are selected at random from the considered population U by simple random sampling with replacement (SRSWR) method. These w_1^2 units are then allocated into w_1 sets each of size w_1. The elements in a particular set are ranked with respect to the values of known auxiliary variable z from 1 (smallest) to w_1 (largest) by a very inexpensive method such as eye inspection, expert opinion etc.. At this stage, the actual values are not observed. Then, the unit holding rank $i\left(i=1,2,\ldots,w_1\right)$ in the i^{th} set is selected and the corresponding values of the study variable x is also noted. This completes the first cycle. The entire process is repeated for k_1 cycles to obtain the desired sample s_n of size $n=w_1k_1$ at the first occasion.

Now, a sample $s_m\left(I\right)$ of size m is to be selected from the sample s_n. For this w_2^2 units are selected at random from the sample s_n by simple random sampling with replacement (SRSWR) method.

These w_2^2 units are then allocated into w_2 sets each of size w_2. The elements in a particular set are ranked with respect to the values auxiliary variable z from 1 (smallest) to w_2 (largest). Then, the unit holding rank $i\left(i=1,2,\ldots,w_2\right)$ in the i^{th} set are measured and the corresponding value of the study variable x is also measured. This completes the first cycle for the sample $s_m\left(I\right)$. The entire process is repeated for k_2 cycles to obtain the desired sample s_m of size $m=w_2k_2$ at first occasion. As this being a matched sample so it will be used at the second occasion as well. Let us name it $s_m\left(II\right)$ at second occasion. In this sample the units, which became the part of the sample finally at first occasion are only remeasures at second occasion to complete the matched sample $s_m\left(II\right)$.

Finally, at second occasion, w_3^2 units are selected at random from the $U-s_n$ by simple random sampling with replacement (SRSWR) method. These w_3^2 units are then allocated into w_3 sets each of size w_3. The elements in a particular set are ranked with respect to the values of known auxiliary variable z from 1 (smallest) to w_3 (largest). Then, the unit holding rank $i\left(i=1,2,\ldots,w_3\right)$ in the i^{th} set is noted and the corresponding value of the study variable y is also measured. This completes the first cycle. The entire process is repeated for k_3 cycles to obtain the desired fresh sample s_u of size $u=k_3w_3$ at second occasion.

Table 1. Sample Structure on two occasions RSSS

Occasion	Sample	Variable	Cycle and measured values
I	s_n	x	Cycle 1: $\left\{z_{1(1)\vert 1},x_{1[1]\vert 1}\right\},\left\{z_{2(2)\vert 1},x_{2[2]\vert 1}\right\},\ldots,\left\{z_{w_1(w_1)\vert 1},x_{w_1[w_1]\vert 1}\right\}$ . . . Cycle r: $\left\{z_{1(1)\vert r},x_{1[1]\vert r}\right\},\left\{z_{2(2)\vert r},x_{2[2]\vert r}\right\},\ldots,\left\{z_{w_1(w_1)\vert r},x_{w_1[w_1]\vert r}\right\}$. . . . Cycle k_1. : $\left\{z_{1(1)\vert k_1},x_{1[1]\vert k_1}\right\},\left\{z_{2(2)\vert k_1},x_{2[2]\vert k_1}\right\},\ldots,\left\{z_{w_1(w_1)\vert k_1},x_{w_1[w_1]\vert k_1}\right\}$.
I	$s_m\left(I\right)$	x	Cycle 1: $\left\{z_{1(1)\vert 1},x_{1[1]\vert 1}\right\},\left\{z_{2(2)\vert 1},x_{2[2]\vert 1}\right\},\ldots,\left\{z_{w_2(w_2)\vert 1},x_{w_2[w_2]\vert 1}\right\}$ . . . Cycle r: $\left\{z_{1(1)\vert r},x_{1[1]\vert r}\right\},\left\{z_{2(2)\vert r},x_{2[2]\vert r}\right\},\ldots,\left\{z_{w_2(w_2)\vert r},x_{w_2[w_2]\vert r}\right\}$ . . . Cycle k_2 : $\left\{z_{1(1)\vert k_2},x_{1[1]\vert k_2}\right\},\left\{z_{2(2)\vert k_2},x_{2[2]\vert k_2}\right\},\ldots,\left\{z_{w_1(w_1)\vert k_2},x_{w_2[w_2]\vert k_2}\right\}$
II	$s_m\left(II\right)$	y	Cycle 1: $\left\{z_{1(1)\vert 1},y_{1[1]\vert 1}\right\},\left\{z_{2(2)\vert 1},y_{2[2]\vert 1}\right\},\ldots,\left\{z_{w_2(w_2)\vert 1},y_{w_2[w_2]\vert 1}\right\}$ . . . Cycle r: $\left\{z_{1(1)\vert r},y_{1[1]\vert r}\right\},\left\{z_{2(2)\vert r},y_{2[2]\vert r}\right\},\ldots,\left\{z_{w_2(w_2)\vert r},y_{w_2[w_2]\vert r}\right\}$ . . . Cycle k_2 : $\left\{z_{1(1)\vert k_2},y_{1[1]\vert k_2}\right\},\left\{z_{2(2)\vert k_2},y_{2[2]\vert k_2}\right\},\ldots,\left\{z_{w_1(w_1)\vert k_2},y_{w_2[w_2]\vert k_2}\right\}$
II	s_u	y	Cycle 1: $\left\{z_{1(1)\vert 1},y_{1[1]\vert 1}\right\},\left\{z_{2(2)\vert 1},y_{2[2]\vert 1}\right\},\ldots,\left\{z_{w_3(w_3)\vert 1},y_{w_3[w_3]\vert 1}\right\}$ . . . Cycle r: $\left\{z_{1(1)\vert r},y_{1[1]\vert r}\right\},\left\{z_{2(2)\vert r},y_{2[2]\vert r}\right\},\ldots,\left\{z_{w_3(w_3)\vert r},y_{w_3[w_3]\vert r}\right\}$ . . . Cycle k_3 : $\left\{z_{1(1)\vert k_2},y_{1[1]\vert k_3}\right\},\left\{z_{2(2)\vert k_2},y_{2[2]\vert k_3}\right\},\ldots,\left\{z_{w_3(w_3)\vert k_3},y_{w_3[w_3]\vert k_3}\right\}$

The ranked set data collected in the sample s_n is given by $\left\{z_{i(i)|r}, x_{i[i]|r}, i=1,2,\ldots,w_1; r=1,2,\ldots,k_1\right\}$ where, $z_{i(i)|r}$ is the i^{th} ordered statistics for the auxiliary variae z of the i^{th} set of the r^{th} cycle and $x_{i[i]|r}$ represents the corresponding judgment ordered statistics for the study variable x at the first occasion.

Similarly, the ranked set data accumuted in the sample $s_m(I), s_m(II)$, and s_u are given by $\left\{z_{i(i)|r}, x_{i[i]|r}, i=1,2,\ldots,w_2; r=1,2,\ldots,k_2\right\}$, $\left\{z_{i(i)|r}, y_{i[i]|r}, i=1,2,\ldots,w_2; r=1,2,\ldots,k_2\right\}$, and $\left\{z_{i(i)|r}, y_{i[i]|r}, i=1,2,\ldots,w_3; r=1,2,\ldots,k_3\right\}$ respectively.

3. FORMULATION OF RSSS ESTIMATORS

For estimation of the population mean at current (second) occasion in two occasions RSSS, two independent samples namely s_u (fresh sample) and $s_m(II)$ (matched sample) are available at the second occasion. Therefore, two different RSS estimators are proposed based on the above two samples. As per the convention of the successive sampling, the information from the sample s_n drawn at the first occasion is used as an additional auxiliary information at the second occasion. For robust estimates at current (second) occasion, the final estimator is considered to be the linear combinations of the two estimators based on the fresh and the matched samples. Following RSSS estimators for estimating the population mean at current (second) occasion are proposed:

$$T_i^{RSSS} = \Omega_i T_{iu}^{RSS} + \left(1-\Omega_i\right) T_{im}^{RSS}; i=1,2 \text{, and } 3 \quad (3.1)$$

where T_{iu}^{RSS} is based on fresh sample s_u and T_{im}^{RSS} is based on matched sample $s_{m(II)}$, and Ω_i is unknown weight parameter, that controls emphasis between two estimators T_{iu}^{RSS} and T_{im}^{RSS}. For RSSS

set up an optimum value of Ω_i is evaluated by minimizing the mean squared error of the estimator T_i^{RSSS}.

3.1 RSS Estimators based on fresh sample s_u at current occasion

When no additional auxiliary variable is used at the estimation stage, then the sample mean estimator is considered as

$$T_{1u}^{RSS} = \overline{y}_u^{RSS} \quad (3.2)$$

where $\overline{y}_u^{RSS} = \frac{1}{k_3}\sum_{r=1}^{k_3} \overline{y}_{[w_3]|r} = \frac{1}{u}\sum_{r=1}^{k_3}\sum_{i=1}^{w_3} y_{i[i]|r}$.

When additional auxiliary variable is used at the estimation stage, and it is known that y and z are highly positively correlated, and the relationship between them is approximately linear through the origin, then the ratio type estimator is considered to be useful. Hence, the second estimator based on the sample s_u is the ranked set ratio type estimator given by

$$T_{2u}^{RSS} = \left(\frac{\overline{y}_u^{RSS}}{\overline{z}_u^{RSS}}\right)\overline{Z} \quad (3.3)$$

where $\overline{z}_u^{RSS} = \frac{1}{k_3}\sum_{r=1}^{k_3} \overline{z}_{(w_3)|r} = \frac{1}{u}\sum_{r=1}^{k_3}\sum_{i=1}^{w_3} z_{i(i)|r}$.

However, if the relationship between the study variable and auxiliary variable is not necessarily linear through the origin, then the use of regression type estimator is recommended as it is more robust to variations in data. Therefore, the next estimator based on the fresh sample s_u is the ranked set regression type estimator given by:

$$T_{3u}^{RSS} = \overline{y}_u^{RSS} + \beta_{yz}\left(\overline{Z} - \overline{z}_u^{RSS}\right) \quad (3.4)$$

It is to be noted that, the above estimator is based on the population regression coefficient $\beta_{yz} = \frac{\sum_{i=1}^{N}\left(y_i - \overline{Y}\right)\left(z_i - \overline{Z}\right)}{\sum_{i=1}^{N}\left(z_i - \overline{Z}\right)^2}$, which is generally unknown. In such situation it may be replaced by the sample regression coefficient $\hat{b}_{yz}(u) = \frac{\sum_{r=1}^{k_3}\sum_{i=1}^{w_3}\left(y_{i[i]|r} - \overline{y}_u^{RSS}\right)\left(z_{i(i)|r} - \overline{z}_u^{RSS}\right)}{\sum_{r=1}^{k_3}\sum_{i=1}^{w_3}\left(z_{i(i)|r} - \overline{z}_u^{RSS}\right)^2} = \frac{s_{yz}^{RSS}(u)}{s_z^{2RSS}(u)}$, based on the sample s_u and the working version of the estimator T_{3u}^{RSS} is denoted and defined as follows:

$$T_{3u}^{*RSS} = \overline{y}_u^{RSS} + \hat{b}_{yz}(u)\left(\overline{Z} - \overline{z}_u^{RSS}\right) \quad (3.5)$$

3.2 RSS Estimators based on fresh sample $s_m(II)$ at current occasion

In successive sampling framework, it is well known that the information from previous occasion may be used as auxiliary information at the current occasion. Exploring this concept, the RSS estimator based on sample s_m at the current occasion is proposed as

$$T_{1m}^{RSS} = \bar{y}_m^{RSS} + \beta_{yx}\left(\bar{X} - \bar{x}_m^{RSS}\right) \tag{3.6}$$

where $\bar{y}_m^{RSS} = \frac{1}{k_2}\sum_{r=1}^{k_2}\bar{y}_{[w_2]|r} = \frac{1}{m}\sum_{r=1}^{k_2}\sum_{i=1}^{w_2} y_{i[i]|r}$,

$\bar{x}_m^{RSS} = \frac{1}{k_2}\sum_{r=1}^{k_2}\bar{x}_{[w_2]|r} = \frac{1}{m}\sum_{r=1}^{k_2}\sum_{i=1}^{w_2} x_{i[i]|r}$, and

$\beta_{yx} = \dfrac{\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)\left(x_i - \bar{X}\right)}{\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2}$. However, in addition to the information from previous occasion, additional auxiliary information may also be utilized for improving the estimates at current occasion. Hence, chain type RSS ratio to regression estimator T_{2m}^{RSS} and chain type RSS regression to regression estimators T_{3m}^{RSS} are respectively proposed as

$$T_{2m}^{RSS} = \bar{y}_m^{\oplus} + \beta_{yx}\left(\bar{x}_n^{\oplus} - \bar{x}_m^{\oplus}\right) \tag{3.7}$$

where $\bar{y}_m^{\oplus} = \left(\frac{\bar{y}_m^{RSS}}{\bar{z}_m^{RSS}}\right)\bar{Z}$, $\bar{x}_n^{\oplus} = \left(\frac{\bar{x}_n^{RSS}}{\bar{z}_n^{RSS}}\right)\bar{Z}$, and

$\bar{x}_m^{\oplus} = \left(\frac{\bar{x}_m^{RSS}}{\bar{z}_m^{RSS}}\right)\bar{Z}$ with $\bar{z}_m^{RSS} = \frac{1}{k_2}\sum_{r=1}^{k_2}\bar{z}_{(w_2)|r} = \frac{1}{m}\sum_{r=1}^{k_2}\sum_{i=1}^{w_2} z_{i(i)|r}$,

$\bar{x}_n^{RSS} = \frac{1}{k_1}\sum_{r=1}^{k_1}\bar{x}_{[w_1]|r} = \frac{1}{n}\sum_{r=1}^{k_1}\sum_{i=1}^{w_1} x_{i[i]|r}$, and

$\bar{z}_n^{RSS} = \frac{1}{k_1}\sum_{r=1}^{k_1}\bar{z}_{(w_1)|r} = \frac{1}{n}\sum_{r=1}^{k_1}\sum_{i=1}^{w_1} z_{i(i)|r}$.

$$T_{3m}^{RSS} = \bar{y}_m^{\ominus} + \beta_{yx}\left(\bar{x}_n^{\ominus} - \bar{x}_m^{\ominus}\right) \tag{3.8}$$

where $\bar{y}_m^{\ominus} = \bar{y}_m^{RSS} + \beta_{yz}\left(\bar{Z} - \bar{z}_m^{RSS}\right)$,

$\bar{x}_n^{\ominus} = \bar{x}_n^{RSS} + \beta_{xz}\left(\bar{Z} - \bar{z}_n^{RSS}\right)$, and

$\bar{x}_m^{\ominus} = \bar{x}_m^{RSS} + \beta_{xz}\left(\bar{Z} - \bar{z}_m^{RSS}\right)$ with

$\beta_{yz} = \dfrac{\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)\left(z_i - \bar{Z}\right)}{\sum_{i=1}^{N}\left(z_i - \bar{Z}\right)^2}$, and

$\beta_{xz} = \dfrac{\sum_{i=1}^{N}\left(x_i - \bar{X}\right)\left(z_i - \bar{Z}\right)}{\sum_{i=1}^{N}\left(z_i - \bar{Z}\right)^2}$.

The estimators T_{2m}^{RSS} and T_{3m}^{RSS} proposed in equations (3.7) and (3.8) respectively involves the

population regression coefficients. In case they are unknown, then for practical applicability of these estimators, the unknown population regression coefficients are replaced by the corresponding sample regression coefficients and the estimators are represented as T_{2m}^{*RSS} and T_{3m}^{*RSS} respectively and are given as

$$T_{2m}^{*RSS} = \bar{y}_m^{\oplus} + \hat{b}_{yx}(m)\left(\bar{x}_n^{\oplus} - \bar{x}_m^{\oplus}\right) \tag{3.9}$$

where $\hat{b}_{yx}(m) = \dfrac{s_{yx}^{RSS}(m)}{s^{2RSS}(m)}$, and

$$T_{3m}^{*RSS} = \bar{y}_m^{\ominus *} + \hat{b}_{yx}(m)\left(\bar{x}_n^{\ominus *} - \bar{x}_m^{\ominus *}\right) \tag{3.10}$$

where $\bar{y}_m^{\ominus *} = \bar{y}_m^{RSS} + \hat{b}_{yz}(m)\left(\bar{Z} - \bar{z}_m^{RSS}\right)$,

$\bar{x}_n^{\ominus *} = \bar{x}_n^{RSS} + \hat{b}_{xz}(n)\left(\bar{Z} - \bar{z}_n^{RSS}\right)$, and

$\bar{x}_m^{\ominus *} = \bar{x}_m^{RSS} + \hat{b}_{xz}(m)\left(\bar{Z} - \bar{z}_m^{RSS}\right)$ with

$\hat{b}_{yz}(m) = \dfrac{s_{yz}^{RSS}(m)}{s_z^{2RSS}(m)}$, $\hat{b}_{xz}(n) = \dfrac{s_{xz}^{RSS}(n)}{s_z^{2RSS}(n)}$, and

$\hat{b}_{xz}(m) = \dfrac{s_{xz}^{RSS}(m)}{s_z^{2RSS}(m)}$.

4. PROPERTIES OF THE PROPOSED RSSS ESTIMATORS

In order to derive the properties of the proposed estimators, following error terms are assumed:

$\bar{y}_u^{RSS} = \bar{Y}(1+\varepsilon_1)$, $\bar{y}_m^{RSS} = \bar{Y}(1+\varepsilon_2)$,

$\bar{x}_n^{RSS} = \bar{X}(1+\varepsilon_3)$, $\bar{x}_m^{RSS} = \bar{X}(1+\varepsilon_4)$, $\bar{z}_u^{RSS} = \bar{Z}(1+\varepsilon_5)$,

$\bar{z}_m^{RSS} = \bar{Z}(1+\varepsilon_6)$, $\bar{z}_n^{RSS} = \bar{Z}(1+\varepsilon_7)$,

$s_{yz}^{RSS}(u) = S_{yz}(1+\varepsilon_8)$, $s_z^{2RSS}(u) = S_z^2(1+\varepsilon_9)$,

$s_{yz}^{RSS}(m) = S_{yz}(1+\varepsilon_{10})$, $s_z^{2RSS}(m) = S_z^2(1+\varepsilon_{11})$,

$s_{yx}^{RSS}(m) = S_{yx}(1+\varepsilon_{12})$, $s_x^{2RSS}(m) = S_x^2(1+\varepsilon_{13})$,

$s_{xz}^{RSS}(n) = S_{xz}(1+\varepsilon_{14})$, $s_z^{2RSS}(n) = S_z^2(1+\varepsilon_{15})$,

$s_{xz}^{RSS}(m) = S_{xz}(1+\varepsilon_{16})$, $\forall\, E(\varepsilon_i) = 0, i = 1,2,\ldots,16.$

The following notations are assumed

$$W_{y[i]}^2(u)=\frac{1}{w_3^2k_3}\sum_{i=1}^{w_3}\left(\mu_{y[i]}-\bar{Y}\right)^2,$$

$$W_{z(i)}^2(u)=\frac{1}{w_3^2k_3}\sum_{i=1}^{w_3}\left(\mu_{z(i)}-\bar{Z}\right)^2,$$

$$W_{yz(i)}(u)=\frac{1}{w_3^2k_3}\sum_{i=1}^{w_3}\left(\mu_{y[i]}-\bar{Y}\right)\left(\mu_{z(i)}-\bar{Z}\right),$$

$$W_{y[i]}^2(m)=\frac{1}{w_2^2k_2}\sum_{i=1}^{w_2}\left(\mu_{y[i]}-\bar{Y}\right)^2,$$

$$W_{x[i]}^2(m)=\frac{1}{w_2^2k_2}\sum_{i=1}^{w_2}\left(\mu_{x[i]}-\bar{X}\right)^2,$$

$$W_{x[i]}^2(n)=\frac{1}{w_1^2k_1}\sum_{i=1}^{w_1}\left(\mu_{x[i]}-\bar{X}\right)^2,$$

$$W_{yx[i]}(n)=\frac{1}{w_1^2k_1}\sum_{i=1}^{w_1}\left(\mu_{y[i]}-\bar{Y}\right)\left(\mu_{x[i]}-\bar{X}\right),$$

$$W_{yx[i]}(m)=\frac{1}{w_2^2k_2}\sum_{i=1}^{w_2}\left(\mu_{y[i]}-\bar{Y}\right)\left(\mu_{x[i]}-\bar{X}\right),$$

$$W_{z(i)}^2(n)=\frac{1}{w_1^2k_1}\sum_{i=1}^{w_1}\left(\mu_{z(i)}-\bar{Z}\right)^2,$$

$$W_{z(i)}^2(m)=\frac{1}{w_2^2k_2}\sum_{i=1}^{w_2}\left(\mu_{z(i)}-\bar{Z}\right)^2,$$

$$W_{yz(i)}(m)=\frac{1}{w_2^2k_2}\sum_{i=1}^{w_2}\left(\mu_{y[i]}-\bar{Y}\right)\left(\mu_{z(i)}-\bar{Z}\right),$$

$$W_{xz(i)}(n)=\frac{1}{w_1^2k_1}\sum_{i=1}^{w_1}\left(\mu_{x[i]}-\bar{X}\right)\left(\mu_{z(i)}-\bar{Z}\right),$$

$$W_{xz(i)}(m)=\frac{1}{w_2^2k_2}\sum_{i=1}^{w_2}\left(\mu_{x[i]}-X\right)\left(\mu_{z(i)}-\bar{Z}\right)$$

where the value of $\mu_{y[i]}$, $\mu_{x[i]}$, and $\mu_{z(i)}$ can be obtained following Arnaldo et al (1993) and depends on ordered statistics from some specific distributions.

4.1 Mean Squared Error of RSS estimators based on fresh sample s_u at current occasion

The estimator T_{1u}^{RSS} is the ranked set sample mean estimator, its variance is given by

$$MSE\left(T_{1u}^{RSS}\right)=\frac{1}{u}S_y^2-W_{y[i]}^2(u) \tag{4.1}$$

The mean squared error of T_{2u}^{RSS} is evaluated in following steps:

$$MSE\left(T_{2u}^{RSS}\right)=E\left(T_{2u}^{RSS}-\bar{Y}\right)^2$$

Now, writing the estimator T_{2u}^{RSS} in terms of error terms, we get

$$MSE\left(T_{2u}^{RSS}\right)=E\left[\bar{Y}\left(1+\varepsilon_1\right)\left(1+\varepsilon_5\right)^{-1}-\bar{Y}\right]^2$$

Simplifying and retaining the terms up to the first order of approximations,

$$MSE\left(T_{2u}^{RSS}\right)=\bar{Y}^2E\left[\varepsilon_1^2+\varepsilon_5^2-2\varepsilon_1\varepsilon_5\right]$$

Taking expectation, and simplifying we get

$$MSE\left(T_{2u}^{RSS}\right)=\frac{1}{u}\left[S_y^2+R^2S_z^2-2RS_{yz}\right]-\left[W_{y[i]}^2(u)+R^2W_{z(i)}^2(u)-2RW_{yz(i)}(u)\right] \tag{4.2}$$

where $R=\dfrac{\bar{Y}}{\bar{Z}}$.

Now, the mean squared error of T_{3u}^{RSS} is evaluated as:

$$MSE\left(T_{3u}^{RSS}\right)=E\left(T_{3u}^{RSS}-\bar{Y}\right)^2$$

Now, writing the estimator T_{3u}^{RSS} in terms of error terms, we get

$$MSE\left(T_{3u}^{RSS}\right)=E\left[\bar{Y}\left(1+\varepsilon_1\right)-\beta_{yz}\bar{Z}\varepsilon_5-\bar{Y}\right]^2$$

Simplifying and retaining the terms up to the first order of approximations,

$$MSE\left(T_{3u}^{RSS}\right)=E\left[\bar{Y}^2\varepsilon_1^2-2\bar{Z}\bar{Y}\beta_{yz}\varepsilon_1\varepsilon_5+\beta_{yz}^2\bar{Z}^2\varepsilon_5^2\right] \tag{4.3}$$

Taking expectation, and simplifying we get

$$MSE\left(T_{3u}^{RSS}\right)=\frac{1}{u}\left[1-\rho_{yz}^2\right]S_y^2-\left[W_{y[i]}^2(u)+\beta_{yz}^2W_{z(i)}^2(u)-2\beta_{yz}W_{yz(i)}(u)\right] \tag{4.4}$$

Similarly, the mean squared error of T_{3u}^{*RSS} is calculated as:

$$MSE\left(T_{3u}^{*RSS}\right)=E\left(T_{3u}^{*RSS}-\bar{Y}\right)^2$$

Expressing the estimator T_{3u}^{*RSS} in terms of error terms, we get

$$MSE\left(T_{3u}^{*RSS}\right)=E\left[\bar{Y}\left(1+\varepsilon_1\right)-\frac{S_{yz}}{S_z^2}(1+\varepsilon_8)\left(1+\varepsilon_9\right)^{-1}\bar{Z}\varepsilon_5-\bar{Y}\right]^2$$

Simplifying and retaining the terms up to the first order of approximations, we get the expression for $MSE\left(T_{3u}^{*RSS}\right)$, same as obtained in equation (4.3). Thus, it is observed that the mean squared error of

the estimator T_{3u}^{*RSS} is same as the mean squared error of the estimator T_{3u}^{RSS} up to the first order of approximations.

4.2 Mean Squared Error of RSS Estimators based on matched sample $s_m(II)$ at current occasion

The mean squared error of T_{1m}^{RSS} is written as:

$$MSE\left(T_{1m}^{RSS}\right)=E\left(T_{1m}^{RSS}-\bar{Y}\right)^2$$

Writing the estimator T_{1m}^{RSS} in terms of error terms, simplifying and retaining the terms up to the first order of approximations, we get

$$MSE\left(T_{1m}^{RSS}\right)=E\Big[\bar{Y}^2\varepsilon_2^2+\bar{X}^2\beta_{yx}^2\left(\varepsilon_3^2+\varepsilon_4^2-2\varepsilon_3\varepsilon_4\right)+2\bar{X}\bar{Y}\beta_{yx}\left(\varepsilon_2\varepsilon_3-\varepsilon_2\varepsilon_4\right)\Big]$$

Taking expectation, and simplifying we get

$$MSE\left(T_{1m}^{RSS}\right)=\frac{1}{m}S_y^2+\left(\frac{1}{m}-\frac{1}{n}\right)\left[\beta_{yx}^2S_x^2-2\beta_{yx}S_{yx}\right]+\left[W_{y[i]}^2(m)-\beta_{yx}^2W_{x[i]}^2(m)+2\beta_{yx}\left(W_{yx[i]}(m)-W_{yx[i]}(n)\right)\right] \tag{4.5}$$

The mean squared error of T_{2m}^{RSS} is written as:

$$MSE\left(T_{2m}^{RSS}\right)=E\left(T_{2m}^{RSS}-\bar{Y}\right)^2$$

Expressing the estimator T_{2m}^{RSS} in terms of error terms, simplifying and retaining the terms up to the first order of approximations, we get

$$MSE\left(T_{2m}^{RSS}\right)=E\left[\bar{Y}\left(\varepsilon_2-\varepsilon_6\right)+\beta_{yx}\bar{X}\left(\varepsilon_3-\varepsilon_7-\varepsilon_4+\varepsilon_6\right)\right]^2$$

Taking expectation, and simplifying we get

$$MSE\left(T_{2m}^{RSS}\right)=\frac{1}{m}A_1+\left(\frac{1}{m}-\frac{1}{n}\right)A_2-A_3-\beta_{yx}^2A_4-2\beta_{yx}A_5 \tag{4.6}$$

where $A_1=S_y^2+R_1^2S_z^2-2R_1S_{yz}$,

$$A_2=\beta_{yx}^2\left(S_x^2+R_2^2S_z^2-2R_2S_{xz}\right)+2\beta_{yx}\left(R_1S_{xz}-R_1R_2S_z^2-S_{yx}+\left\{R_1-R_2\right\}S_{yz}\right),$$

$$A_3=W_{y[i]}^2(m)+R_1^2W_{z(i)}^2(m)-2R_1W_{yz(i)}(m),$$

$$A_4=W_{x[i]}^2(m)-W_{x[i]}^2(n)+R_2^2\left\{W_{z(i)}^2(m)-W_{z(i)}^2(n)\right\}+2R_2\left\{W_{xz(i)}(m)-W_{xz(i)}(n)\right\},$$

$$A_5=R_1\left\{W_{xz(i)}(m)-W_{xz(i)}(n)\right\}-R_1R_2\left\{W_{z(i)}^2(m)-W_{z(i)}^2(n)\right\}-\left\{W_{yx[i]}(m)-W_{yx(i)}(n)\right\}+R_1W_{yz(i)}(m)-R_2W_{yz(i)}(n).$$

The mean squared error of T_{3m}^{RSS} is written as:

$$MSE\left(T_{3m}^{RSS}\right)=E\left(T_{3m}^{RSS}-\bar{Y}\right)^2$$

Expressing the estimator T_{3m}^{RSS} in terms of error terms, simplifying and retaining the terms up to the first order of approximations, we get

$$MSE\left(T_{3m}^{RSS}\right)=E\Big[\bar{Y}\varepsilon_2-\beta_{yz}\bar{Z}\varepsilon_6+\beta_{yx}\left\{\bar{X}\left(\varepsilon_3-\varepsilon_4\right)+\beta_{xz}\bar{Z}\left(\varepsilon_6-\varepsilon_7\right)\right\}\Big]^2 \tag{4.7}$$

Taking expectation, and simplifying we get

$$MSE\left(T_{3m}^{RSS}\right)=\frac{1}{m}B_1+\left(\frac{1}{m}-\frac{1}{n}\right)B_2+B_3+B_4+B_5+B_6+B_7+B_8 \tag{4.8}$$

where $B_1=\left(1-\rho_{yz}^2\right)S_y^2$,

$$B_2=\left\{2\rho_{yx}\rho_{xz}\rho_{yz}-\rho_{yx}^2\left(1+\rho_{xz}^2\right)\right\}S_y^2,$$

$$B_3=-W_{y[i]}^2(m)+\beta_{yz}^2W_{z(i)}^2(m)+2\beta_{yz}W_{yz(i)}(m),$$

$$B_4=-\beta_{yx}^2\left\{-W_{x[i]}^2(m)-W_{x[i]}^2(n)\right\},$$

$$B_5=-\left\{\beta_{yx}^2\beta_{xz}^2-2\beta_{yx}\beta_{yz}\beta_{xz}\right\}\left\{W_{z(i)}^2(m)-W_{z(i)}^2(n)\right\},$$

$$B_6=2\beta_{yx}\left\{W_{yx[i]}(m)-W_{yx[i]}(n)\right\},$$

$B_7 = -2\beta_{yx}\beta_{xz}\left\{W_{yz(i)}(m) - W_{yz(i)}(n)\right\}$, and

$B_8 = \left\{2\beta_{yx}^2\beta_{xz} - 2\beta_{yx}\beta_{yz}\right\}\left\{W_{xz(i)}(m) - W_{xz(i)}(n)\right\}$.

Now we compute the mean squared error of T_{3m}^{*RSS} is written as:

$$MSE\left(T_{3m}^{*RSS}\right) = E\left(T_{3m}^{*RSS} - \bar{Y}\right)^2$$

Expressing the estimator T_{3m}^{*RSS} in terms of error terms, we get

$$MSE\left(T_{3m}^{*RSS}\right) = E\left[\bar{Y}\varepsilon_2 - \frac{S_{yz}}{S_z^2}(1+\varepsilon_{10})(1+\varepsilon_{11})^{-1}\bar{Z}\varepsilon_6 + \frac{S_{yx}}{S_x^2}(1+\varepsilon_{12})(1+\varepsilon_{11})^{-1}\left\{\bar{X}(\varepsilon_3 - \varepsilon_4) + \frac{S_{xz}}{S_z^2}\bar{Z}\left\{(1+\varepsilon_{14})(1+\varepsilon_{11})^{-1}\varepsilon_6 - (1+\varepsilon_{16})(1+\varepsilon_{11})^{-1}\varepsilon_7\right\}\right\}\right]^2 \tag{4.9}$$

Simplifying and retaining the terms up to the first order of approximations, we get the expression for the mean squared error of T_{3m}^{*RSS} same as the mean square error of the estimator T_{3m}^{RSS} as obtained in the equation (4.8).

Theorem 1. *The optimum mean squared error of the estimators* $\mathrm{T}_i^{\mathrm{RSSS}}$ *is given by*

$$MSE\left(T_i^{RSSS}\right)_{min.} = \frac{MSE\left(T_{iu}^{RSS}\right) \times MSE\left(T_{im}^{RSS}\right)}{MSE\left(T_{iu}^{RSS}\right) + MSE\left(T_{im}^{RSS}\right)} \quad \forall i = 1, 2, and\, 3. \tag{4.10}$$

Proof: The mean squared error of the estimators T_i^{RSSS} is computed as:

$$MSE\left(T_i^{RSSS}\right) = E\left(T_i^{RSSS} - \bar{Y}\right)^2$$

Substituting the value of T_i^{RSSS} from equation (3.1). we get

$$MSE\left(T_i^{RSSS}\right) = E\left(\Omega_i T_{iu}^{RSS} + (1-\Omega_i)T_{im}^{RSS} - \bar{Y}\right)^2$$

Simplifying, we get

$$MSE\left(T_i^{RSSS}\right) = \Omega_i^2 E\left[T_{iu}^{RSS} - \bar{Y}\right]^2 + (1-\Omega_i)^2 E\left[T_{im}^{RSS} - \bar{Y}\right]^2 + 2\Omega_i(1-\Omega_i)E\left[\left(T_{iu}^{RSS} - \bar{Y}\right)\left(T_{im}^{RSS} - \bar{Y}\right)\right] \tag{4.11}$$

Since, the samples s_u and s_m are independent of each other, therefore it is reasonable to assume that the covariance term $E\left[\left(T_{iu}^{RSS} - \bar{Y}\right)\left(T_{im}^{RSS} - \bar{Y}\right)\right] = 0 \,\forall i = 1, 2,$ and 3 in the above equation (4.11). So, the mean squared error of T_i^{RSSS} becomes

$$MSE\left(T_i^{RSSS}\right) = \Omega_i^2 MSE\left(T_{iu}^{RSS}\right) + (1-\Omega_i)^2 MSE\left(T_{im}^{RSS}\right) \quad \forall i = 1, 2, \text{ and } 3 \tag{4.12}$$

It is to be noted that the $MSE\left(T_i^{RSSS}\right)$ in the above equation (4.12) is a function of unknown constant Ω_i, so the mean squared error of T_i^{RSSS} is optimized with respect to Ω_i. For this we compute $\Omega_i = \Omega_i^{opt.}$ (say) by solving the following equation

$$\frac{d\, MSE\left(T_i^{RSSS}\right)}{d\,\Omega_i} = 0 \,\forall i = 1, 2, \text{ and } 3.$$

$$\Omega_i^{opt.} = \frac{MSE\left(T_{im}^{RSS}\right)}{MSE\left(T_{iu}^{RSS}\right) + MSE\left(T_{im}^{RSS}\right)} \,\forall i = 1, 2, \text{ and } 3 \tag{4.13}$$

This critical value of $\Omega_i = \Omega_i^{opt.}$ would minimize the $MSE\left(T_i^{RSSS}\right)$ iff

$$\left.\frac{d^2 MSE\left(T_i^{RSSS}\right)}{d\Omega_i^2}\right|_{\Omega_i = \Omega_i^{opt.}} > 0$$

Substituting the optimum value of $\Omega_i = \Omega_i^{opt.}$ In equation (4.12), we get the minimum mean squared error of T_i^{RSSS} $\forall i = 1, 2,$ and 3 as given in equation (4.10).

5. NUMERICAL ILLUSTRATION

In order to justify the practical applicability and comparison of proposed RSSS estimators, both real and simulated populations have been considered.

Population I (Real Data): The data presented in Priyanka *et al.* (2023) have been used here for illustration. In the first occasion the RSS sample s_n has been drawn with set size w_1 = 5 and k_1 = 5

replications, so that the sample size is n = 25. From the sample s_n, RSS sub sample s_m is drawn with set size w_2 = 3 with k_2 = 5 replications so that the sample size m = 15. This sample is retained to be used at the second (current) occasion also. However, a fresh RSS sample s_u has been drawn from U- s_n with set size w_3 = 5 with k_3 = 2 replications so that the sample size is u = 10. The mean squared errors of the three proposed RSSS estimators have been computed for Ω = 0.1, 0.2, ..., 0.9. The process is repeated five times and the data is recorded and presented in Table 2. However, the minimum mean squared errors of these estimators are computed for optimum value of Ω. The results are represented in Table 3. In order to identify the effectiveness of RSSS estimators over simple random sampling successive sampling estimators, a comparison has been carried out. If we consider the simple random sampling, then the estimator T_3^{RSSS} reduces to T_3, which is Singh and Priyanka (2008) . Comparison has been carried out at optimum conditions and the percent relative efficiencies has been computed by following formula and results are presented in Table 4.

$$PRE_i = \frac{MSE\left(T_i\right)_{min.}}{MSE\left(T_i^{RSSS}\right)_{min.}} \times 100; i = 1,2, \text{ and } 3. \quad (5.1)$$

5.1 Simulation - Based Evaluation

Simulation is a powerful tool to check whether the proposed techniques will be applicable to real life problems. So, each proposed estimator is simulated and applied to the real population (Population I) as well as artificially simulated populations (Population II and Population III). The simulation has been carried out using MATLAB software.

Two artificial populations each of size N = 550 have been generated following Singh and Horn (1998):

$$x_i = 15 + \left(\sqrt{1-\rho_{xz}^2}\right)x_i^* + \rho_{xz}\frac{S_x}{S_z}z_i^* \quad (5.2)$$

$$y_i = 13 + \left(\sqrt{1-\rho_{yz}^2}\right)y_i^* + \rho_{yz}\frac{S_y}{S_z}z_i^* \quad (5.3)$$

$$z_i = 3.7 + z_i^* \quad (5.4)$$

Let us assume S_x = 4.5, S_y = 4.3, and S_z = 2.5 for illustration. As z is the auxiliary variable, which is correlated to the study variables x and y respectively. So, for further illustration we consider the following two cases:

Case a (High correlation): Let us assume ρ_{xz} = 0.9, ρ_{yz} = 0.86.

Case b (Moderate correlation): Let us assume ρ_{xz} = 0.55, ρ_{yz} = 0.58.

Population II (Simulated Population): This is generated from transformations given in equations (5.2) - (5.4), where the study and auxiliary variables follow exponential distribution given by

$x_i^* \sim Exp\left(x_i;\lambda_1\right)$;

$y_i^* \sim Exp\left(y_i;\lambda_2\right)$;

$z_i^* \sim Exp\left(z_i;\lambda_3\right)$; where

$$Exp\left(t_i;\lambda_i\right) = \begin{cases} \lambda_i e^{-\lambda_i t_i} & t_i > 0 \\ 0 & otherwise \end{cases}$$

where $t_i \in \{x_i, y_i, z_i\}$ and $\lambda_i \in \{\lambda_1, \lambda_2, \lambda_3\}$.

Let λ_1 = 7.2, λ_2 = 7.0, λ_3 = 5.3, S_x = 4.5, S_y = 4.3, and S_z = 2.5 for generation of simulated population II.

Population II (a) (Simulated Population, high correlation case): The observed values of ρ_{xz}, ρ_{yz}, and ρ_{yx} from the generated populations are 0.9372, 0.9099, and 0.8577 respectively.

Population II (b) (Simulated Population, moderate correlation case): The observed values of

ρ_{xz}, ρ_{yz}, and ρ_{yx} from the generated populations are 0.6843, 0.6839, and 0.4689 respectively.

Population III (Simulated Population): For generating this population, it is assumed that the study and auxiliary variable follow Normal Distribution as

$x_i^* \sim N\left(x_i;\mu_1,\sigma_1\right)$;

$y_i^* \sim N\left(y_i;\mu_2,\sigma_2\right)$;

$z_i^* \sim N\left(z_i;\mu_3,\sigma_3\right)$;

where $N(t_i;\mu_i,\sigma_i)=\frac{1}{\sigma_i\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{t_i-\mu_i}{\sigma_i}\right)^2}$

where $t_i \in \{x_i, y_i, z_i\}$ and $\mu_i \in \{\mu_1,\mu_2,\mu_3\}$, and $\sigma_i \in \{\sigma_1,\sigma_2,\sigma_3\}$.

For generating population, let us assume $[\mu_1,\mu_2,\mu_3]=[35.5, 33.6, 20.0]$, and $[\sigma_1,\sigma_2,\sigma_3]=[2,3,4]$.

Population III (a) (Simulated Population, high correlation case): The observed values of ρ_{xz}, ρ_{yz}, and ρ_{yx} from the generated populations are 0.9821, 0.9717, and 0.9568 respectively.

Population III (b) (Simulated Population, moderate correlation case): The observed values of ρ_{xz}, ρ_{yz}, and ρ_{yx} from the generated populations are 0.7622, 0.6011, and 0.4940 respectively.

For simulation study, the RSSS parameter assumed are [w_1, w_2, w_3] = [5, 3, 5] and [k_1, k_2, k_3] = [5, 5, 2] so that the sample sizes are [*n*, *m*, *u*] = [25, 15, 10]. The simulation process has been repeated 10,000 times and the simulated mean squared errors of the proposed RSSS estimators have been computed for varying values of Ω = {0.1, 0.2, ..., 0.9} as

$$MSE_i(l)=\frac{1}{10{,}000}\sum_{i=1}^{10{,}000} MSE\left(T_{il}^{RSSS}\right); i=1,2, \text{ and }$$

3; $l \in \{II(a), II(b), III(a), III(b)\}$, where l represents simulated populations.

The simulation results are presented in Figure 1 and Figure 2 for Population II, and Figure 3 and Figure 4 for Population III.

The percent relative efficiencies of the proposes RSSS estimator with the corresponding successive sampling estimators, in which the samples are drawn with simple random sampling without replacement have also been computed for different values of Ω, and results are vindicated in Figure 5 and Figure 6 for population II and Figure 7 and Figure 8 for Population III respectively.

Table 2. Mean squared error of the proposed RSSS estimators for $\Omega_i = \Omega \,\forall\, i = 1,2,$ and 3 for Population I

Sample	Ω	$MSE\left(T_1^{RSSS}\right)$	$MSE\left(T_2^{RSSS}\right)$	$MSE\left(T_3^{RSSS}\right)$
1	0.1	16.2173	0.3520	0.1728
	0.2	29.6597	0.6725	0.3353
	0.3	40.5874	0.9632	0.4879
	0.4	49.2605	1.2253	0.6309
	0.5	55.9392	1.4605	0.7645
	0.6	60.8837	1.6701	0.8892
	0.7	64.3542	1.8557	1.0052
	0.8	66.6110	2.0187	1.1127
	0.9	67.9142	2.1605	1.2123
2	0.1	19.8779	0.4718	0.3087
	0.2	36.1192	0.8731	0.5693
	0.3	49.0921	1.2094	0.7859
	0.4	59.1608	1.4866	0.9624
	0.5	66.6951	1.7103	1.1031
	0.6	72.0611	1.8863	1.2119
	0.7	75.6259	2.0203	1.2930
	0.8	77.7506	2.1179	1.3504
	0.9	78.8201	2.1851	1.3882
3	0.1	17.0542	0.1784	0.0356
	0.2	29.7375	0.2969	0.0604
	0.3	38.6958	0.3677	0.0765
	0.4	44.5752	0.4029	0.0857
	0.5	48.0215	0.4149	0.0899
	0.6	49.6807	0.4159	0.0910
	0.7	50.1988	0.4181	0.0911
	0.8	50.2218	0.4338	0.0918
	0.9	50.3955	0.4751	0.0953
4	0.1	18.5962	0.3672	0.2224
	0.2	31.9987	0.6265	0.4055
	0.3	41.0562	0.7966	0.5532
	0.4	46.6172	0.8963	0.6694
	0.5	49.5303	0.9441	0.7577
	06	50.6442	0.9590	0.8220
	0.7	50.8075	0.9507	0.8661
	0.8	50.8687	0.9648	0.8938
	0.9	51.6764	0.9931	0.9089
5	0.1	23.2159	0.5220	0.4003
	0.2	42.5019	0.9792	0.7381
	0.3	58.2221	1.3568	1.0186
	0.4	70.7406	1.6802	1.2472
	0.5	80.4216	1.9480	1.4292
	0.6	87.6294	2.1653	1.5699
	0.7	92.7280	2.3374	1.6746
	0.8	96.0817	2.4696	1.7486
	0.9	98.0545	2.5672	1.7973

Table 3. Optimum value of $\Omega_i, i = 1,2,$ and 3 and minimum mean squared errors of the proposed RSSS estimators for Population I

Sample		$i = 1$	$i = 2$	$i = 3$
1	$\Omega_i^{opt.}$ $MSE\left(T_i^{RSSS}\right)_{min.}$	0.9693 0.7950	0.7590 0.1721	0.6664 0.1188
2	$\Omega_i^{opt.}$ $MSE\left(T_i^{RSSS}\right)_{min.}$	0.9850 0.4450	0.9284 0.0595	0.9419 0.0308
3	$\Omega_i^{opt.}$ $MSE\left(T_i^{RSSS}\right)_{min.}$	0.9579 0.5562	0.7597 0.0088	0.8572 0.0021
4	$\Omega_i^{opt.}$ $MSE\left(T_i^{RSSS}\right)_{min.}$	0.90111 0.9434	0.8754 0.0209	0.9769 0.0080
5	$\Omega_i^{opt.}$ $MSE\left(T_i^{RSSS}\right)_{min.}$	0.9664 1.2571	0.8978 0.0983	0.9425 0.0395

Table 4. Comparison of proposed RSSS estimators with corresponding traditional successive sampling estimators at respective optimum conditions for Population I

Sample	PRE_1	PRE_2	PRE_3
1	160.58	104.07	500.85
2	450.94	150.08	319.16
3	216.92	546.33	810.36
4	490.12	387.25	594.13
5	378.85	983.00	625.35

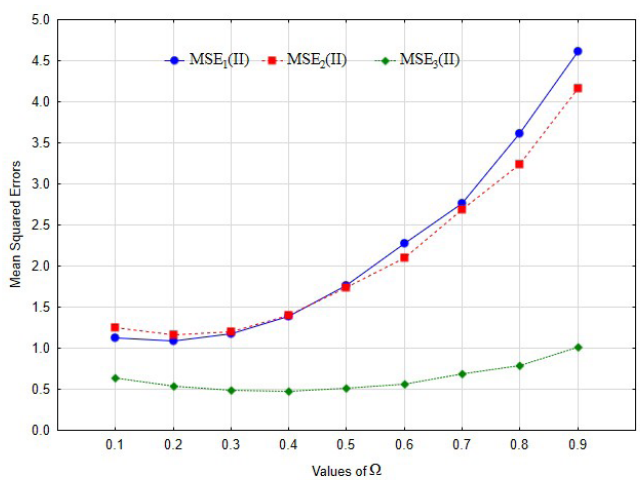

Fig. 1. Mean squared errors of the proposed RSSS estimators for Population II(a)

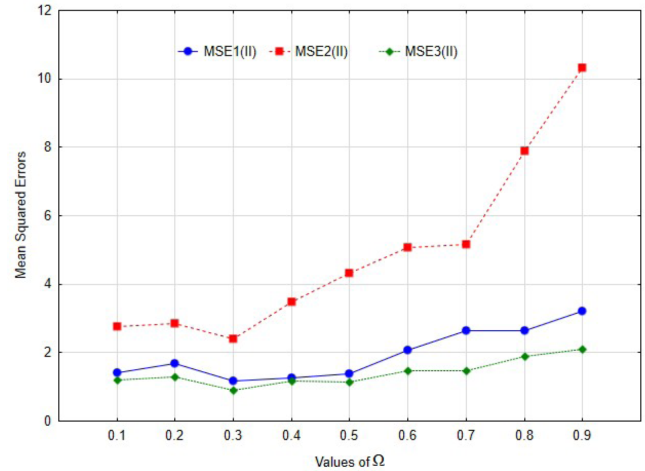

Fig. 2. Mean squared errors of the proposed RSSS estimators for Population II(b)

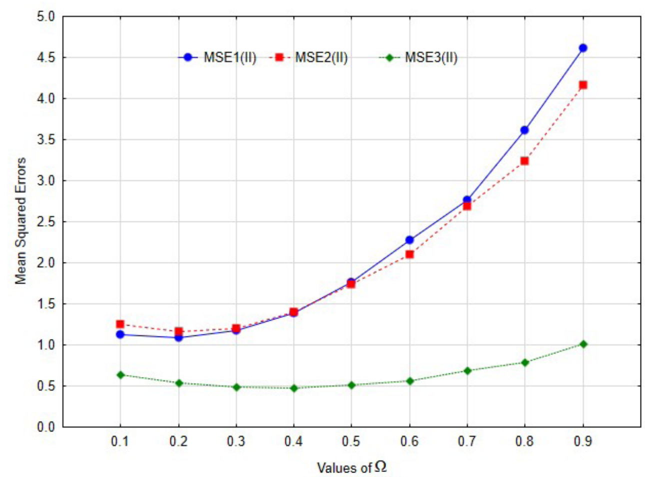

Fig. 3. Mean squared errors of the proposed RSSS estimators for Population III(a)

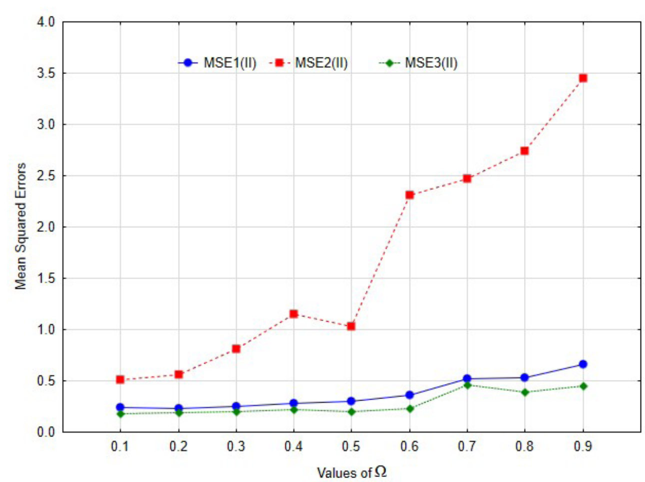

Fig. 4. Mean squared errors of the proposed RSSS estimators for Population III(b)

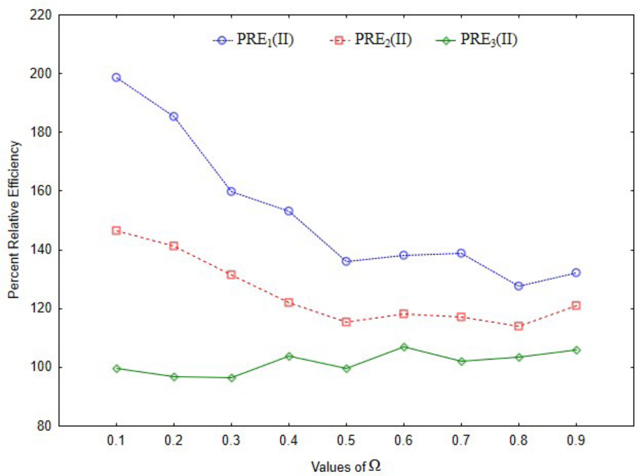

Fig. 5. Percent Relative Efficiency of proposed RSSS estimators for Population II(a)

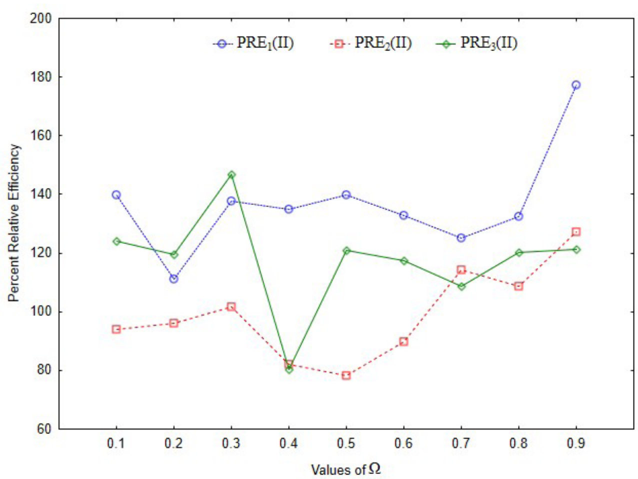

Fig. 6. Percent Relative Efficiency of proposed RSSS estimators for Population II(b)

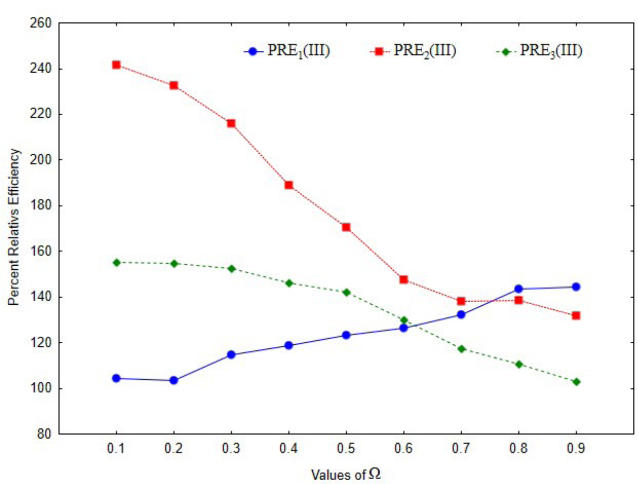

Fig. 7. Percent Relative Efficiency of proposed RSSS estimators for Population III(a)

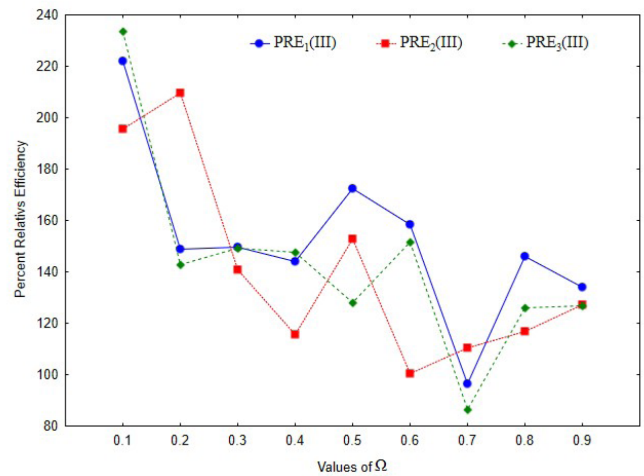

Fig. 8. Percent Relative Efficiency of proposed RSSS estimators for Population III(b)

6. DISCUSSION OF RESULTS

In order to see the behavior of the proposed estimators, detailed numerical study has been carried out using three populations. The Population I considered is a natural population; however, Population II and Population III have been artificially generated from exponential distribution and normal distributions respectively. The mean squared errors of the three RSSS estimators namely T_1^{RSSS}, T_2^{RSSS}, and T_3^{RSSS} have been computed for Population I for varying values of Ω_1, Ω_2, and Ω_3 assumed as Ω, which varies from 0.1, 0.2, …, 0.9. The result presented in Table 2 indicates that, out of the three RSSS estimators $MSE\left(T_1^{RSSS}\right) > MSE\left(T_2^{RSSS}\right) > MSE\left(T_3^{RSSS}\right)$ for all values of Ω and for all samples considered. This reflects that the estimator T_3^{RSSS} has least mean squared error among three proposed. Also, the value of mean squared errors of the proposed RSSS estimators, $MSE\left(T_i^{RSSS}\right)$ increases with increase in the value of Ω_i, $\forall\, i = 1,2$, and 3. The results presented in Table 3, shows that the optimum value of Ω_i exists for all three proposed RSSS estimators as a result their minimum mean squared errors have been computed at the respective optimum values of Ω. It is observed that $MSE\left(T_3^{RSSS}\right)_{min.} < MSE\left(T_2^{RSSS}\right)_{min.} < MSE\left(T_1^{RSSS}\right)_{min.}$.

This reflects that the among the proposed estimators, the estimator T_3^{RSSS} has least minimum mean squared error for all five different samples considered. Then, in order to check the behavior of proposed RSSS over traditional successive sampling method, where samples at different occasions may be drawn by simple random sampling, for population I corresponding similar estimators are defined as $T_1, T_2,$ and T_3 respectively. The percent relative efficiencies PRE_i for i = 1, 2, and 3 at their respective optimum conditions has been computed that are available in Table 4 for Population I. It is observed that the value of $PRE_i > 100, i = 1,2$, and 3, this shows that all the three proposed RSSS estimators are better than the corresponding non RSSS estimators for Population I.

In order to further confirm the performance of proposed RSSS estimators, two artificial populations have been generated from exponential and normal distribution named as Population II and Population III respectively. The simulation process has been carried out on both the populations with 10,000 iterations and the results are presented in Figure 1 - Figure 4. Similar performance has been observed in simulation for Population II and Population III as observed for the Population I. The estimator T_3^{RSSS} performs better than the estimators T_1^{RSSS} and T_2^{RSSS}. By simulation process, the percent relative efficiencies of RSSS estimators with the corresponding non RSSS successive sampling estimators has been evaluated as $PRE_i(II)$ and $PRE_i(III)$ for i = 1, 2, and 3 for the Population II and Population III respectively and the results are vindicated in Figure 5 - Figure 8 respectively. In Figure 5 - Figure 8, it is further observed that the percent relative efficiencies for all values of Ω are more than 100, which reflects that the proposed RSSS estimators are better than the corresponding classical successive sampling estimators. It is to be noted that the estimator T_3^{RSSS} is compared with Singh and Priyanka (2008), which is the estimator T_3 but non ranked set sampling case. It is observed that T_3^{RSSS} is better than Singh and Priyanka (2008) estimator.

7. CONCLUSION

In the present study, ranked set sampling (RSS) has been applied in conjunction with successive sampling, leading to the proposed RSSS. Three RSSS estimators were proposed, and their properties were discussed in detail. For the assessment of the practical applicability and analyze the performances of the RSSS estimators, three populations were explored, including the use of a real dataset. The findings support the utility of RSSS in sampling strategies, contributing to efficient estimation methods. Overall, the study demonstrates the potential of RSSS in enhancing estimation accuracy in various population settings. The estimator T_1^{RSSS} does not utilizes additional auxiliary information, however, T_2^{RSSS} and T_3^{RSSS} leverage additional auxiliary information at the estimation stage, leading to better performance. Among the proposed estimators, T_3^{RSSS} performs the best. The proposed estimators have also been compared with the similar corresponding non RSSS estimators including Singh and Priyanka (2008). It is observed the best performing RSSS estimator T_3^{RSSS} is better than Singh and Priyanka (2008), which is non-RSS successive sampling estimators. Comparisons with similar non-RSSS estimator by Singh and Priyanka (2008) show that the best-performing RSSS estimator, T_3^{RSSS} out performs its counterparts. Thus, the proposed RSSS framework is concluded to be more efficient than the classical successive sampling framework. It is recommended that further studies may be conducted in this direction and the proposed estimators may be applied practically where feasible.

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