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Some Series of Cyclic Partially Balanced Designs

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SUMMARY

Some methods of constructions of cyclic balanced incomplete block (BIB) designs, group divisible designs and rectangular designs are described. As a particular case, cyclic solution of the BIB designwith parameters: $v = 45, r = 11, k = 5, b = 99, \lambda = 1$, and listed as H51 in the table of Hall (1998), is obtained.The present solution is different from the earlier solutions reported in Dey (1986) and Hall (1998).

Keywords: Cyclic solution; Balanced incomplete block design; Group divisible design; Dihedral group; Galois field.

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1. INTRODUCTION

A balanced incomplete block (BIB) design is an arrangement of v treatments into b blocks such that

(i) Every block contains $k(<v)$ distinct treatments;

(ii) Every treatment is replicated r times and any pair of distinct treatments occurs together in exactly λ blocks.

The positive integers: v, b, r, k, λ are known as parameters of the BIB design and they are connected by the relations: $bk = vr, r(k-1) = \lambda(v-1), b = \dfrac{\lambda v(v-1)}{k(k-1)}$.

Let $v = mn$ treatments be arranged in $m \times n$ array A in the natural order as follows:

$$A = \begin{matrix} 1 & 2 & 3 & \cdots & n \\ n+1 & n+2 & n+3 & \cdots & 2n \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ (m-1)n+1 & (m-1)n+2 & (m-1)n+3 & \cdots & mn \end{matrix}.$$

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Then a rectangular design (RD) is an arrangement of the $v = mn$ treatments into b blocks, each block containing $k(<v)$ treatments, such that

(i) Every treatment occurs at most once in a block;

(ii) Every treatment is replicated r times;

(iii) Every pair of treatments, which are in the same row (column) of the array A, occurs together in $\lambda_1(\lambda_2)$ blocks, respectively, and in λ_3 blocks otherwise.

The non–negative integers: $v = mn, b, r, k, \lambda_1$, λ_2 and λ_3 are referred to as parameters of the RD and they are connected by the relations: $bk = vr$, $(n-1)\lambda_1 + (m-1)\lambda_2 + (n-1)(m-1)\lambda_3 = r(k-1)$.

An RD with $v = mn$ treatments reduces to a *group divisible* (*GD*) *design* if $\lambda_2 = \lambda_3$ (by combining the second and the third associate classes) with the defining array of the GD design has m rows and n columns or $\lambda_1 = \lambda_3$ (by combining the first and the third associate classes) with the defining array of the GD design has n rows and m columns [see Saurabh (2024)].

Further, the GD design is singular (*S*) if $r-\lambda_1=0$; it is semi–regular (*SR*) if $r-\lambda_1>0$ and $rk-v\lambda_2=0$ and for $r-\lambda_1>0$ and $rk-v\lambda_2>0$, it is regular (*R*)[see Clatworthy (1973)].

The definition of cyclic design may be found in Dey (1986). Here, some series of cyclic BIB designs, GD designs and RDs are described. As a particular case, cyclic solution of the BIB design with parameters: $v=45, r=11, k=5, b=99, \lambda=1$, and listed as H51 in the table of Hall (1998), is obtained by juxtaposing the solutions of two suitable GD designs. Dey (1986) and Hall (1998) also reported a cyclic solution of this design but the present solution is different from the earlier solutions (see Table 1). A series of cyclic RGD designs is obtained using dihedral groups whereas cyclic RDs are obtained using cyclic BIB designs. R*X* numbers used here are from Clatworthy (1973).

2. RECENT DEVELOPMENTS IN PBIB DESIGNS

There is an extensive literature on partially balanced incomplete block (PBIB) designs. A detailed survey on research done in India during the period 1938–2012 in the field of experimental designs can be found in Dey and Mukerjee (2012). RDs and GD designs are 3–associate class and 2–associate class PBIB designs respectively.

Recently Singh and Saurabh (2023) classified RDs into following types: singular, semi–regular, regular, Latin semi–regular, Latin regular and obtained some series of such designs. GD designs have been generalized to GD$t-$designs. Generalized GD $t-$designs are further generalization of GD designs which include both generalized $t-$designs and GD$t-$designs [Liu *et al.* (2023)]. Wang *et al.* (2026) introduced generalized GD covering designs, which unify the classes of generalized covering designs, GD covering designs and another classes of combinatorial designs.

Nested RDs and GD designs have been studied by Longyear (1986), Satpati and Prasad (2004) and Prasad (2019), among others. Recently Mandal and Gupta (2023) applied linear integer programming in design of experiments and sample surveys. A method of constructing PBIB(3) designs based on polygonal association scheme is described in Jaggi *et al.* (2023). A brief literature survey related to cyclic designs and their applications in statistics and information theory are given in Section 4.

3. THE METHODS

3.1 Cyclic BIB designs using GD designs

Let 'm' groups of a GD design with parameters: $v=mk, b=mr, r, k, \lambda_1=0, \lambda_2=1, m, n=k$ be treated as blocks where: $r(k-1)=k(m-1)$. Then annexing these 'm' blocks to 'b' blocks of the GD design, we obtain:

Proposition 1: The existence of a GD design with parameters: $v=mk, b=mr, r, k, \lambda_1=0, \lambda_2=1, m, n=k$ satisfying $r(k-1)=k(m-1)$ implies the existence of a BIB design with parameters: $v'=mk,\ b'=m(r+1),\ r'=r+1,\ k'=k,\ \lambda=1.$

Further if the source GD design is cyclic then the resulting BIB design is also cyclic.

Table 1. Cyclic Solution

No.	Initial blocks	Source
1.	$(1_1,2_1,x_3,2x_3,0_2);((2x+1)_1,(x+2)_1,(2x+2)_3,(x+1)_3,0_2);$ $(1_2,2_2,x_4,2x_4,0_3);((2x+1)_2,(x+2)_2,(2x+2)_4,(x+1)_4,0_3);$ $(1_3,2_3,x_5,2x_5,0_4);((2x+1)_3,(x+2)_3,(2x+2)_5,(x+1)_5,0_4);$ $(1_4,2_4,x_1,2x_1,0_5);((2x+1)_4,(x+2)_4,(2x+2)_1,(x+1)_1,0_5);$ $(1_5,2_5,x_2,2x_2,0_1);((2x+1)_5,(x+2)_5,(2x+2)_2,(x+1)_2,0_1);$ $(0_1,0_2,0_3,0_4,0_5)$ where x satisfies minimum function x^2+x+2 over a Galois field $GF(9)$.	Dey (1986)
2.	Treatments: $(x,y,z)(mod(3,3,5))$; Initial blocks: $[(0,1,0);(0,2,0);(1,0,2);(2,0,2);(0,0,1)](mod(3,3,5))$; $[(2,1,0);(1,2,0);(2,2,2);(1,1,2);(0,0,1)](mod(3,3,5))$; $[(0,0,0);(0,0,1);(0,0,2);(0,0,3);(0,0,4)](mod(3,3,-))$	Hall (1998)
3.	$A:[(1,3,11,23,26)_9;(2,3,10,22,25)_9;(1,2,12,24,27)_9;$ $(5,6,19,13,25)_9;(4,6,14,20,26)_9;(4,5,15,21,27)_9;$ $(8,9,16,19,22)_9;(9,7,17,20,23)_9;(7,8,18,21,24)_9;$ $(8,6,10,23,27)_9;(4,9,11,24,25)_9;(5,7,12,22,26)_9;$ $(2,9,13,21,26)_9;(3,7,14,19,27)_9;(1,8,15,20,25)_9;$ $(3,5,16,20,24)_9;(2,4,18,19,23)_9;(1,6,17,21,22)_9]$ $(mod 45)$ and $B:[(1,10,19,28,37)_1](mod 45)$	Table 2

Example 1: A cyclic solution of BIB design with parameters: $v=45,\ r=11,\ k=5,\ b=99,\ \lambda=1$ using the above proposition is presented below, where $(\theta_1,\theta_2,\ldots,\theta_k)_p$ are the blocks obtained by adding: $p, 2p, \ldots$ successively to the initial block $(\theta_1,\theta_2,\ldots,\theta_k)\, mod v.$

The initial blocks in A when developed under $mod\, 45$ represent a RGD design(R163) with parameters [see Clatworthy (1973)]: $v=45,\ b=90,\ r=10,\ k=5,\ \lambda_1=0,\ \lambda_2=1,\ m=9,\ n=5$ whereas the initial blocks in B when developed under $mod\, 45$ represent a singular GD design with parameters: $v=45, b=9, r=1, k=5, \lambda_1=1, \lambda_2=0, m=9, n=5$ whose blocks represent groups of the GD design.

Juxtaposing these solutions, we obtain a cyclic solution of the above BIB design whose blocks are given below in Table 2:

Table 2. Cyclic solution of the BIB design H51

$(1,3,11,23,26)_9$	$(2,3,10,22,25)_9$	$(1,2,12,24,27)_9$	$(5,6,19,13,25)_9$
(1, 3, 11, 23, 26) (10, 12, 20, 32, 35) (19, 21, 29, 41, 44) (28, 30, 38, 5, 8) (37, 39, 2, 14, 17)	(2, 3, 10, 22, 25) (11, 12,19, 31, 34) (20, 21, 28, 40, 43) (29, 30, 37, 4, 7) (38, 39, 1, 13, 16)	(1, 2, 12, 24, 27) (10, 11, 21, 33, 36) (19, 20, 30, 42, 45) (28, 29, 39, 6, 9) (37, 38, 3, 15, 18)	(5, 6, 19, 13, 25) (14, 15, 28, 22, 34) (23, 24, 37, 31, 43) (32, 33, 1, 40,7) (41, 42, 10, 4, 16)
$(4,6,14,20,26)_9$	$(4,5,15,21,27)_9$	$(8,9,16,19,22)_9$	$(9,7,17,20,23)_9$
(4, 6, 14, 20, 26) (13, 15, 23, 29, 35) (22, 24, 32, 38, 44) (31, 33, 41, 2, 8) (40, 42, 5, 11, 17)	(4, 5, 15, 21, 27) (13, 14, 24, 30, 36) (22, 23, 33, 39, 45) (31, 32, 42, 3, 9) (40, 41, 6, 12, 18)	(8, 9, 16, 19, 22) (17, 18, 25, 28, 31) (26, 27, 34, 37, 40) (35, 36, 43, 1, 4) (44, 45, 7, 10, 13)	(9, 7, 17, 20, 23) (18, 16, 26, 29, 32) (27, 25, 35, 38, 41) (36, 34, 44, 2, 5) (45, 43, 8, 11, 14)
$(7,8,18,21,24)_9$	$(8,6,10,23,27)_9$	$(4,9,11,24,25)_9$	$(5,7,12,22,26)_9$
(7, 8, 18, 21, 24) (16, 17, 27, 30, 33) (25, 26, 36, 39, 42) (34, 35, 45, 3, 6) (43, 44, 9, 12, 15)	(8, 6, 10, 23, 27) (17, 15, 19, 32, 36) (26, 24, 28, 41, 45) (35, 33, 37, 5, 9) (44, 42, 1, 14, 18)	(4, 9, 11, 24, 25) (13, 18, 20, 33, 34) (22, 27, 29, 42, 43) (31, 36, 38, 6, 7) (40, 45, 2, 15, 16)	(5, 7, 12, 22, 26) (14, 16, 21, 31, 35) (23, 25, 30, 40, 44) (32, 34, 39, 4, 8) (41, 43, 3, 13, 17)
$(2,9,13,21,26)_9$	$(3,7,14,19,27)_9$	$(1,8,15,20,25)_9$	$(3,5,16,20,24)_9$
(2, 9, 13, 21, 26) (11, 18, 22, 30, 35) (20, 27, 31, 39, 44) (29, 36, 40, 3, 8) (38, 45, 4, 12, 17)	(3, 7, 14, 19, 27) (12, 16, 23, 28, 36) (21, 25, 32, 37, 45) (30, 34, 41, 1, 9) (39, 43, 5, 10, 18)	(1, 8, 15, 20, 25) (10, 17, 24, 29, 34) (19, 26, 33, 38, 43) (28, 35, 42, 2, 7) (37, 44, 6, 11, 16)	(3, 5, 16, 20, 24) (12, 14, 25, 29, 33) (21, 23, 34, 38, 42) (30, 32, 43, 2, 6) (39, 41, 7, 11, 15)
$(2,4,18,19,23)_9$	$(1,6,17,21,22)_9$	$(1,10,19,28,37)_1$	
(2, 4, 18, 19, 23) (11, 13, 27, 28, 32) (20, 22, 36, 37, 41) (29, 31, 45, 1, 5) (38, 40, 9, 10, 14)	(1, 6, 17, 21, 22) (10, 15, 26, 30, 31) (19, 24, 35, 39, 40) (28, 33, 44, 3, 4) (37, 42, 8, 12, 13)	(1, 10, 19, 28, 37); (6, 15, 24, 33, 42) (2, 11, 20, 29,38); (7, 16, 25, 34, 43) (3, 12, 21, 30, 39); (8, 17, 26, 35, 44) (4, 13, 22, 31, 40); (9, 18, 27, 36, 45) (5, 14, 23, 32, 41)	

Remark 1: Some cyclic solutions of GD designs which satisfy the conditions of Proposition 1 are given in Clatworthy (1973) which can be used to obtain a cyclic solution of the corresponding BIB designs.

3. 2. Cyclic RGD designs using dihedral groups

A dihedral group (D_n) is the group of symmetries of a regular $n-$ gon. It contains $2n$ elemnts and it is also represented using relations and generators as follows:

$$D_n = r,\ s|r^n = s^2 = e,$$

$$sr = r^{n-1}s = \{e, r, r^2, \ldots, r^{n-1}, s, sr, sr^2, \ldots, sr^{n-1}\},$$

where r represents a rotation and s represents a reflection. We label the vertices of the $n-$ gon starting with 1 and moving clockwise to $2,3,\ldots,n$. Then r is the rotation of the $n-$ gon by $2\pi/n$ radians and s is a reflection across the line which connects 1 to the center of the $n-$ gon. Some series of GD designs using dihedral groups may be found in Saurabh and Sinha (2023).

Proposition 2: A cyclic solution of RGD design with parameters: $v=b=2n,\ r=k=n+1,\ \lambda_1=n,\ \lambda_2=2,\ m=2,\ n$ always exists. (1)

The initial block $\mathcal{B}_0 = r \cup \{s\} = \{e, r,\ r^2,\ \cdots, r^{n-1}\} \cup \{s\} = \{e, r,\ r^2,\ \cdots, r^{n-1},\ s\} \subset D_n$, when multiplied by $'2n'$ group treatments of D_n, generates blocks of the RGD design with parameters (1) i. e. the set $S = \{s^i r^j \mathcal{B}_0 : 0 \le i, j \le n-1\}$ yields $2n$ blocks.

The arrangement of $v=2n$ treatments in $2\times n$ array is as follows: $\begin{matrix} e & r & r^2 & \cdots & r^{n-1} \\ s & sr & sr^2 & \cdots & sr^{n-1} \end{matrix}.$

Example 2: Consider a RGD design R94 with parameters: $v=b=6,\ r=k=4,\ \lambda_1=3,\ \lambda_2=2, m=2,\ n=3$ and a dihedral group:

$$D_3 = \langle r, s|r^3 = s^2 = e, sr = r^2 s\rangle = \{e, r, r^2, s, sr, sr^2\}.$$

The initial block $\mathcal{B}_0 = r \cup \{s\} = \{e, r,\ r^2, s\}$ generates the blocks of R94 as follows:

$$\mathcal{B}_0 = \{e, r, r^2, s\}, \mathcal{B}_2 = r\mathcal{B}_0 = \{e, r, r^2, rs = sr^2\},$$

$$\mathcal{B}_3 = r^2\mathcal{B} = \{e, r, r^2, r^2 s = sr\}, \quad \mathcal{B}_4 = s\mathcal{B}_0 = \{e, s, sr, sr^2\},$$

$$\mathcal{B}_5 = sr\mathcal{B}_0 = \{s, sr, sr^2, srs = r^2\},$$

$$\mathcal{B}_6 = sr^2\mathcal{B}_0 = \{s, sr, sr^2, sr^2 s = r\}.$$

The arrangement of $v = 6$ treatments in 2×3 array is as follows: $\begin{matrix} e & r & r^2 \\ s & sr & sr^2 \end{matrix}$.

Remark 2: For $n = 4,5,6,7,8$ and 9 in Proposition 2, we obtain cyclic solutions of RGD designs: R133, R166, R173, R187, R195 and R206 respectively. Although cyclic solutions of these RGD designs are reported in Clatworthy (1973), here the solutions are obtained using algebraic approach.

Remark 3: A generalized cyclic solution of the RGD design R58: $v = 8, b = 24, r = 9, k = 3, \lambda_1 = 2, \lambda_2 = 3, m = 2, n = 4$ is obtained by adding 0, 2, 4 and 6 successively to the initial blocks: $(1,3,4), (1,6,8), (1,7,8), (1,4,6), (1,5,6), (1,4,8)$ which is not mentioned in Clatworthy (1973).

3. 3. Cyclic RDs using BIB designs

The following series of RDs is a particular case of Theorem 7 of Singh and Saurabh (2023). Here, it is shown that the design is cyclic.

Proposition 3: Let $4t - 1$ be a prime or prime power. Then there existsa cyclic RD with parameters: $v = b = 5(4t-1), r = k = 8t - 3, \lambda_1 = 4(t-1), \lambda_2 = 2t - 1, \lambda_3 = 3t - 1, m = 5, n = 4t - 1.$ (2)

Proof: It is well known (see Dey, 1986) that there exists a cyclic BIB design with parameters: $v = b = 4t - 1, r = k = 2t - 1, \lambda = t - 1$ for $4t - 1$ being a prime or prime power, whose initial block is: $(1, x^2, x^4, \ldots, x^{4t-4})$ or $(x, x^3, x^5, \ldots, x^{4t-3})$ where x is a primitive element of Galois (finite) field of order $4t - 1$.

The above RD is obtained by developing the following five initial blocks mod $(4t-1)$:

$$\big(A_0, B_1, B_{x^2}, B_{x^4}, \ldots, B_{x^{4t-4}}, C_1, C_{x^2}, C_{x^4}, \ldots, C_{x^{4t-4}}, D_x, D_{x^3}, D_{x^5}, \ldots, D_{x^{4t-3}}, E_x, E_{x^3}, E_{x^5}, \ldots, E_{x^{4t-3}}\big),$$

$$\big(B_0, C_1, C_{x^2}, C_{x^4}, \ldots, C_{x^{4t-4}}, D_1, D_{x^2}, D_{x^4}, \ldots, D_{x^{4t-4}}, E_x, E_{x^3}, E_{x^5}, \ldots, E_{x^{4t-3}}, A_x, A_{x^3}, A_{x^5}, \ldots, A_{x^{4t-3}}\big),$$

$$\big(C_0, D_1, D_{x^2}, D_{x^4}, \ldots, D_{x^{4t-4}}, E_1, E_{x^2}, E_{x^4}, \ldots, E_{x^{4t-4}}, A_x, A_{x^3}, A_{x^5}, \ldots, A_{x^{4t-3}}, B_x, B_{x^3}, B_{x^5}, \ldots, B_{x^{4t-3}}\big),$$

$$\big(D_0, E_1, E_{x^2}, E_{x^4}, \ldots, E_{x^{4t-4}}, A_1, A_{x^2}, A_{x^4}, \ldots, A_{x^{4t-4}}, B_x, B_{x^3}, B_{x^5}, \ldots, B_{x^{4t-3}}, C_x, C_{x^3}, C_{x^5}, \ldots, C_{x^{4t-3}}\big),$$

$$\big(E_0, A_1, A_{x^2}, A_{x^4}, \ldots, A_{x^{4t-4}}, B_1, B_{x^2}, B_{x^4}, \ldots, B_{x^{4t-4}}, C_x, C_{x^3}, C_{x^5}, \ldots, C_{x^{4t-3}}, D_x, D_{x^3}, D_{x^5}, \ldots, D_{x^{4t-3}}\big).$$

This design can also be represented as:

$$N = \begin{pmatrix} I & M & M & M^T & M^T \\ M & I & M^T & M^T & M \\ M & M^T & I & M & M^T \\ M^T & M^T & M & I & M \\ M^T & M & M^T & M & I \end{pmatrix},$$

where M is the incidence matrix of the above BIB design and I is the identity matrix of order $4t - 1$.

Example 3: The following initial blocks: $(A_0, B_1, C_1, D_2, E_2), (B_0, C_1, D_1, E_2, A_2), (C_0, D_1, E_1, A_2, B_2), (D_0, E_1, A_1, B_2, C_2), (E_0, A_1, B_1, C_2, D_2)$

when developed under mod 3 represent a cyclic RD with parameters: $v = b = 15, r = k = 5, \lambda_1 = 0, \lambda_2 = 1, \lambda_3 = 2, m = 5, n = 3.$

4. DISCUSSION AND CONCLUSION

Here, some series of cyclic BIB designs, GD designs and RDs are obtained. As a particular case, we obtain a cyclic solution of the BIB design with parameters: $v = 45,\ r = 11,\ k = 5,\ b = 99,\ \lambda = 1.$ The present solution is different from the earlier solutions reported by Dey (1986) and Hall (1998). Further, two series of cyclic RGD designs and RDs are obtained using dihedral groups and cyclic BIB designs respectively.

RDs are useful as factorial experiments, which have balance as well as orthogonality [see Gupta and Mukerjee (1989)]. In addition, RDs having $\lambda_3 > \lambda_1, \lambda_2$

when used as the $m \times n$ complete confounded experiments, the loss of information on the main effects is small [see Suen (1989)]. An application of RDs and GD designs in cluster sampling is described in Raghavarao and Singh (1975). GD designs are useful in group testing experiments and intercropping experiments [see Raghavarao and Padgett (2005)]. RDs and GD designs are also applicable in agricultural field trials.

A recent survey on cyclic designs is available in Saurabh and Sinha (2022) and some tables of cyclic designs with $r = k$ are given in Lamacraft and Csiro (1982). Some constructions and more details on cyclic designs and their efficiencies are provided in Saha (1975) and Rizvi *et al.* (1976). Some methods of constructions of cyclic GD designs and RDs are described in Freeman (1976) and Sinha *et al.* (2002) respectively.

A primary benefit of the cyclic design is that field experimenters only need to utilize a single initial block of treatments. The remaining blocks are easily derived using the initial block. Cyclic designs are useful when there are too many treatments (e.g., crop varieties, drug dosages) to test in a single block. They allow for efficient comparisons with equal precision while minimizing required resources. Cyclic designs are also useful as partial confounding in factorial experiments and in the constructions of low–density parity–check (LDPC) codes [see John (1973), Dean and Lewis (1980), Nigam *et al.* (1988), Vasic and Milenkovic (2004) and Amirzade *et al.* (2025)].

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