https://doi.org/10.56093/JISAS.V80I3.3
Available online at https://isas.live/newsite
JOURNAL OF THE INDIAN SOCIETY OF AGRICULTURAL STATISTICS 80(3) 2026 419–426

Evaluating Accuracy in Mean Estimation with Post-Stratified Sampling

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SUMMARY

Accurately estimating the population mean under post-stratification can be difficult particularly in presence of auxiliary information. This study introduces t_c a separate ratio–cum–product estimator that uses two auxiliary variables, x and z to improve estimation precision. First-order approximations are used to calculate bias and mean squared error and comparisons with conventional ratio, product, and Lone and Tailor (2014, 2015) estimators. Empirical simulations confirm that t_c consistently reduces MSE and enhances efficiency, demonstrating its practical value for statistical applications in post-stratified surveys.

Keywords: MSE; Bias; Post Stratification.

1. INTRODUCTION

In conventional stratified random sampling, it is usually believed that a sampling frame and the stratum sizes are easily accessible. However, in many real-world applications, this assumption does not hold true. For instance, in household surveys, the total number of families in a region may be known, but their exact distribution across various localities or strata may not be clearly documented or up to date. In such situations, post-stratification serves as an effective alternative. This method first uses SRS to select a sample of a predefined size. Once the sample has been drawn, the selected units are then classified into relevant strata according to an auxiliary factor or characteristic. Even in situations where an accurate sampling frame for each stratum is not known beforehand, this method enables researchers to take the advantages of stratification.

Several researchers have developed separate estimators in post-stratification using supplementary data to increase estimation effectiveness. Ige and Tripathi (1989) developed ratio as well as product estimators for estimation of population mean using post-stratification. Srivenkataramana (1980) proposed transformation-based dual estimators which inspired further developments in this area. Several notable investigations such as Holt and Smith (1979), Jagers (1968), Agrawal and Panday (1993), Singh and Vishwakarma (2008), Tailor and Lone (2014), Lone and Tailor (2014, 2015) and Lone *et al.* (2016) contributed in research related to post-stratification. Hilal *et al.* (2020), Verma *et al.* (2023), Choudhury (2025), Yadav and Kumar (2025), Kumar *et al.* (2026) and Singh *et al.* (2026) also contributed in estimation of population mean using auxiliary information.

Extending this line of research, the current work proposes a modified separate ratio–cum–product estimator under post-stratification that utilizes information from two auxiliary variables, aiming to achieve higher efficiency relative to existing methods.

Let Y be the study variable and X is the auxiliary variable which ispositively correlated with the study variable and Z is the auxiliary variable which has

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negative correlation with the study variable. Let y_{hi} be the observation on i^{th} unit of h^{th} stratum for study variable Y and x_{hi} and z_{hi} be the observation on i^{th} unit of h^{th} stratum for auxiliary variables X and Z respectively then

$\bar{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$: h^{th} stratum mean of auxiliary variable x

$\bar{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{hi}$: h^{th} stratum mean of study variable y

$\bar{Z}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$: h^{th} stratum mean (auxiliary variable z)

$\bar{X} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} x_{hi} = \sum_{h=1}^{L} W_h \bar{X}_h$: Population mean (auxiliary variable x)

$\bar{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi} = \sum_{h=1}^{L} W_h \bar{Y}_h$: Population mean (study variable y)

$\bar{Z} = \sum_{h=1}^{L} W_h \bar{Z}_h$: Population mean (auxiliary variable z)

$\bar{x}_{st} = \sum_{h=1}^{L} W_h \bar{x}_h$: Population mean (auxiliary variable x) in stratified sampling

$\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h$: Population mean (study variable y) in stratified sampling

$\bar{z}_{st} = \sum_{h=1}^{L} W_h \bar{z}_h$: Population mean (auxiliary variable z) in stratified sampling

$S_{yh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(y_{hi} - \bar{Y}_h\right)^2$: h^{th} stratum population mean square (study variable y)

$S_{xh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(x_{hi} - \bar{X}_h\right)^2$: h^{th} stratum population mean square (auxiliary variable x)

$S_{zh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(z_{hi} - \bar{Z}_h\right)^2$: h^{th} stratum population mean square (auxiliary variable z)

$W_h = \frac{N_h}{N}$: h^{th} stratum weight

From the population U a random sample of size n is drawn by SRSWOR. Let n_h be the sample size selected from h^{th} stratum such that $\sum_{h=1}^{L} n_h = n$. Now we assume that n is sufficiently large such that possibility of n_h being zero is negligible.

The estimator of population mean in post-stratificationis given by

$$\bar{y}_{ps} = \sum_{h=1}^{L} W_h \bar{y}_h$$

The variance of estimator $\bar{y}_{ps}$ is given by

$$\mathrm{Var}(\bar{y}_{ps}) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h S_{yh}^2 + \frac{1}{n^2}\sum_{h=1}^{L}(1 - W_h) S_{yh}^2 \quad (1.1)$$

The traditional separate ratio & product estimators are

$$t_1 = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{X}_h}{\bar{x}_h}\right) \quad (1.2)$$

$$t_2 = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{z}_h}{\bar{Z}_h} \right) \tag{1.3}$$

The MSE of t_1 and t_2 are given by

$$MSE(t_1) = \left(\frac{1}{n} - \frac{1}{N} \right) \sum_{h=1}^{L} W_h \left[S_{yh}^2 + R_{1h}^2 S_{xh}^2 - 2R_{1h} S_{yxh} \right] \tag{1.4}$$

$$MSE(t_2) = \left(\frac{1}{n} - \frac{1}{N} \right) \sum_{h=1}^{L} W_h \left[S_{yh}^2 + R_{2h}^2 S_{zh}^2 + 2R_{2h} S_{yzh} \right] \tag{1.5}$$

where $R_{1h} = \frac{\bar{Y}_h}{\bar{X}_h}$ and $R_{2h} = \frac{\bar{Y}_h}{\bar{Z}_h}$

On the lines of Srivenkataramana (1980) and Bandhyopadhyay (1980), Lone and Tailor (2015) defined dual to separate ratio type estimator as

$$t_3 = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{x}_h^*}{\bar{X}_h} \right), \tag{1.6}$$

$$MSE(t_3) = \left(\frac{1}{n} - \frac{1}{N} \right) \left[\sum_{h=1}^{L} W_h S_{yh}^2 + \sum_{h=1}^{L} W_h R_{1h}^2 a_h^2 S_{xh}^2 - 2 \sum_{h=1}^{L} W_h a_h R_{1h} S_{yxh} \right] \tag{1.7}$$

Where $\bar{x}_h^* = \frac{N_h \bar{X}_h - n_h \bar{x}_h}{N_h - n_h}$ and $a_h = \frac{n_h}{N_h - n_h}$

The ratio type exponential estimator developed by Chouhan (2012) can be expressed as

$$t_4 = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{X}_h - \bar{x}_h}{\bar{X}_h + \bar{x}_h} \right). \tag{1.8}$$

$$MSE(t_4) = \left(\frac{1}{n} - \frac{1}{N} \right) \sum_{h=1}^{L} W_h \left(S_{yh}^2 + \frac{1}{4} R_{1h}^2 S_{xh}^2 - R_{1h} S_{yxh} \right). \tag{1.9}$$

In case of post-stratification Lone and Tailor (2014) defined the dual to separate ratio type exponential estimator as

$$t_5 = \sum_{h=1}^{L} W_h \bar{y}_h \, exp\left(\frac{\bar{x}_h^* - \bar{X}_h}{\bar{x}_h^* + \bar{X}_h} \right) \tag{1.10}$$

$$MSE(t_5) = \left(\frac{1}{n} - \frac{1}{N} \right) \left[\sum_{h=1}^{L} W_h S_{yh}^2 + \frac{1}{4} \sum_{h=1}^{L} W_h a_h^2 R_{1h}^2 S_{xh}^2 - \sum_{h=1}^{L} W_h a_h R_{1h} S_{yxh} \right] \tag{1.11}$$

Dual of separate product type estimators were proposed by Lone and Tailor (2015) is

$$t_6 = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{X}_h}{\bar{x}_h^*} \right) \tag{1.12}$$

$$MSE(t_6) = \left(\frac{1}{n} - \frac{1}{N} \right) \left[\sum_{h=1}^{L} W_h S_{yh}^2 + \sum_{h=1}^{L} W_h R_{1h}^2 a_h^2 S_{xh}^2 + 2 \sum_{h=1}^{L} W_h a_h R_{1h} S_{yxh} \right] \tag{1.13}$$

The product type exponential estimator developed by Chouhan (2012) can be expressed as

$$t_7 = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{x}_h - \bar{X}_h}{\bar{x}_h + \bar{X}_h} \right) \tag{1.14}$$

$$MSE(t_7) = \left(\frac{1}{n} - \frac{1}{N} \right) \sum_{h=1}^{L} W_h \left(S_{yh}^2 + \frac{1}{4} R_{1h}^2 S_{xh}^2 + R_{1h} S_{yxh} \right) \tag{1.15}$$

Dual to separate ratio exponential type estimator proposed by Lone and Tailor (2015) in case of post stratification is given by

$$t_8 = \sum_{h=1}^{L} W_h \bar{y}_h \, exp\left(\frac{\bar{X}_h - \bar{x}_h^*}{\bar{X}_h + \bar{x}_h^*} \right) \tag{1.16}$$

$$MSE(t_8) = \left(\frac{1}{n} - \frac{1}{N} \right) \left[\sum_{h=1}^{L} W_h S_{yh}^2 + \frac{1}{4} \sum_{h=1}^{L} W_h a_h^2 R_{1h}^2 S_{xh}^2 + \sum_{h=1}^{L} W_h a_h R_{1h} S_{yxh} \right]. \tag{1.17}$$

Where $S_{xh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(x_{hi} - \bar{X}_h \right)^2$ and

$S_{yxh} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(y_{hi} - \bar{Y}_h \right) \left(x_{hi} - \bar{X}_h \right).$

2. PROPOSED ESTIMATOR

Following Singh (1967) and Tailor *et al.* (2012) the proposed estimator isgiven by

$$t_C = \overline{y}_{ps}\left(\frac{X_u}{x_u}\right)\left(\frac{Z_v}{z_v}\right) \tag{2.1}$$

Where $X_u = \overline{X}_h + C_{xh}$, $x_u = \overline{x}_h + C_{xh}$, $Z_v = \overline{Z}_h + C_{zh}$, $z_v = \overline{z}_h + C_{zh}$, C_{xh} and C_{zh} are the CV (Coefficient of variation) of the auxiliary variables X and Z in the h^{th} stratum.

Equation (2.1) in terms of $e_{ih}'s$ after neglecting the higher order terms can be expressed as

$$t_C = \sum_{h=1}^{L} W_h\overline{Y}_h\left[1-\lambda_h e_{1h}+\lambda_h{}^2 e_{1h}{}^2-\psi_h e_{2h}+\psi_h{}^2 e_{2h}{}^2+\lambda_h\psi_h e_{1h}e_{2h}+e_{0h}-\lambda_h e_{0h}e_{1h}-\psi_h e_{0h}e_{2h}\right]$$

$$\left(t_C-\sum_{h=1}^{L} W_h\overline{Y}_h\right)=\sum_{h=1}^{L} W_h\overline{Y}_h\left[e_{0h}-\lambda_h e_{1h}-\psi_h e_{2h}-\lambda_h e_{0h}e_{1h}-\psi_h e_{0h}e_{2h}+\lambda_h{}^2 e_{1h}{}^2+\psi_h{}^2 e_{2h}{}^2+\lambda_h\psi_h e_{1h}e_{2h}\right] \tag{2.2}$$

where $\lambda_h = \dfrac{\overline{X}_h}{\overline{X}_h + C_{xh}}$ and $\psi_h = \dfrac{\overline{Z}_h}{\overline{Z}_h + C_{zh}}$.

The bias of the proposed estimator t_C after taking expectations of both sides of (2.2) is given by

$$B(t_C)=\left(\frac{1}{n}-\frac{1}{N}\right)\left[\sum_{h=1}^{L}\left\{\frac{S_{xh}^2\lambda_h^2 R_{1h}}{\overline{X}_h}+\frac{S_{zh}^2\psi_h^2 R_{2h}}{\overline{Z}_h}+\frac{\lambda_h\psi_h R_{1h}S_{xzh}}{\overline{Z}_h}-\frac{S_{xyh}}{\overline{X}_h}-\frac{\psi_h S_{yzh}}{\overline{Z}_h}\right\}\right] \tag{2.3}$$

The mean square error of the estimator t_C up to the first degree approximation as given by

$$MSE(t_C)=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L} W_h\left[S_{yh}^2+R_{1h}^2\lambda_h^2 S_{xh}^2+R_{2h}^2\psi_h^2 S_{zh}^2-2\left(R_{1h}\lambda_h S_{yxh}+R_{2h}\psi_h S_{yzh}-R_{1h}R_{2h}\lambda_h\psi_h S_{xzh}\right)\right] \tag{2.4}$$

3. EFFICIENCY COMPARISON

From the Comparisons of equations (1.1), (1.4), (1.5), (1.7), (1.9), (1.11), (1.13), (1.15), (1.17) and (2.4), it has been observed that the proposed estimator t_C is more efficient than

(i) $\overline{y}_{ps}$ if

$$\sum_{h=1}^{L} W_h\left[R_{1h}^2\lambda_h^2 S_{xh}^2+R_{2h}^2\psi_h^2 S_{zh}^2-2\left(R_{1h}\lambda_h S_{yxh}+R_{2h}\psi_h S_{yzh}-R_{1h}R_{2h}\lambda_h\psi_h S_{xzh}\right)\right]<0 \tag{3.1}$$

(ii) t_1 if

$$\sum_{h=1}^{L} W_h\left[R_{1h}^2 S_{xh}^2\left(\lambda_h^2-1\right)+R_{2h}^2\psi_h^2 S_{zh}^2-2\left(R_{1h}S_{yxh}(\lambda_h-1)+R_{2h}\psi_h S_{yzh}-R_{1h}R_{2h}\lambda_h\psi_h S_{xh}\right)\right]<0 \tag{3.2}$$

(iii) t_2 if

$$\sum_{h=1}^{L} W_h\left[R_{1h}^2\lambda_h^2 S_{xh}^2+R_{2h}^2 S_{zh}^2\left(\psi_h^2-1\right)-2\left(R_{1h}\lambda_h S_{yxh}+R_{2h}S_{yzh}\left(\psi_h+1\right)-R_{1h}R_{2h}\lambda_h\psi_h S_{xh}\right)\right]<0 \tag{3.3}$$

(iv) t_3 if

$$\sum_{h=1}^{L} W_h\left[R_{1h}^2 S_{xh}^2\left(\lambda_h^2-a_h^2\right)+R_{2h}^2\psi_h^2 S_{zh}^2-2\left(R_{1h}S_{yxh}\left(\lambda_h-a_h\right)+R_{2h}\psi_h S_{yzh}-R_{1h}R_{2h}\lambda_h\psi_h S_{xh}\right)\right]<0 \tag{3.4}$$

(v) t_4 if

$$\sum_{h=1}^{L} W_h\left[R_{1h}^2 S_{xh}^2\left(\lambda_h^2-\frac{1}{4}\right)+R_{2h}^2\psi_h^2 S_{zh}^2-2\left(R_{1h}S_{yxh}\left(\lambda_h-\frac{1}{2}\right)+R_{2h}\psi_h S_{yzh}-R_{1h}R_{2h}\lambda_h\psi_h.S_{xh}\right)\right]<0 \tag{3.5}$$

(vi) t_5 if

$$\sum_{h=1}^{L} W_h\left[R_{1h}^2 S_{xh}^2\left(\lambda_h^2-\frac{1}{4}a_h^2\right)+R_{2h}^2\psi_h^2 S_{zh}^2-2\left(R_{1h}S_{yxh}\left(\lambda_h-a_h\right)+R_{2h}\psi_h S_{yzh}-R_{2h}\lambda_h\psi_h S_{xh}\right)\right]<0 \tag{3.6}$$

(vii) t_6 if

$$\sum_{h=1}^{L} W_h\left[R_{1h}^2 S_{xh}^2\left(\lambda_h^2-a_h^2\right)+R_{2h}^2\psi_h^2 S_{zh}^2-2(R_{1h}S_{yxh}\left(\lambda_h+a_h\right)+R_{2h}\psi_h S_{yzh}-R_{1h}R_{2h}\lambda_h\psi_h S_{xh})\right]<0 \tag{3.7}$$

(viii) t_7 if ,

$$\sum_{h=1}^{L} W_h \left[R_{1h}^2 S_{xh}^2 \left(\lambda_h{}^2 - \frac{1}{4} \right) + R_{2h}^2 \psi_h^2 S_{zh}^2 - 2\left(R_{1h} S_{yxh} \left(\lambda_h + \frac{1}{2} \right) + R_{2h} \psi_h S_{yzh} - R_{1h} R_{2h} \lambda_h \psi_h S_{xh} \right) \right] < 0 \quad (3.8)$$

(ix) t_8 if

$$\sum_{h=1}^{L} W_h \left[R_{1h}^2 S_{xh}^2 \left(\lambda_h^2 - \frac{1}{4} a_h^2 \right) + R_{2h}^2 \psi_h^2 S_{zh}^2 - 2\left(R_{1h} S_{yxh} \left(\lambda_h + \frac{1}{2} a_h \right) \right) + R_{2h} \psi_h S_{yzh} - R_{1h} R_{2h} \lambda_h \psi_h S_{xh}) \right] < 0 \quad (3.9)$$

4. SIMULATION STUDY

Three simulated population data sets were used in the study to show how well the suggested estimators performed in comparison to other estimators. In the first simulated population, a total of 700 units were generated and equally divided into two strata, A and B. Using Python (Google Colab), random samples of sizes $n_1 = 100$ and $n_2 = 100$ were drawn from these strata, and the corresponding estimators were computed to obtain their Mean Squared Error (MSE) values. Similarly, a second simulated population consisting of 1800 units was generated and also divided equally into two strata, A and B. From this population, samples of sizes $n_1 = 200$ and $n_2 = 200$ were drawn, in third simulated population $n_1 = 250$ and $n_2 = 350$ and the corresponding estimators were again computed to determine their MSE values. This dual-population framework enabled in evaluation of the efficiency and robustness of proposed estimator under varying sample sizes and population conditions.

Population-I:

Constants	Stratum 1	Stratum 2
n_h	100	100
N_h	350	350
$\bar{X}_h$	50.00	60.39
$\bar{Y}_h$	99.68	112.71
$\bar{Z}_h$	153.55	158.88
S_{yh}	20.31	21.67
S_{xh}	9.56	12.16
S_{zh}	28.91	34.56
S_{yh}^2	412.57	469.56
S_{xh}^2	91.41	147.81
S_{zh}^2	837.75	1194.30
S_{yxh}	-16.63	-0.39
S_{yzh}	0.53	16.52

Population-II:

Constants	Stratum 1	Stratum 2
n_h	200	200
N_h	900	900
$\bar{X}_h$	55.22	60.08
$\bar{Y}_h$	106.40	108.54
$\bar{Z}_h$	145.11	154.70
S_{yh}	17.94	19.77
S_{xh}	11.77	14.57
S_{zh}	21.41	23.69
S_{yh}^2	321.86	390.95
S_{xh}^2	138.55	212.41
S_{zh}^2	458.32	561.13
S_{yxh}	-3.66	8.04
S_{yzh}	0.75	7.75

Population-III:

Constants	Stratum 1	Stratum 2
n_h	250	350
N_h	1400	2200
$\bar{X}_h$	51.15	67.63
$\bar{Y}_h$	93.65	99.33
$\bar{Z}_h$	141.60	165.08
S_{yh}	8.89	11.35
S_{xh}	8.37	10.71
S_{zh}	18.35	24.82
S_{yh}^2	78.98	128.84
S_{xh}^2	70.08	114.65
S_{zh}^2	336.64	616.25
S_{yxh}	43.72	53.12
S_{yzh}	-92.55	-187.70

Table 1. Conditions on proposed estimator to be more efficient

Efficiency	Conditions	Numerical Conditions		
		Population I	Population II	Population III
$MSE(t_c) < V(\bar{y}_{ps})$ if	$\sum_{h=1}^{L} W_h \left[R_{1h}^2 \lambda_h^2 S_{xh}^2 + R_{2h}^2 \psi_h^2 S_{zh}^2 - 2\left(R_{1h}\lambda_h S_{yxh} + R_{2h}\psi_h S_{yzh} - R_{1h}R_{2h}\lambda_h \psi_h S_{xzh} \right) \right] < 0$	-0.000129 < 0	-0.000063 < 0	-0.000082 < 0
$MSE(t_c) < MSE(t_1)$ if	$\sum_{h=1}^{L} W_h \left[R_{1h}^2 S_{xh}^2 \left(\lambda_h^2 - 1 \right) + R_{2h}^2 \psi_h^2 S_{zh}^2 - 2\left(R_{1h} S_{yxh} (\lambda_h - 1) + R_{2h}\psi_h S_{yzh} - R_{1h}R_{2h}\lambda_h \psi_h S_{xh} \right) \right] < 0$	-0.583936 < 0	-0.584736 < 0	-0.594736 < 0
$MSE(t_c) < MSE(t_2)$ if	$\sum_{h=1}^{L} W_h \left[R_{1h}^2 \lambda_h^2 S_{xh}^2 + R_{2h}^2 S_{zh}^2 \left(\psi_h^2 - 1 \right) - 2\left(R_{1h}\lambda_h S_{yxh} + R_{2h} S_{yzh} \left(\psi_h + 1 \right) - R_{1h}R_{2h}\ddot{u}_h \psi_h S_{xh} \right) \right] < 0$	-2.724843 < 0	-2.734843 < 0	-2.845731 < 0
$MSE(t_c) < MSE(t_3)$ if	$\sum_{h=1}^{L} W_h \left[R_{1h}^2 S_{xh}^2 \left(\lambda_h^2 - a_h^2 \right) + R_{2h}^2 \psi_h^2 S_{zh}^2 - 2\left(R_{1h} S_{yxh} \left(\lambda_h - a_h \right) + R_{2h}\psi_h S_{yzh} - R_{1h}R_{2h}\lambda_h \psi_h S_{xh} \right) \right] < 0$	-0.007509 < 0	-0.007509 < 0	-0.008214 < 0
$MSE(t_c) < MSE(t_4)$ if	$\sum_{h=1}^{L} W_h \left[R_{1h}^2 S_{xh}^2 \left(\lambda_h^2 - \frac{1}{4} \right) + R_{2h}^2 \psi_h^2 S_{zh}^2 - 2\left(R_{1h} S_{yxh} \left(\lambda_h - \frac{1}{2} \right) + R_{2h}\psi_h S_{yzh} - R_{1h}R_{2h}\lambda_h \psi_h . S_{xh} \right) \right] < 0$	-0.052554 < 0	-0.053564 < 0	-0.054821 < 0
$MSE(t_c) < MSE(t_5)$ if	$\sum_{h=1}^{L} W_h \left[R_{1h}^2 S_{xh}^2 \left(\lambda_h^2 - \frac{1}{4} a_h^2 \right) + R_{2h}^2 \psi_h^2 S_{zh}^2 - 2\left(R_{1h} S_{yxh} \left(\lambda_h - a_h \right) + R_{2h}\psi_h S_{yzh} - R_{2h}\lambda_h \psi_h \ S_{xh} \right) \right] < 0$	-0.006836 < 0	-0.007936 < 0	-0.007614 < 0
$MSE(t_c) < MSE(t_6)$ if	$\sum_{h=1}^{L} W_h \left[R_{1h}^2 S_{xh}^2 \left(\lambda_h^2 - a_h^2 \right) + R_{2h}^2 \psi_h^2 S_{zh}^2 - 2R_{1h} S_{yxh} \left(\lambda_h - a_h \right) + R_{2h}\psi_h S_{yzh} - R_{1h}R_{2h}\lambda_h \psi_h S_{xh} \right] < 0$	-0.007480 < 0	-0.007380 < 0	-0.007925 < 0
$MSE(t_c) < MSE(t_7)$ if	$\sum_{h=1}^{L} W_h \left[R_{1h}^2 S_{xh}^2 \left(\lambda_h^2 - \frac{1}{4} \right) + R_{2h}^2 \psi_h^2 S_{zh}^2 - 2\left(R_{1h} S_{yxh} \left(\lambda_h + \frac{1}{2} \right) + R_{2h}\psi_h S_{yzh} - R_{1h}R_{2h}\lambda_h \psi_h S_{xh} \right) \right] < 0$	-0.469443 < 0	-0.46944 < 0	-0.482517 < 0
$MSE(t_c) < MSE(t_8)$ if	$\sum_{h=1}^{L} W_h \left[R_{1h}^2 S_{xh}^2 \left(\lambda_h^2 - \frac{1}{4} a_h^2 \right) + R_{2h}^2 \psi_h^2 S_{zh}^2 - 2\left(R_{1h} S_{yxh} \left(\lambda_h + \frac{1}{2} a_h \right) + R_{2h}\psi_h S_{yzh} - R_{1h}R_{2h}\lambda_h \psi_h S_{xh} \right) \right] < 0$	-0.018507 < 0	-0.018507 < 0	-0.019104 < 0

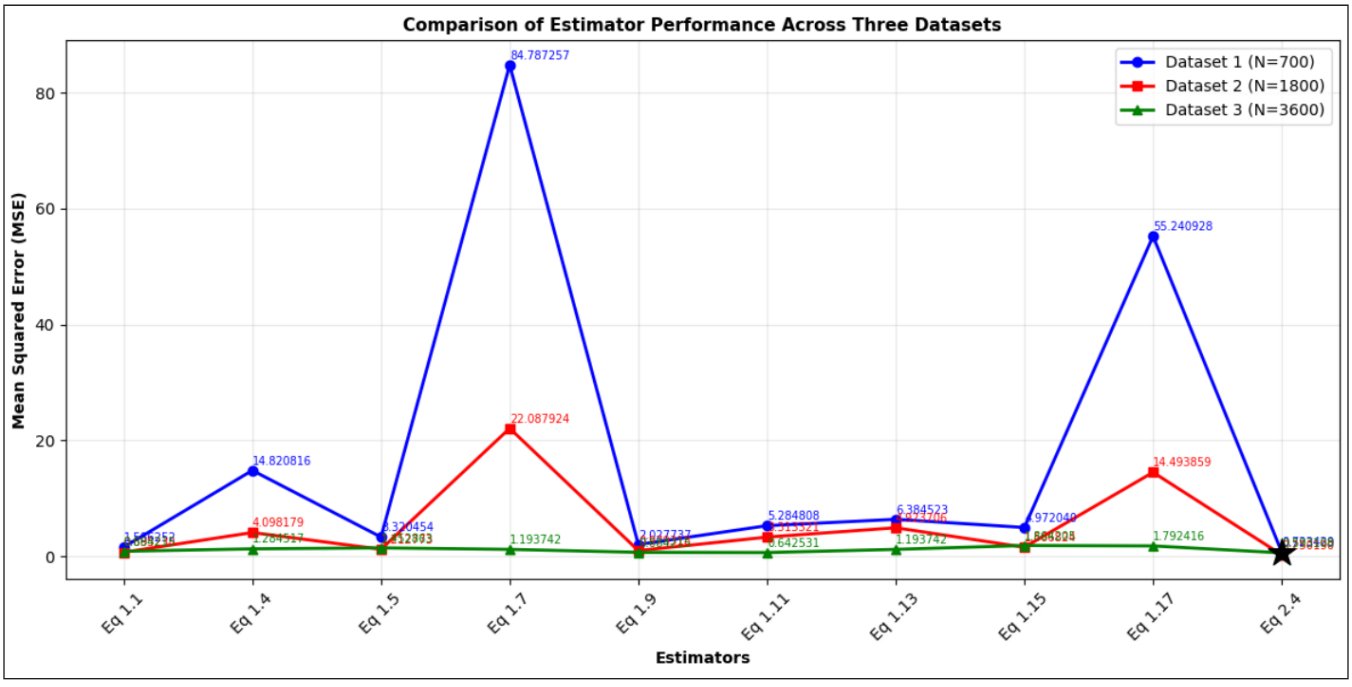


Fig. 1. Graphical representation of MSE's of different estimators

Table 2. MSE based evaluation of estimator performance for the three simulated populations

Estimator	Mean Square Error		
	Population-I	Population-II	Population-III
$\bar{y}_{ps}$	1.586252	0.695238	0.864215
t_1	14.820816	4.098179	1.284517
t_2	3.320454	1.212773	1.452863
t_3	84.787257	22.087924	1.193742
t_4	2.027737	0.979318	0.684215
t_5	5.284808	3.313321	0.642531
t_6	6.384523	4.923706	1.193742
t_7	4.972040	1.566824	1.864205
t_8	55.240928	14.493859	1.792416
t_c	0.723429	0.250190	0.593108

5. CONCLUSION

The comparative evaluation of Mean Square Error (MSE) for a set of estimators across three simulated populations was carried out to assess their relative precision and stability. The results demonstrate clear variations in the performance of different estimators for all three Populations. Notably, the proposed estimator t_c consistently achieved the smallest MSE in both populations, establishing it as the most efficient estimator among those examined. Its performance remained stable even when population size and sample size changed, demonstrating minimal bias and superior precision. The theoretical conditions derived for estimator superiority were also validated numerically, confirming that the proposed estimator is more efficient than the existing estimators under a broad range of scenarios.

Thus, the findings of this comparative analysis affirm that the proposed estimator t_c provides the most reliable and robust estimation, offering a significant improvement in accuracy over traditional and existing modified estimators. Overall, the study highlights the effectiveness of the proposed estimator in enhancing efficiency and reducing sampling error for all populations.

ACKNOWLEDGEMENTS

The authors are thankful to referee for his valuable comments for improvement of the original version of the paper.

DECLARATIONS

- **Conflict of Interest**

  The author declares that he has no conflict of interest.

- **Data Availability**

  Not Applicable.

- **Ethical Approval**

  Ethical approval was not required for this study because it did not involve human participants, animals, or collection of identifiable personal information.

- **Clinical Trial Number**

  Not Applicable. This study did not involve a clinical trial.

- **Consent To Publish**

  Not applicable. This study did not involve the recruitment or participation of human subjects.

- **Consent To Participate**

  This study did not involve the recruitment or participation of human subjects.

- **Funding**

  The author received no specific funding or financial support for this study.

  The authors confirm that the paper is not under review or in publication in full or partial in any other Journal.

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